Industrial Policies, Production Networks, and Oligopolistic Competition: Econometric Evaluation of the U.S. Semiconductor Subsidy*

Ko Sugiura†

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Abstract

This paper develops a general equilibrium multisector model of a production network and oligopolistic firms to study *ceteris paribus* causal effects of an industrial policy. The key mechanism is that when firm-level production functions exhibit constant returns to scale, the production network compounds changes in firms' marginal profits not only with respect to their own actions but also with respect to competitors' choices (i.e., strategic complementarities), while the latter is absent in monopolistic competition models. My empirical illustration using the U.S. CHIPS and Science Act suggests that accounting for firms' strategic interactions even flips the sign of the policy effect.

JEL: E61, E65, F13, F41, L13, L16

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†Sugiura: Ghent University, Sint-Pietersplein 6, Ghent 9000, Belgium, ko.sugiura@ugent.be.

1 Introduction

Over the past few decades, industrial policies — policies targeted at particular industries — have been at the forefront of economic policy debates in a range of contexts.[1] For example, the CHIPS and Science Act of 2022 aims to make nearly \$53 billion of investment in the semiconductor industry.[2] At the centerpiece of policy debates in such contexts are the following questions: How much financial support should be provided to which industries? How large are the causal effects of subsidizing particular industries on an economy's well-being?

While there has been a surge in the literature studying industrial policies, the state of the art leaves two issues unexplored. One thing is that recent theoretical studies highlight the role of a production network under the premise of perfectly or monopolistically competitive firms (e.g., Liu, 2019; Lashkaripour and Lugovskyy, 2023), whereas a parallel literature has documented growing evidence about firms engaging in oligopolistic competition (e.g., Atkeson and Burstein, 2008; De Loecker et al., 2021). It is thus natural to ask whether the existing theories about industrial policies in a networked economy still apply to the case of oligopolistic firms.[3] The other is that the existing theoretical research abstracts away from causal effects. Although a more empirically-oriented approach addresses causal policy questions, it is tailored for *ex post* assessment of a particular event with partial coverage:[4] It is silent about *ex ante* evaluation of a universal policy reform that has never previously been experimented.

To fill these gaps, this paper develops a new econometric policy evaluation framework by making two contributions.[5] First, I develop a general-equilibrium multisector model featuring a production network and firms' strategic interactions. This model is used to define a causal policy parameter as a *ceteris paribus* difference in outcome variables in response to an additional sectoral input subsidy.[6] A key mechanism of my model is that when firms'

[1]For a recent review of industrial policies, see Rodrik (2008) and Juhász et al. (2023).

[2]See Appendix G.1 for details.

[3]Indeed, this question is legitimate for the CHIPS and Science Act — the motivating example of this paper — because the government at the time of enactment recognized the importance of supply chains and the prevalence of market concentration. See Appendix G.1 for the details.

[4]See Lane (2020) and Juhász et al. (2023) for review.

[5]The literature on econometric policy evaluation dates at least as far back as Haavelmo (1943, 1944) and has been attempted in various disguises (see Dawkins et al., 2001), with receiving renewed interest in the early 2000s (e.g., Heckman and Vytlacil, 2005, 2007).

[6]It is essential to emphasize that the notion of "randomization" is not *necessary* for *defining* a causal policy effect; it is only *occasionally useful* for *identifying* a causal effect. See Heckman and Vytlacil (2007) and Deaton (2010) for a discussion. *Ceteris paribus* causal effects are one of the most widely accepted notions of causal effects in economics ever since Alfred Marshall (Marshall, 1890). It is worth stressing that statistical treatment effects are a special case of this class of causal effects. My paper puts forth an alternative to treatment effects, which is another special case of *ceteris paribus* causal effects. For identification, I exploit

production functions exhibit constant returns to scale, the production network compounds not only the responses of firms' marginal profits with respect to their own choices but also those with respect to competitors' (i.e., strategic complementarities), with the latter being absent in monopolistic models.

Second, to empirically quantify the policy relevance of this theoretical property, I establish a new nonparametric identification methodology that accounts for firms' strategic interactions as well as the production network. The proposed set of identifying assumptions nests many specifications commonly used in the macroeconomics and international trade literature. The identification analysis exploits variation in firms' input variables, rather than variation in policy variables per se. As a consequence, my framework can be used for *ex ante* evaluation of an unprecedentedly large universal policy intervention, as long as the support conditions regarding firms' input variables are satisfied. I then take my model to study one part of the CHIPS subsidy: The estimate suggests the empirical relevance of the joint existence of a production network and firms' strategic interactions.

My theoretical model builds on Liu (2019) to study a general equilibrium multisector model of a production network by assuming that each sector is populated by a finite number of heterogeneous oligopolistic firms.[7] The government helps firms to purchase sectoral intermediate goods through an ad-valorem subsidy specific to the purchaser sector. The market distortions in this model arise from both the firms' market power and policies in place. The model does not impose any parametric functional-form assumptions beyond constant-returns-to-scale firm-level production functions. I use this model to define a policy effect as the change in GDP due to a shift in the sector-specific subsidy with other things being equal, i.e., a *ceteris paribus* causal effect (Marshall, 1890). Notably, this causal estimand is inclusive of firms' strategic interactions, peer effects along a production network, and general equilibrium feedback effects, all of which are typically precluded in the empirical treatment effect literature.[8]

I show that policy effect spillovers along the production network are augmented by sectoral measures of the level of market competitiveness, which may amplify, weaken, or even reverse the overall effect. This characterization result delivers two important implications for

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variation in firms' input variables (see Section 4).

[7]The primary focus of this paper is on understanding the "effects of causes," a distinct task from investigating the "causes of effects" (see Heckman and Vytlacil, 2007). For the latter, the modeling choice of this paper is motivated by the voluminous literature documenting the empirical salience of sectoral production networks and firms' strategic interactions in each sector. Consequently, this paper is not concerned with judging which specification should be preferred.

[8]In Section 2.7, I make the case that in the presence of a production network and firms' strategic interactions, even if a policy is targeted at a single industry, its effect propagates along the production network while being pronounced or dampened by the firms' strategic forces in each sector. See also Heckman and Vytlacil (2007) for the relevance of these channels.

empirical policy evaluation. First, the configuration of policy spillovers crucially hinges on the type of market competition as well as the production network. This observation points to the practical importance of jointly accommodating production networks and firms' strategic interactions, a feature that has attracted little to no attention in the existing literature. Second, the actual signs and magnitudes of the spillovers additionally depend on the shapes of supply and demand functions, which are generally unknown *a priori* to policymakers.[9] For empirical policy evaluation to be agnostic about the underlying model specification, the identification analysis should hence be accomplished under a minimal set of assumptions.

Motivated by these considerations, this paper further provides a new nonparametric identification methodology. A difficulty in the identification arises from the fact that in strategic interaction models, individual firms have the potential to exert a nonnegligible influence over sectoral outcomes; thus, the policy parameter cannot be characterized by aggregate variables alone (Gaubert and Itskhoki, 2020). This invalidates the aggregate sufficient statistics approach, a method increasingly used in recent macroeconomics and international trade literature.[10] To circumvent this problem, I first rewrite the policy parameter in terms of responses of sector- and firm-level comparative statics. I then recover these by leveraging firm-level data and techniques of the production function estimation (e.g., Gandhi et al., 2019; Kasahara and Sugita, 2020). In doing so, my approach accounts for firms' strategic interactions by imposing three sets of additional assumptions. The first assumption restricts the firm-level production function to exhibit Hicks-neutral productivity. The second set of assumptions pertains to the "demand function": The sectoral aggregator takes the form of a homothetic demand system with a single aggregator (HSA; Matsuyama and Ushchev, 2017). Under these assumptions, the firms' equilibrium choices are shown to depend on competitors' productivities only through some aggregates. The last set of assumptions, combined with the first two, ensures that this equilibrium quantity function is "invertible" in the firm's own productivity. It should be emphasized that these assumptions are flexible enough to accommodate the specifications commonly used in the macroeconomics and international trade literature. Moreover, my identification strategy exploits variation in firms' input variables instead of variation in policy variables, thereby allowing policymakers to consider *ex ante* evaluation of an unprecedentedly large universal policy intervention, as long as the input variables remain within the historical variation.[11] This identification analysis

[9]For example, in oligopolistic competition under the commonly used parametric specification of Cobb-Douglas firm-level production functions and constant-elasticity-of-substitution (CES) demand functions, (i) firms' revenues do not change, and neither do sectoral revenues; and (ii) the general equilibrium feedback through the change in wage is muted (see Appendix C.9).

[10]See, for example, Arkolakis et al. (2012), Adão et al. (2017), Arkolakis et al. (2019), and Adão et al. (2020) for applications in macroeconomics.

[11]This feature becomes particularly relevant when it comes to the CHIPS and Science Act, the motivating

is constructive, so that a nonparametric estimator for the policy effect can be obtained by reading these procedures in reverse order.

My framework differs from the conventional structural approach for counterfactual predictions in macroeconomics in four important ways. For instance, policy analysis in the computational general equilibrium models typically proceeds in five steps: (*i*) specify models in detail, which often involves a large number of parameters; (*ii*) preset some parameter values on the basis of prior or external knowledge (e.g., parameter estimates from the preceding research); (*iii*) simulate (or calibrate) the model to match the data in terms of some criteria of researcher's choice, yielding values for the remaining parameters with assuming away from any random variation in the data generating process; (*iv*) and conditioning on the obtained parameter values, simulate again the model under a counterfactual state and compare outcomes generated by these two simulations.[12] In contrast, (*i*)' my approach specifies the model primitives only up to a class of functions and recovers only a limited number of comparative statics, thereby the subsequent empirical analysis being more robust against misspecification and less computationally burdensome.[13] (*ii*)' Estimation in my framework does not require any external information and thus can be performed in a self-contained fashion, obviating the arbitrariness inherent to the parameter preselection.[14] (*iii*)' Loss functions in my estimation naturally arise from the preceding identification argument, which eliminates the arbitrariness in the choice of the estimation criteria. (*iv*)' My approach is designed to directly recover the causal effect in a single procedure with admitting sampling variation.[15]

Finally, in order to quantify the empirical relevance of firms' strategic forces compounding through the production network, I bring my model to the U.S. firm-level data and evaluate the economic impacts of the CHIPS and Science Act, which was enacted in 2022 and selectively promotes the semiconductor industry. My framework serves as a plausible policy-evaluation tool for this policy episode because the U.S. government at the time of enactment acknowledged the prevalence of market concentration and the importance of supply chains.[16] I consider a hypothetical policy experiment of shifting the ad-valorem subsidy on the computer and electronic products industry from the 2021 level, which is 15.43%, to an alternative level of 18.43% — equivalent to \$2.02 billion. The estimate accounting for strategic interactions, as well as the production network, predicts a fall in GDP of \$0.0378

example of this paper.

[12]See, e.g., Dawkins et al. (2001) and Adão et al. (2017).

[13]In the econometric policy evaluation literature, this strategy is called *Marschak's Maxim* (Heckman and Vytlacil, 2007).

[14]The advantage of this feature becomes particularly acute when the model under consideration has never previously been studied in the literature, as is the case with my paper.

[15]This provides a ground for statistical hypothesis testing about the causal effect.

[16]See Appendix G.1.

billion, whereas the estimate based on monopolistic competition suggests a rise of $0.5581 billion. Comparing these two estimates underlines the policy relevance of correctly specifying market competition in the presence of a production network.[17]

1.1 Related literature

This paper contributes to four strands of the literature. First, the framework put forth in this paper is directly related to the literature on *ex ante* counterfactual predictions of economic shocks (e.g., trade costs, productivity), such as Arkolakis et al. (2012), Melitz and Redding (2015), Adão et al. (2017), and Adão et al. (2020). These papers are based on perfectly competitive or monopolistic firms, and thus express an aggregate outcome in terms of aggregate variables — aggregate sufficient statistics. In contrast, my paper explicitly accounts for firms' strategic interactions by building up an aggregate outcome from firm-level variables — firm-level sufficient statistics. To recover these firm-level variables, I propose a new identification procedure.

Second, this paper advances the literature on industrial policies on both theoretical and empirical grounds. The theory of optimal industrial policy in a multisector environment is explored in Itskhoki and Moll (2019) and Liu (2019) for exogenous market distortions; in Lashkaripour and Lugovskyy (2023) for endogenous but constant markups; and in Bartelme et al. (2021) for endogenously varying market distortions. In my model, market distortions arise from both policies in place and oligopolistic competition, and can endogenously vary according to the firms' strategic interactions. On the empirical front, my paper intersects with the literature studying causal policy effects. The treatment effect approach typically rules out the possibilities of agents' strategic interactions, peer effects through a network, and general equilibrium feedback.[18] In the spirit of the econometric policy evaluation literature (e.g., Heckman and Vytlacil, 2007), this paper puts forth an alternative policy parameter that is inclusive of all these spillover effects, while retaining a causal interpretation in the sense of Marshall (1890).[19] The identification approach in my paper supplements the existing literature by exploiting variation in firms' input variables, instead of variation in policy

[17] Although my model is developed without reference to any particular functional-form assumptions, and thus its implications apply fairly generally, the subsequent empirical analysis is constrained by the data limitation and additional identifying assumptions, as is the case with any empirical analysis. In light of this, my empirical estimates may not necessarily be an accurate gauge of the "actual" policy effects. Rather, the empirical illustration of this paper is tailored to examine the quantitative relevance of the wedge in policy effects, created by jointly accommodating firms' strategic interactions and a production network.

[18] See Lane (2020) and Juhász et al. (2023) for a review.

[19] In a similar vein, Rotemberg (2019) investigates the aggregate effects, taking into account the general equilibrium effects, and Sraer and Thesmar (2019) derive formulas that are able to counterfactually expand firm-level treatment effects to the aggregate level. Their methodologies are, however, *ex post* by nature, whereas my framework can be used for *ex ante* policy evaluation.

variables per se. As a consequence, my framework can analyze an unprecedentedly large universal policy reform, as long as firms' input variables remain within the historically observed supports.

Third, this paper connects the literature documenting the empirical relevance of oligopolistic competition (e.g., Atkeson and Burstein, 2008; Amiti et al., 2019; Gaubert and Itskhoki, 2020; De Loecker et al., 2021) to recent macroeconomics literature on production networks (e.g., Baqaee and Farhi, 2020, 2022; Bigio and La'O, 2020).[20] I show that the transmission of policy effects is dictated by firms' strategic complementarities accruing through the production network. This feature is absent in the existing literature on industrial policies under perfectly competitive or monopolistic competition, such as Liu (2019) and Lashkaripour and Lugovskyy (2023). Grassi (2017) also studies the case of oligopolistic competition, but his focus is on positive analysis under a parametric specification of production and demand functions. My paper is concerned with evaluating the policy effects with a minimal set of functional form assumptions.

Lastly, outside the domain of the macroeconomics literature, my method is tightly linked to the industrial organization literature on the identification of firms' production functions. In particular, the existing work has customarily assumed perfect competition (e.g., Ackerberg et al., 2015; Gandhi et al., 2019) or monopolistic competition (e.g., Kasahara and Sugita, 2020). My paper applies these approaches to the case of strategic interactions by adapting the notion of sufficient statistics for competitors' decisions and productivity. There have been recent studies that adopt analogous approaches, such as Blum et al. (2023), Ackerberg and De Loecker (2024), Doraszelski and Jaumandreu (2024).[21] Their methodologies, however, are established under the premise that firm-level prices and/or quantities are observable. In my framework, in contrast, revenue is the only available firm-level outcome variable, while firms' prices and quantities are recovered within my methodology.

2 Model

The goal of this section is to define a causal policy parameter that i) compares aggregate variables between the baseline (e.g., status quo) environment and an alternative policy regime and ii) includes firms' strategic interactions, peer effects through a production network, and general equilibrium effects.

[20] These works are principally interested in characterizing welfare loss due to misallocation in the presence of production networks: they start from an efficient economy and then focus on the consequence of adding a policy as a source of distortion. My paper admits market distortions in the initial state of the economy, including the policy itself, and then investigates a welfare-improving policy prescription.

[21] Appendix C.11 provides careful comparisons of my work with the existing literature.

To define such a parameter, this section spells out a general equilibrium closed-economy multisector model of oligopolistic competition among heterogeneous firms under a sectoral production network. The model is akin to Liu (2019), who considers the optimal industrial policy in the presence of a production network when there are exogenous market distortions. I depart from his setup by replacing the exogenous wedges with endogenously variable firms' markups. In my model, the markups can arise from oligopolistic competition among a finite number of heterogeneous firms and the non-CES specification of the residual inverse demand functions faced by the firms.[22]

It is postulated that as a way to neutralize the market distortions induced by the endogenous markups as well as the status-quo policies, the government manipulates sector-specific policy instruments $\boldsymbol{\tau} \coloneqq \{\tau_i\}_{i=1}^N$, where τ_i is understood as an ad-valorem subsidy on sector i's purchase of sectoral intermediate goods if it is positive, and a tax otherwise. I restrict my attention to the short-run policy effects, abstracting away from the firms' endogenous entry and exit decisions.[23]

The model is static, and there is no uncertainty. The economy consists of a representative household, a government, and N production sectors, indexed by $i \in \mathscr{N} \coloneqq \{1, \ldots, N\}$. Each sector i is populated by a finite number N_i of heterogeneous oligopolistic firms, indexed by $k \in \mathscr{N}_i \coloneqq \{1, \ldots, N_i\}$, each of which produces a single horizontally differentiated good. There is a sectoral aggregator that aggregates the firms' products in the same sector into a single intermediate good. Sectoral goods are further combined to produce a final consumption good. Both the final and sectoral aggregators operate in perfectly competitive markets.

Firm-level production uses labor and sectoral intermediate goods as inputs. The firm's transaction of sectoral goods forms the sectoral input-output linkages, denoted by $\boldsymbol{\Omega} \coloneqq [\omega_{i,j}]_{i,j\in\mathscr{N}}$ with $\omega_{i,j}$ being the share of sector j's intermediate good in sector i's expenditure for inputs.[24]

2.1 Market Distortions and Industrial Policy

Let $\boldsymbol{\tau}^0$ denote the policy regime currently in place. Suppose that the policymaker wishes to learn how much GDP would increase or decrease by moving to an alternative policy regime

[22] Arkolakis et al. (2019) consider a model of variable markups under monopolistic competition with a flexible class of non-CES demand functions. My paper introduces an additional source of endogenous markups; that is, strategic interactions.

[23] This simplifying assumption is often posited in the literature (e.g., Wang and Werning, 2022). Technically, accommodating the firm's endogenous entry and exit decisions requires another layer of the fixed-point problem concerning the free-entry condition, which in general is very hard to solve. Extending the theory to a long-run analysis is left for future work.

[24] Likewise, I write $\boldsymbol{\omega}_L \coloneqq [\omega_{i,L}]_{i=1}^N$ with $\omega_{i,L}$ meaning the labor share in sector i's cost.

$\boldsymbol{\tau}^1$.[25] In particular, the policymaker is interested in changing only the subsidy on sector n while keeping the subsidies on the other sectors (i.e., an industrial policy on sector n).[26] Thus, the policy parameter is defined as the change in GDP due to a policy reform from τ_n^0 to τ_n^1, which is denoted by $\Delta Y(\tau_n^0, \tau_n^1)$.

To grant this policy parameter a causal interpretation, I impose the following assumptions.

Assumption 2.1 (Policy Invariance). *Throughout the policy reform from $\boldsymbol{\tau}^0$ to $\boldsymbol{\tau}^1$, the following elements remain unchanged: (i) the index set for sectors $\mathscr{N}$, (ii) the index set for firms in each sector $\mathscr{N}_i$ for all $i \in \mathscr{N}$, (iii) each sectoral aggregator, (iv) every firm-level production function in each sector, and (v) the shape of the input-output linkages $\boldsymbol{\omega}_L$ and $\boldsymbol{\Omega}$.*

Assumption 2.1 (*i*) is consistent with the focus of this study on ad-valorem subsidies, excluding other competition interventions. Invariance condition (*ii*) assumes away from endogenous entry and exit in response to the policy change, which is implied by the short-run scope of this paper. Conditions (*iii*) and (*iv*) jointly mean that the policy reform does not alter the firms' operating environments, which in turn rules out both direct and indirect impacts of the policy reform on firms' productivities. Part (*v*) states that the input-output linkages $\boldsymbol{\omega}_L$ and $\boldsymbol{\Omega}$ do not reshape in reaction to the policy reform. This again accords with the scope of my analysis and also resonates with the existing literature that assumes the production network to be stable over a period of time (e.g., Baqaee and Farhi, 2020).

2.2 Household

Consider a representative household that consumes a final consumption good and inelastically supplies labor across sectors. The household owns all firms so that it receives firms' profits as dividends. The household derives utility only from consumption of the final good, with the utility function being the standard.

Assumption 2.2 (Utility Function). *The consumer's utility function is strictly monotonic and continuously differentiable in the final consumption good.*

Assumption 2.2 means that there exists a one-to-one mapping between the utility level and consumption of the final good. Based on this preference, the household chooses the utility-maximizing quantity of the final consumption good subject to the binding budget

[25] The current policy $\boldsymbol{\tau}^0$ might not yet be optimized but rather can be a part of the market distortions.

[26] That is, $\tau_n^0 \neq \tau_n^1$ and $\tau_{n'}^0 = \tau_{n'}^1$ for all $n' \neq n$. In the example of the CHIPS and Science Act, sector n corresponds to the semiconductor industry.

constraint:

$$C = WL + \Pi - T, \tag{1}$$

where Π is the firm's total profit, and T indicates the tax payment to the government in the form of a lump-sum transfer. I let the price index of the final consumption good be the numeraire.

2.3 Technologies

Economy-wide and sectoral aggregations.— The economy-wide aggregator $\mathcal{F} : \mathbb{R}_+^N \to \mathbb{R}_+$ collects sectoral intermediate goods to produce a final consumption good Y:

$$Y = \mathcal{F}(\{X_i\}_{i \in \mathscr{N}}), \tag{2}$$

where X_i represents sector i's intermediate good used for the production of the final consumption good. In each sector $i \in \mathscr{N}$, firm-level products are aggregated into a single sectoral good Q_i according to

$$Q_i = F_i(\{q_{ik}\}_{k \in \mathscr{N}_i}), \tag{3}$$

where $F_i : \mathbb{R}_+^{N_i} \to \mathbb{R}_+$ represents the sector-specific aggregator that collects firms' products in sector i and q_{ik} denotes the quantity of firm k's product.[27] Both the economy-wide and sectoral aggregators operate in perfectly competitive markets under the following standard assumptions.

Assumption 2.3 (Economy-Wide and Sectoral Aggregators)**.** *(i) The economy-wide aggregation function $\mathcal{F}(\cdot)$ is increasing and concave in each of its arguments. (ii) For each $i \in \mathscr{N}$, the sectoral aggregator $F_i(\cdot)$ is a) twice continuously differentiable and b) increasing and concave in each of its arguments.*

Each sectoral aggregator solves the cost-minimization problem, which delivers the price index of sector i's good P_i. A sectoral aggregator serves two purposes. First, it is a useful modeling device that unites firms' differentiated goods into a single homogeneous good (Bigio and La'O, 2020; La'O and Tahbaz-Salehi, 2022), and helps isolate the firm's input choices from the strategic considerations. Second, from the perspective of an individual firm, the

[27] To economize on notation, I use the same notation q_{ik} to mean the demand for firm k's good and firm k's output quantity. This is innocuous as the sectoral aggregator is the sole buyer of firms' output.

sectoral aggregator acts as a "demand function" through which the firms' strategic forces interact.

Firm-level production.— The firm-level production process combines labor and material inputs, where the latter is a composite of sectoral intermediate goods along the production network. It is assumed that all inputs are variable (i.e., firms do not incur fixed costs). To focus on the short-run behavior, I do not model the firms' entry decisions; instead, I assume that each sector is populated by an exogenously fixed number of heterogeneous firms.

In the output market of each sector, firms are heterogeneous in productivity and engage in a Cournot competition of complete information,[28] while they are perfectly competitive in the input markets. Thus, each firm first chooses its output quantity so as to maximize its profits in the Cournot competition, followed by input decisions based on cost-minimization problems under the constraint of output quantity.

The production technology for firm k in sector i is described by

$$q_{ik} = f_i(\ell_{ik}, m_{ik}; z_{ik}) \qquad \textit{with} \qquad m_{ik} = \mathcal{G}_i(\{m_{ik,j}\}_{j\in\mathscr{N}}), \tag{4}$$

where q_{ik}, ℓ_{ik}, and m_{ik} denote, respectively, the quantity of gross output, labor input, and material input; z_{ik} is firm-specific productivity; $m_{ik,j}$ represents the input demand for sector j's intermediate good; and $f_i : \mathbb{R}^2_+ \to \mathbb{R}_+$ and $\mathcal{G}_i : \mathbb{R}^N_+ \to \mathbb{R}_+$ indicate, respectively, the firm-level production technology and material aggregator.[29] Note that $\mathcal{G}_i(\cdot)$ reflects the input-output linkages $\mathbf{\Omega}$. Notice moreover that both $f_i(\cdot)$ and $\mathcal{G}_i(\cdot)$ are only traced by sector index i, meaning that firms in the same sector i have access to the same production technologies up to the idiosyncratic productivity.[30] These aggregators are assumed to be neoclassical in the sense of the following assumption:[31]

Assumption 2.4 (Firm-Level Production Functions)**.** *For each sector $i \in \mathscr{N}$, aggregators $f_i(\cdot)$ and $\mathcal{G}_i(\cdot)$ (i) display constant returns to scale (CRS), (ii) are twice continuously differentiable in all arguments, (iii) are increasing and concave in each of its arguments, and (iv) satisfy $f_i(0,0) = 0$ and $\mathcal{G}_i(\mathbf{0}) = 0$.*

[28] The case of Bertrand competition can also be analyzed analogously.

[29] I abstract away capital accumulation in order to stick to a static environment. Extending my framework to a dynamic setup is technically challenging and is reserved for future work (See Appendix D.1).

[30] This also implies that producer-side heterogeneity pertaining to product differentiation (e.g., quality) is encoded in the productivity term z_{ik}. In my setup, differentiated goods are produced by heterogeneous firms, so that the level at which product differentiation is defined is the same as that at which firm heterogeneity is defined. Thus, the notion of firm coincides with that of variety.

[31] This assumption is prevalent in the literature (e.g., Bigio and La'O, 2020). In particular, the CRS production functions are customarily assumed by recent works on firm-level macroeconomic models — for example, Atkeson and Burstein (2008) in an oligopolistic competition model of international trade and Baqaee and Farhi (2022) in a multi-country model of international trade in the presence of production networks.

Importantly, when a firm decides the quantity of output, it also takes into account its input decisions in a forward-looking way. Thus, the firm's decision problem proceeds backward in effect. First, taking the quantities of output and material input and the sectoral price indices as given, the firm's optimal demand for sectoral intermediate goods is given by

$$\{m^*_{ik,j}\}_{j\in\mathcal{N}} \in \underset{\{m_{ik,j}\}_{j\in\mathcal{N}}}{\arg\min} \sum_{j=1}^{N}(1-\tau_i)P_j m_{ik,j} \quad s.t. \quad \mathcal{G}_i(\{m_{ik,j}\}_{j\in\mathcal{N}}) \geq \bar{m}_{ik}, \tag{5}$$

where $m^*_{ik,j}$ denotes the optimal level of purchase of sector j's good, and $\bar{m}_{ik}$ indicates the level of material input corresponding to a given quantity of output. Note that the associated unit cost condition defines the cost index of material input P_i^M gross of the policy $\boldsymbol{\tau}$.

Second, taking the output quantity and input prices as given, the optimal input quantities for firm k in sector i are given by

$$\{\ell^*_{ik}, m^*_{ik}\} \in \underset{\ell_{ik}}{\arg\min}\left\{\min_{m_{ik}|\ell_{ik}} W\ell_{ik} + P_i^M m_{ik} \quad s.t. \quad f_i(\ell_{ik}, m_{ik}; z_{ik}) \geq \bar{q}_{ik}\right\}, \tag{6}$$

where W denotes the wage[32] and $\bar{q}_{ik}$ is a given level of output quantity.[33] Implicit in this expression is the timing assumption that every firm chooses its labor input prior to material input. An economic intuition behind this is that labor is easier to obtain compared to material.[34] This assumption is imposed only for the purpose of econometric analysis, and its theoretical implication remains the same even if it is replaced by a simultaneous choice of labor and material inputs.[35] The cost-minimization problem (6) is assumed to have an interior solution.[36]

Third, taking the competitors' quantity choices and aggregate variables as given, firm k in sector i chooses the quantity of output $q_{ik} \in \mathscr{S}_i \coloneqq \mathbb{R}_+ \cup \{+\infty\}$ to maximize its profit.[37] Let $\pi_{ik} : \mathscr{S}_i \times \mathscr{S}_i^{N_i-1} \to \mathbb{R}$ represent firm k's profit function that maps its own quantity choice q_{ik} and competitors' choices $\mathbf{q}_{i,-k} \coloneqq \{q_{ik'}\}_{k'\in\mathcal{N}_i\setminus\{k\}}$ to the profit under the information

[32] Since the labor force is assumed to be frictionlessly mobile across sectors, the wage W is common for all sectors.

[33] Input decisions (5) and (6) are separated purely for expositional purposes. These two problems could be collapsed.

[34] Since my model is static, and assumes away from firm's endogenous entry and exit, my model can be interpreted as a long-run approximation, in which every firm behaves just like a "continuing" firm. For such firms, labor input is as easy as maintaining the existing employment relationship.

[35] See Ackerberg et al. (2015) and Gandhi et al. (2019).

[36] This assumption can be stated in terms of the firm's production function. See Appendix A.2.

[37] The firm's profit here is defined as revenue minus variable costs.

set $\mathcal{I}_i$:

$$\mathcal{I}_i \coloneqq \{Y, \{X_j\}_{j\in\mathscr{N}}, \{Q_j\}_{j\in\mathscr{N}\setminus\{i\}}, W, P_i^M, \{z_{ik}\}_{k\in\mathscr{N}_i}, \boldsymbol{\omega}_L, \boldsymbol{\Omega}, \boldsymbol{\tau}\}.$$

The construction of $\mathcal{I}_i$ reflects the fact that when firms in sector i make quantity decisions, they take these aggregate variables as fixed while internalizing the possibility of the sectoral aggregate quantity Q_i and the associated price index P_i varying as a result of their own decisions.[38] Note that the sectoral cost index for material input P_i^M is taken as given. All sectoral price indices $\{P_j\}_{j\in\mathscr{N}}$ are determined to be consistent with all sectoral material cost indices $\{P_j^M\}_{j\in\mathscr{N}}$ in the aggregate equilibrium.[39] The inclusion of the firms' productivities $\{z_{ik}\}_{k\in\mathscr{N}_i}$ partly embodies the complete information structure of the strategic interaction. For each sector $i \in \mathscr{N}$, the Cournot-Nash equilibrium quantities $\mathbf{q}_i^* \coloneqq \{q_{ik}^*\}_{k\in\mathscr{N}_i}$ must satisfy the following system of equations:[40] For each firm $k \in \mathscr{N}_i$,

$$q_{ik}^* \in \arg\max_q \quad \pi_{ik}(q, \mathbf{q}_{i,-k}^*; \mathcal{I}_i). \tag{7}$$

In what follows, the dependence on the information set $\mathcal{I}_i$ is made implicit, and it is understood as being absorbed by the sector i subscript.[41]

2.4 Government

The government sets the level of subsidies $\boldsymbol{\tau}$ under the balanced budget. Government expenditures consist of two components. First, the government purchases the final consumption good, which can be conceived as public spending G. The second element refers to the total policy expenditure S_i in sector i. The residual between these two expenditures is charged to the representative consumer in the form of a lump-sum tax T. Hence, the government's

[38] Note that, as seen in (12), government spending G can be dropped under (1), (8), and (9).

[39] It might seem to be natural to consider a situation where firms recognize their impacts on input prices as well as output prices. In such a case, firms' strategic interactions prevail across sectors through input uses along the production network. This entails two additional theoretical complications: *i*) all firms engage in a single very large strategic competition across sectors, and *ii*) firms have oligopsony power in the input markets. The causal mechanism of this paper, on the other hand, is motivated by existing research that points to the prevalence of *i*)' within-sector strategic interactions and *ii*)' oligopolistic competition in the output markets. To keep the focus of the analysis consistent with the motivating literature, I maintain the sectoral aggregator (3), which effectively safeguards the input markets against the firms' strategic forces. Exploring the case of oligopsony across sectors is left for future work.

[40] The existence of Cournot-Nash equilibria in each sector immediately follows from the Debreu-Glicksberg-Fan theorem.

[41] Strictly speaking, each step of the firm's decision is based on different information sets. For instance, the information set at the time of input decision should be $\mathcal{I}_i' \coloneqq \mathcal{I}_i \cup \{q_{ik'}^*\}_{k'=1}^{N_i}$. The i index should thus be understood as conditioning on the appropriate information set.

budget constraint is

$$G + \sum_{i=1}^{N} S_i = T \qquad where \qquad S_i := \sum_{k=1}^{N_i} \sum_{j=1}^{N} \tau_i P_j m_{ik,j}. \tag{8}$$

2.5 Equilibria

Market Clearing.— The market clearing conditions are standard:

$$\text{[Final consumption good]} \qquad Y = C + G \tag{9}$$

$$\text{[Sectoral intermediate goods]} \qquad Q_j = X_j + \sum_{i=1}^{N} \sum_{k=1}^{N_i} m_{ik,j} \qquad \forall j \in \mathscr{N} \tag{10}$$

$$\text{[Labor]} \qquad L = \sum_{i=1}^{N} \sum_{k=1}^{N_i} \ell_{ik} \tag{11}$$

The resource constraints (9) and (10) hold, respectively, because the final consumption good is either consumed by the household or purchased by the government, and because the sectoral intermediate goods are used either for producing the final consumption good or as input in an individual firm's production.[42] In the labor market clearing condition (11), labor L is assumed to be inelastically supplied, fully employed, and frictionlessly mobile across sectors and firms. Lastly, substituting (1) and (8) into (9), it follows that

$$Y = WL + \Pi - \sum_{i=1}^{N} S_i, \tag{12}$$

which is nothing but the income accounting identity of GDP.

Equilibria Defined.— I assume that subsidies $\boldsymbol{\tau}$ are externally manipulated (by the government). Under Assumption 2.1, the number of sectors and firms, and firms' productivities, as well as the network structure, are invariant to a policy shift, while other aggregate and firm-level variables are endogenously determined in equilibrium. Defining the equilibria in this model amounts to finding a fixed point in these endogenous variables. I use the symbol $*$ to denote the equilibrium values.

Definition 2.1 (General Equilibria)**.** *Given the realization of firms' productivities* $\{\{z_{ik}\}_{k \in \mathscr{N}_i}\}_{i \in \mathscr{N}}$*, sector-specific subsidies* $\boldsymbol{\tau}$*, and the input-output linkages* $\boldsymbol{\omega}_L$ *and* $\boldsymbol{\Omega}$*, the general equilibria of this model are defined as fixed points that solve the following problems:*

[42]The market clearing condition for individual firms' products is straightforward, as firm-level products are only used by the sectoral aggregator. Thus, it is already implicitly applied in the exposition.

Sectoral equilibria: *For each sector i, given the information set $\mathcal{I}_i$, the solution to the quantity-setting game* (7) *yields a vector of sectoral Cournot-Nash equilibrium quantities $\{q^*_{ik}\}_{k\in\mathcal{N}_i}$, followed by the cost-minimization problems* (5) *and* (6) *to derive the optimal labor and material inputs $\{\ell^*_{ik}, m^*_{ik}\}_{k\in\mathcal{N}_i}$, and input demand for sectoral intermediate goods $\{\{m^*_{ik,j}\}_{j\in\mathcal{N}}\}_{k\in\mathcal{N}_i}$.*

Aggregate equilibria: *Given a collection of sectoral equilibrium quantities $\{q^*_{ik}, \ell^*_{ik}, m^*_{ik}, \{m^*_{ik,j}\}_{j\in\mathcal{N}}\}_{i,k}$, an aggregate equilibrium is referenced by the set of aggregate quantities $\{Y^*, \{X^*_j, Q^*_j\}_{j\in\mathcal{N}}\}$ together with the set of aggregate prices $\{W^*, \{P^*_j\}_{j\in\mathcal{N}}\}$, such that i) the household maximizes its utility subject to* (1)*, ii) the market clearing conditions for composite intermediate goods* (10) *and labor* (11) *are satisfied, and iii) the income accounting identity* (12) *holds.*

2.6 The Object of Interest

Recall from Section 2.1 that the policymaker hopes to learn how much GDP would change due to the policy reform from τ^0_n to τ^1_n. Let $Y^{\boldsymbol{\tau}}$ be the economy's GDP in equilibrium under policy regime $\boldsymbol{\tau}$. From (11) and (12), it follows that

$$Y^{\tau} = \sum_{i=1}^{N} Y_i(\boldsymbol{\tau}) \quad where \quad Y_i(\boldsymbol{\tau}) \coloneqq \sum_{k=1}^{N_i} \left(W^* \ell^*_{ik} + \pi^*_{ik} - \sum_{j=1}^{N} \tau_i P^*_j m^*_{ik,j} \right), \tag{13}$$

where π_{ik} stands for firm k's profit. In (13), $Y_i(\boldsymbol{\tau})$ can be viewed as sector i's GDP.

Now the object of interest $\Delta Y(\tau^0_n, \tau^1_n)$ is defined as

$$\Delta Y(\tau^0_n, \tau^1_n) \coloneqq \sum_{i=1}^{N} Y_i(\boldsymbol{\tau}^1) - \sum_{i=1}^{N} Y_i(\boldsymbol{\tau}^0). \tag{14}$$

While a variety of "causal effects" of an industrial policy have been proposed in the empirical treatment effect literature, they do not necessarily speak to policy-relevant questions such as those considered in this paper.[43] The policy parameter (14) directly compares the economy's GDP under $\boldsymbol{\tau}^0$ to that under $\boldsymbol{\tau}^1$ and therefore answers the important macroeconomic question. A virtue of this parameter is that under Assumption 2.1, it represents an *intensive-margin causal effect* of the policy reform in the sense of a *ceteris paribus* change in an outcome variable across different policy regimes (Marshall, 1890).[44] Moreover, the target

[43] See Lane (2020) and Juhász et al. (2023).

[44] For the long-run analysis, wherein the firm's endogenous entry and exit are allowed, the *extensive-margin causal effect* can be defined analogously (Appendix D.2).

parameter (14) includes the contributions arising from firms' strategic interactions, network spillovers, and general equilibrium feedback, all of which are typically assumed away in the treatment effect literature.[45,46]

Remark 2.1. *While I confine attention to the causal effect of an industrial policy on GDP, my model can be used to define various other (both aggregate and distributional) causal parameters, to analyze changing subsidies to multiple sectors (i.e., universal treatments), and to formulate an optimal policy problem. See Appendix D.*

2.7 Properties of the Policy Parameter $\Delta Y(\tau_n^0, \tau_n^1)$

Under Assumptions 2.2–2.4, the object of interest (14) is differentiable over the domain of definition of the model[47] and thus can equivalently be rewritten as

$$\Delta Y(\tau_n^0, \tau_n^1) = \sum_{i=1}^{N} \int_{\tau_n^0}^{\tau_n^1} \frac{dY_i(\cdot)}{d\tau_n} d\tau_n, ^{48} \tag{15}$$

where

$$\left.\frac{dY_i(s)}{ds}\right|_{s=\boldsymbol{\tau}} = \sum_{k=1}^{N_i} \left\{ \frac{dp_{ik}^*}{d\tau_n} q_{ik}^* + p_{ik}^* \frac{dq_{ik}^*}{d\tau_n} - \sum_{j=1}^{N} \left(\frac{dP_j^*}{d\tau_n} m_{ik,j}^* + P_j^* \frac{dm_{ik,j}^*}{d\tau_n} \right) \right\}.^{49} \tag{16}$$

In the remainder of this section, I investigate the determination of the comparative statics in (16) using a simplified version of the model, while a full description is delegated to Appendix A.

2.7.1 Macro and Micro Complementarities

To highlight how (16) depends on the firms' strategic forces accruing through the sectoral production network, I focus on the comparative statics of firm-level output quantity and sectoral price index as well as material cost index. For the sake of simplicity, I assume away from the general equilibrium effects, i.e., wage is invariant to a policy change. Let

[45] This shares the same spirit with the policy-relevant treatment effect (Heckman and Vytlacil, 2005, 2007).

[46] There have been recent advancements in the treatment effect literature to accommodate these elements (see, e.g., Rotemberg (2019) and Sraer and Thesmar (2019)). However, no existing work takes into consideration all of these elements simultaneously.

[47] The domain of definition is not necessarily the same as the empirical support of data. This is discussed in Section 4.

[48] Note that subsidies to other sectors $\{\tau_j\}_{j\neq n}$ are fixed constant throughout the integral, so that $Y_i(\cdot)$ can effectively be treated as a univariate function of τ_n. In light of this, I write $dY_i(\cdot)/d\tau_n = \partial Y_i(\cdot)/\partial\tau_n$.

[49] With a slight abuse of notation, for an equality $V^* = V(s)$, I write $(dV(s)/ds)|_{s=\boldsymbol{\tau}} = dV^*/d\tau_n$.

$\mathcal{P}_i^M(\cdot)$ and $\mathcal{P}_i(\cdot)$ denote, respectively, functions such that $P_i^{M*} = \mathcal{P}_i^M(\{P_j^*\}_{j=1}^N, \tau_i)$ and $P_i^* = \mathcal{P}_i(\{q_{ik'}\}_{k'=1}^{N_i})$. Define moreover $a_{i,j}^M := \partial\mathcal{P}_i^M(\cdot)^*/\partial P_j$, and $b_{i,n}^M := \partial\mathcal{P}_i^M(\cdot)^*/\partial\tau_n$. Note that $b_{i,n}^M$ allows for the interpretation as the initial (or direct) impact of the policy change.

The structural model described above can be solved to yield the following "reduced-form" expressions: (i) $dP_i^{M*}/d\tau_n = h_{i,n}^M b_{i,n}^M$, (ii) $dP_i^*/d\tau_n = \bar{\lambda}_{i\cdot}^M(dP_i^{M*}/d\tau_n)$, and (iii) $dq_{ik}^*/d\tau_n = \bar{\lambda}_{ik}^M(dP_i^{M*}/d\tau_n)$.[50] The coefficient $\bar{\lambda}_{i\cdot}^M$ is a weighted sum of the $\bar{\lambda}_{ik}^M$'s in sector i, where each $\bar{\lambda}_{ik}^M$ represents the firm k's contribution to the sector i's overall strategic complementarity.[51] By construction, $\bar{\lambda}_{i\cdot}^M$ can be conceived as a sectoral measure of the firms' strategic complementarities. The coefficient $h_{i,n}^M$ is given by the (i, n) entry of the matrix $(\mathbf{I} - \mathbf{\Gamma})^{-1}$ where $\mathbf{I}$ stands for the identitiy matrix and $\mathbf{\Gamma} := [a_{i,j}^M \bar{\lambda}_{j\cdot}^M]_{i,j=1}^N$.[52] These coefficients are equilibrium objects and capture the comovement patterns between the comparative statics. In (i), the $h_{i,n}^M$ can be understood as a multiplier measuring the extent to which the initial policy impact affects the material cost index. The change in the material cost then transmits to the sectoral output price index and firm-level output quantity according to (ii) and (iii), respectively, with the $\bar{\lambda}_{i\cdot}^M$ and $\bar{\lambda}_{ik}^M$ dictating the degree of pass-through. I refer to $\{\bar{\lambda}_{j\cdot}^M\}_{j=1}^N$ as the *micro complementarities* and $\{h_{j,n}^M\}_{j=1}^N$ as the *macro complementarities.*[53,54]

The macro complementarity $h_{i,n}^M$ represents downstreamness of sector i relative to sector n.[55] For instance, when $n \neq i$, it is given by

$$\underbrace{\bar{\lambda}_{n\cdot}^M a_{i,n}^M}_{dP_n^M \to dP_n \to dP_i^M} + \sum_{j=1}^N \underbrace{\bar{\lambda}_{n\cdot}^M a_{j,n}^M \bar{\lambda}_{j\cdot}^M a_{i,j}^M}_{dP_n^M \to dP_n \to dP_j^M \to dP_j \to dP_i^M} + \sum_{j=1}^N \sum_{j'=1}^N \underbrace{\bar{\lambda}_{n\cdot}^M a_{j,n}^M \bar{\lambda}_{j\cdot}^M a_{j',j}^M \bar{\lambda}_{j'\cdot}^M a_{i,j'}^M}_{dP_n^M \to dP_n \to dP_j^M \to dP_j \to dP_{j'}^M \to dP_{j'} \to dP_i^M} + \dots. \tag{17}$$

[50] Analogous expressions can be derived for $dp_{ik}^*/d\tau_n$ and $dm_{ik,j}^*/d\tau_n$ as well.

[51] This coefficient is defined as a ratio whose denominator takes the form of a linear combination of the responsivenesses of all firms' marginal profits with respect to all firms, and whose numerator is given by another linear combination of the responsivenesses of all firms' marginal profits in response to all firms' quantities but for firm k's quantity. The denominator can be regarded as a measure of the sector's overall strategic complementarity. Since the numerator does not involve the firm k's quantity adjustment, this coefficient backs out the extent to which firm k affects the sectoral measure of strategic complementarity. See Appendix A.1 for details.

[52] It is assumed that $(\mathbf{I} - \mathbf{\Gamma})^{-1}$ exists.

[53] These terminologies are inspired by Klenow and Willis (2016).

[54] Even in the absence of strategic competition, such as in monopolistic competition, the micro complementarities generally do not vanish because $\{\bar{\lambda}_{jk}\}_{k=1}^{N_i}$ involve the responsiveness of firms' marginal profits with respect to their own quantity adjustments, which are not necessarily zero. See Example A.3 in Appendix A.

[55] Notice that $\mathcal{P}_i^M(\cdot)$ involves the information about the production network carried over from the aggregator $\mathcal{G}_i(\cdot)$, and so are its partial derivatives $\{a_{i,j}^M\}_{i,j\in\mathcal{N}}$. In light of this, $\{h_{i,n}^M\}_{i\in\mathcal{N}}$ can be viewed as a version of the downstreamness measure of the type defined as the Leontief inverse of $\mathbf{\Omega}$ (e.g., Carvalho and Tahbaz-Salehi, 2019).

It is evident in (17) that $h_{i,n}^M$ captures the indirect effects due to changes in other sectors' price indices accumulated through the production network. In each term, the micro complementarity $\bar{\lambda}_{j\cdot}^M$ designates the pass-through from the material cost index to the output price index within sector j, while $a_{j',j}^M$ represents the change in the sector j''s material cost index caused by the change in the sector j's output price index. For instance, the first term represents an indirect effect coming through the purchase of intermediate goods from the targeted sector. The second and third terms capture feedback effects coming through multiple rounds of input purchases by other sectors. Each round of these indirect effects is augmented by the source sectors' micro complementarities $\{\bar{\lambda}_{j\cdot}^M\}_{j=1}^N$. This way, the macro complementarity compounds the micro complementarities along the production network.

Clearly, different specifications about functional form relationships, market competition, and a production network generally lead to different values of the macro and micro complementarities, which in turn alter the policy conclusion. To fix the idea, I now explore these two pass-through coefficients using a special case of my model. This exercise distills the motivation for the empirical policy evaluation under a minimal set of identifying assumptions.

Remark 2.2. *In an attempt to map micro shocks to macroeconomic fluctuations, Liu (2019) and Baqaee and Farhi (2020) study Domer weights near efficiency, and Bigio and La'O (2020) push this line forward by departing from efficiency.*[56] *In these works, market competition is assumed to be perfectly or monopolistically competitive. The macro complementarity of this paper thus takes one step further by considering strategic competition.*

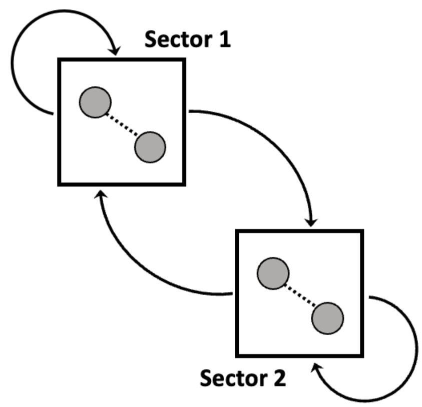
2.7.2 An Illustrative Example: Two Sectors and Two Firms

Suppose that the economy consists of two sectors, i.e., $\mathscr{N} = \{1, 2\}$. Each sector is populated by two firms, i.e., $\mathscr{N}_i = \{1, 2\}$ for all $i \in \mathscr{N}$. Without loss of generality, firm 1 is assumed to be more productive than firm 2, i.e., $z_{i1} > z_{i2}$. In each sector, firms engage in strategic competition over quantity in the output market (i.e., Cournot duopoly). Consider an industrial policy targeted at sector 1, i.e., $n = 1$.

The economy-wide aggregator $\mathcal{F}(\cdot)$ is given by a Cobb-Douglas production function. The sectoral aggregator $F_i(\cdot)$ takes the form of a constant elasticity of substitution (CES) production function with an elasticity of substitution $\sigma_i > 1$ (i.e., firms' products are substitutes). Each firm produces a differentiated good using a Cobb-Douglas production function $f_i(\cdot)$ with Hicks-neutral productivity. The material aggregator $\mathcal{G}_i(\cdot)$ is once again given by a Cobb-Douglas production function, with the input share of sector j's intermediate good

[56] Domer weights in the network economy are given as a combination of the network structure and sectoral expenditure shares.

$\gamma_{i,j}$ reflecting the production network $\mathbf{\Omega}$. It is assumed that $\gamma_{i,j} > 0$ for all $i, j \in \mathscr{N}$, so that every firm purchases positive quantities of intermediate goods from both sectors 1 and 2 (see Figure 1). The associated unit cost condition determines the material cost index: $P_i^{M*} = \prod_{j \in \mathscr{N}} (1/\gamma_{i,j}^{\gamma_{i,j}}) \{(1-\tau_j) P_j^*\}^{\gamma_{i,j}}$, thereby yielding $a_{i,j}^M = \gamma_{i,j}(P_i^{M*}/P_j^*)$ and $b_{i,n}^M = -P_i^{M*}/(1-\tau_i)\mathbb{1}_{\{n=i\}}$.



Note: This figure illustrates the two-sector economy studied in Section 2.7.2. Black square borders stand for sectors. Two gray circles entrenched in each of the squares represent duopoly firms with dotted lines indicating strategic interactions between them. Circular arrows designate input purchases along the production network. For example, the circular arrow from sector 1 to 2 means the purchase of sector 1's intermediate goods by firms in sector 2.

Figure 1: Duopoly in Two-Sector Economy

Micro complementarity.— In Appendix A.4, I show that in equilibrium, firm 1's quantity choice is a strategic complement to firm 2's choice, whereas firm 2's choice is a strategic substitute for firm 1's choice. I further demonstrate that if firm 2's product is a "relatively strong" strategic substitute, then the sectoral measure of strategic complementarity $\bar{\lambda}_{i\cdot}^M$ is positive.[57] The contrapositive of this claim suggests that negative micro complementarity is evidence of firm 2's being a "relatively modest" strategic substitute.

Macro complementarity.— In this two-sector economy, (17) reduces to

$$h_{2,1}^M = \bar{\lambda}_{1\cdot}^M \left(\gamma_{2,1} \frac{P_2^{M*}}{P_1^*} + \gamma_{1,1} \frac{P_1^{M*}}{P_1^*} \bar{\lambda}_{1\cdot}^M \gamma_{2,1} \frac{P_2^{M*}}{P_1^*} + \gamma_{2,1} \frac{P_2^{M*}}{P_1^*} \bar{\lambda}_{2\cdot}^M \gamma_{2,2} \frac{P_2^{M*}}{P_2^*} + \dots \right). \tag{18}$$

Intuitively, the $\bar{\lambda}_{1\cdot}^M$ in front of the round bracket indicates the "initial" response of sector 1's sectoral price index to the "initial" change in the material cost index. The first term inside the bracket measures the shift in sector 2's material cost index due to direct purchases from

[57]Let $MP_{ik}(\cdot) \coloneqq \partial \pi_{ik}(\cdot)^*/\partial q_{ik}$ for each $k \in \{1, 2\}$. I say that firm 2's product is said to be a "relatively strongly" strategic substitute if $(\partial MP_{i2}(\cdot)/\partial q_{i1})/(\partial MP_{i1}(\cdot)/\partial q_{i1}) \in \left(\frac{z_{i1}}{z_{i2}}, \infty\right)$, and a "relatively modest" strategic substitute otherwise. See Appendix A.4 for details.

sector 1 (the circular arrow from sector 1 to 2 in Figure 1). The second keeps track of sector 1's good that is first used by firms in sector 2 before purchased by firms in sector 2 (the circular arrow from sector 1 to 1, followed by the one from sector 2 to 1 in Figure 1), while the third records sector 1's good that is used to produce sector 2's good, which in turn is used as another input by sector 2 (the circular arrow from sector 1 to 2, followed by the one from sector 2 to 2 in Figure 1).

Here, suppose for a moment that firm 2 in each sector is only a relatively strong strategic substitute, so that $\bar{\lambda}^M_{1\cdot} > 0$ and $\bar{\lambda}^M_{2\cdot} > 0$. In this case, it is immediate to see $h^M_{2,1} > 0$, i.e., a positive macro complementarity. Next, suppose instead that firm 2 in each sector is a relatively modest strategic substitute, and thus $\bar{\lambda}^M_{1\cdot} < 0$ and $\bar{\lambda}^M_{2\cdot} < 0$. In this case, the sign of the equilibrium value of $h^M_{2,1}$ becomes ambiguous, as the first term inside the round bracket of (18) takes a positive value while the second and third terms are negative. Likewise, the remaining terms also exhibit sign switching. Hence, the sign and magnitude of $h^M_{2,1}$ — the effective location of the sector on the production network — are essentially empirical matters.

Key implications.— The observation drawn here is of direct policy relevance as it means that even if a policy is targeted at a particular sector, policy effects can propagate along the production network; moreover, such propagation may be amplified, weakened, or even reversed by firms' strategic interactions in each sector. This insight brings about two implications for empirical policy evaluation. First, to accurately evaluate the policy parameter $\Delta Y(\tau^0_n, \tau^1_n)$ warrants the joint consideration of the production network and firms' strategic interactions. Second, the identification of $\Delta Y(\tau^0_n, \tau^1_n)$ should be accomplished under a minimal set of assumptions about the shapes of supply and demand functions, so that the analysis can remain agnostic about the configurations of the policy effect spillovers.[58] These observations motivate my nonparametric approach for identification and estimation (Section 4), and the subsequent empirical analysis (Section 5). To prepare the ground, the next section describes data available to the policymaker.

3 Data

This section briefly describes the dataset used in my empirical analysis and the procedures by which I construct the empirical counterparts to the variables in my model.[59] My dataset

[58]For instance, in a spcial case of my model, in which firm-level production uses a Cobb-Douglas production function with Hicks-neutral productivity, and the sectoral aggregators take the form of a CES production function — a setup commonly adopted in the literature (e.g., Grassi, 2017; Gaubert and Itskhoki, 2020), it is shown that in conjunction with the macro and micro complementarities, (i) firms' revenues do not change, and neither do sectoral revenues; and (ii) the general equilibrium feedback through the change in wage is muted (see Appendix C.9).

[59]The details are provided in Appendix B.

spans between 2010 and 2021, but I do not exploit its time-series feature; rather, I regard it as a collection of snapshots of the same economy with varying levels of subsidies. In this way, I can construct "repeated samples." Consistent with the static nature of my model, the firm-level functions (e.g., technology, demand) are posited to be, conditional on an array of sector-level and aggregate variables, the same across these snapshots.[60] I assume that the observations are generated from an equilibrium (see Assumption 4.1).

Wage and Price Indices.— Data on wage and labor hours worked are taken from the U.S. Bureau of Labor Statistics (BLS) through the Federal Reserve Bank of St. Louis (FRED) at an annual frequency. Consistent with my conceptual framework, I use the average hourly earnings of all employees as my data counterpart for the wage W^*.[61] I obtain data on sectoral price index P_i^* from the GDP by industry data at the Bureau of Economic Analysis (BEA), wherein the industries in the BEA data are used as the empirical counterparts of sectors in my framework.

Input-Output Tables.— I adopt the annual U.S. input-output data from the BEA. The data contain industrial output and input for 66 industries and cover the period from 1995 to 2021. Following Baqaee and Farhi (2020), I omit the government, noncomparable imports, and second-hand scrap industries. I also follow Bigio and La'O (2020) in excluding the finance, insurance, real estate, rental, and leasing (FIRE) industries. I further follow Gutiérrez and Philippon (2017) in segmenting the industries into coarser categories, leaving me with 26 industries.

Each input-output account comes with two distinct tables: the use and supply tables. The use table reports the amounts of commodities used by each industry as intermediate inputs and by final users, and the value added by each industry. The value-added section of the use table includes compensation of employees and taxes on products less subsidies for each purchaser industry. Each cell in the supply table indicates the amount of each commodity produced by each industry.

To transform the use table into an industry-by-industry format, I make the following assumption: Each product has its own specific sales structure, irrespective of the industry where it is produced (Assumption B.1). Here, the sales structure refers to the shares of the respective intermediate and final users in the sales of a commodity. Under this assumption, I can convert the commodity-by-industry use table to the industry-by-industry table, thereby conforming to my conceptual model of the production network $\mathbf{\Omega}$ (see Appendix B.2.1 for details).[62] The transformed input-output table can further be used to back out data for

[60]This aligns with the approach adopted by Ackerberg and De Loecker (2024).

[61]Recall that labor is assumed to be frictionlessly mobile across sectors, which implies that the wage is the same everywhere in the economy.

[62]Using the compensation of employees, I can also construct data for $\boldsymbol{\omega}_L$. Throughout the transformation,

$\boldsymbol{\tau}$ as a value-added net subsidy, which is understood as an amalgamate of sales and input subsidies.

Compustat Data.— The dataset for firm-level variables is Compustat, which is assembled by S&P and provided by Wharton Research Data Services (WRDS). The Compustat data record information about firm-level financial statements, such as sales, input expenditure, capital stock information, and detailed industry activity classifications, from 1950 to 2021. From this data, in conjunction with the data on aggregate variables, I construct measurements for firm-level labor and material inputs as well as revenue.

Since the dataset does not offer a further breakdown of material input, I need to apportion the expenditure on material input to generate separate information about the demand for sectoral intermediate goods. This requires an explicit functional-form assumption on the material input aggregator $\mathcal{G}_i(\cdot)$ in (4). In this paper, I employ a Cobb-Douglas production function:

$$m_{ik} = \prod_{j=1}^{N} m_{ik,j}^{\gamma_{i,j}}, \tag{19}$$

where $m_{ik,j}$ is sector j's intermediate good demanded by firm k in sector i and $\gamma_{i,j}$ denotes the input share of sector j's intermediate good with $\sum_{j=1}^{N} \gamma_{i,j} = 1$.[63] A virtue of this specification is that the production network across sectoral intermediate goods $\{\omega_{i,j}\}_{j \in \mathcal{N}}$ is directly reflected in the output elasticity parameters $\{\gamma_{i,j}\}_{j \in \mathcal{N}}$, which are constant.[64] This property is plausible in light of the particular focus of this paper on the short-run effects of the policies.[65] Under this specification, the equilibrium input demand for sector j's good $m_{ik,j}^*$ is given by

$$m_{ik,j}^* = \gamma_{i,j} \frac{P_i^{M*}}{(1-\tau_i) P_j^*} m_{ik}^*, \tag{20}$$

where $P_i^{M*} m_{ik}^*$ indicates the expenditure on material input gross of subsidies, which can be obtained in the data.

I admit the possibility that the data on firm-level revenues are subject to measurement

the value-added section of the use table remains intact.

[63]In principle, the functional form assumption (19) is necessitated in order to compensate for the shortcoming of the dataset at hand. In general, this assumption could be replaced to the extent that the information about demand for sectoral intermediate goods are recovered. Moreover, this assumption could even be completely dispensed if the econometrician (or the policymaker) has access to detailed data on firm's input purchase, such as firm-to-firm trade.

[64]The Cobb-Douglas production function has routinely been used in the macroeconomics literature (e.g., Grassi, 2017; Bigio and La'O, 2020).

[65]See Assumption 2.1.

errors.[66] Importantly, the Compustat data do not provide information about output quantity and price. To recover these variables from the error-contaminated revenue data, I leverage a methodology that has recently been developed in the industrial organization literature.

4 Identification and Estimation

This section discusses identification of the object of interest (14) based on the model laid out in Section 2 and the dataset described in Section 3. The identification results are constructive, thereby validating the use of nonparametric plug-in estimators.

To simplify the identification analysis, I make two sets of assumptions. First, in order to sidestep the concern about the multiplicity of equilibria, I impose assumptions on the equilibrium selection probability. Second, I assume that the firms' labor and material input quantities stay within their historically observed supports throughout the policy reform. Let $\mathscr{L}_i$ and $\mathscr{M}_i$, respectively, denote the observed supports of labor and material inputs.

Assumption 4.1 (Equilibrium Selection). *(i) The observations in the data are generated from a single equilibrium. (ii) The equilibrium that is played does not change over the course of the policy reform.*

Assumption 4.2 (Support Condition). *For each $i \in \mathscr{N}$ and each $k \in \mathscr{N}_i$, $\ell_{ik}(\tau_n) \in \mathscr{L}_i$ and $m_{ik}(\tau_n) \in \mathscr{M}_i$ for all $\tau_n \in [\tau_n^0, \tau_n^1]$, where $\ell_{ik}(\tau_n)$ and $m_{ik}(\tau_n)$, respectively, represent labor and material inputs under τ_n.*[67]

Assumption 4.1 (i) states that the equilibrium selection probability is degenerated to a single equilibrium, and part (ii) means that it is this single equilibrium that will be chosen in the policy counterfactuals. Assumption 4.1 is widely used in the literature of discrete choice models (Aguirregabiria and Mira, 2010).[68] Assumption 4.2 excludes the scenario that the new policy sends firms' labor and material input quantities outside their empirically observed supports.[69] As will become clear below, my identification methodology exploits variation in firms' input variables, rather than variation in the policy variable per se. Under this assumption, the policymaker can analyze a policy that has never previously been implemented, as long as the input variables remain within the historically observed support.

[66] I assume additive separability in terms of log variables.

[67] In general, labor and material inputs also depend on subsidies to other industries $\{\tau_j\}_{j \neq n}$. The dependencies to other subsidies are made implicit for the interest of brevity.

[68] Notice that Assumption 4.1 only restricts the equilibrium selection probability and does not exclude the possibility of multiple equilibria per se.

[69] Assumptions 4.1 and 4.2 could jointly be relaxed at the expense of additional assumptions, as studied in Canen and Song (2022).

This assumption is testable and verified to be the case throughout the subsequent empirical illustration in Section 5.

To solve the evaluation problem, it is essential to distinguish the policymaker's (or the observing econometrician's) information set from the agent's information set.[70] Four remarks are in order. First, the former includes $\boldsymbol{\tau}^1$, reflecting the premise that the policy variables are externally manipulated by the policymaker. Second, the firm's productivity z_{ik} is not observed by the policymaker, while firms have complete information (Section 2). Third, the firm's equilibrium revenue r_{ik}^* is not known to the policymaker, and the observed firm's revenue r_{ik} is contaminated by measurement error. Lastly, the firm's equilibrium output price p_{ik}^* and quantity q_{ik}^* are not included in the policymaker's information set due to the limitation of the data (Section 3).

4.1 Identification Strategy

My identification strategy builds on (15) and aims to identify the integrand $dY_i(s)/ds$ for all $s \in [\boldsymbol{\tau}^0, \boldsymbol{\tau}^1]$.[71] The existing approach to recover (16) is to characterize its left-hand side in terms of aggregate variables that are directly observed in the data (e.g., Arkolakis et al., 2012, 2019; Adão et al., 2020). Their aggregation results crucially hinge on the modeling assumption of a mass of continuum of firms. Under this assumption, individual firms are infinitesimally small and thus inconsequential to the aggregate variables owing to the law of large numbers (Gaubert and Itskhoki, 2020). In contrast, my framework embraces only a finite number of firms, in which case firm-level idiosyncrasies are not washed away even in the aggregate. My approach is rather to recover each of the firm-level responses on the right-hand side of (16). In doing so, I apply the literature on production function estimation (e.g., Gandhi et al., 2019; Kasahara and Sugita, 2020) and rely on firms' input variables for the source of identifying variation. As a consequence, my framework can be used to analyze an unprecedentedly large policy reform as long as Assumption 4.2 is satisfied.

4.2 Identification

To recover (16) requires the identification of firm-level prices and quantities, and comparative statics, with the latter further calling for the identification of derivatives of firm-level production and inverse demand functions. The equilibrium values of these variables and

[70]It is tacitly assumed that as far as the information set is concerned, the government, which is one component of the model, is identical to the econometrician outside the model.

[71]Notice that the left-hand side of (16) alone may be of limited practical relevance. A common practice of setting $\boldsymbol{\tau}^0 = \mathbf{0}$ (e.g., Liu, 2019; Baqaee and Farhi, 2022) is not sufficient to recover the target parameter (15). Note also that my approach differs from exact hat algebra. I discuss these in Appendix C.11.

responses act as firm-level sufficient statistics — a nonparametric analog of policy-invariant parameters — in my framework. Notice, however, that *a*) firm-level prices and quantities are not observed in my dataset (see Section 3), and *b*) derivatives of the firm-level production and inverse demand functions are not known by definition (see Section 2). To keep track of these variables from the policymaker's viewpoint, I leverage the techniques of the industrial organization literature by imposing three sets of additional assumptions.

First, I assume that the firm-level production function exhibits Hicks-neutral productivity.

Assumption 4.3 (Hicks-Neutral Productivity)**.** *In each sector* $i \in \mathscr{N}$ *and each firm* $k \in \mathscr{N}_i$, $q_{ik} = z_{ik} g_i(\ell_{ik}, m_{ik})$, *where* $g_i : \mathscr{L}_i \times \mathscr{M}_i \to \mathscr{S}_i$ *is a sector-specific production technology.*

This assumption is routinely employed in the macroeconomics literature (e.g., Baqaee and Farhi, 2020; Bigio and La'O, 2020). Notably, this assumption, together with the specification (19), includes the nested Cobb-Douglas production function of the kind studied in Bigio and La'O (2020).

Second, in order to make the model amenable to empirical analysis while maintaining flexibility, I restrict the sectoral aggregator to take the form of a *homothetic demand system with a single aggregator* (HSA; Matsuyama and Ushchev, 2017).

Assumption 4.4 (HSA Inverse Demand Function)**.** *In each sector* $i \in \mathscr{N}$, *the sectoral aggregator* $F_i(\cdot)$ *exhibits an HSA inverse demand function; that is, the inverse demand function faced by firm* $k \in \mathscr{N}_i$ *is given by*

$$p_{ik} = \frac{\Phi_i}{q_{ik}} \Psi_i\left(\frac{q_{ik}}{A_i(\mathbf{q}_i)}\right) \qquad \textit{with} \qquad \sum_{k'=1}^{N_i} \Psi_i\left(\frac{q_{ik'}}{A_i(\mathbf{q}_i)}\right) = 1, \tag{21}$$

where Φ_i *is a constant indicating the expenditure by sector i's aggregator,* $\Psi_i(\cdot)$ *represents the share of firm k's good in the expenditure of sector i's aggregator, and* $A_i(\mathbf{q}_i)$ *denotes the aggregate quantity index capturing interactions between firms' choices with* $\mathbf{q}_i \coloneqq \{q_{ik'}\}_{k' \in \mathscr{N}_i}$.

From an individual firm's perspective, the quantity index $A_i(\mathbf{q}_i)$ in (21) summarizes the firm's interactions in sector i, and this is the only channel through which other firms' choices matter to the firm's own decision. The HSA specification (21) is broad enough to accommodate a wide variety of aggregators — for example, constant elasticity of substitution (CES) and the flexible class of non-CES homothetic aggregators explored in Arkolakis et al. (2019).[72]

Under Assumption 4.4, I show that for each $i \in \mathscr{N}$, there exists a constant $M_i \in \mathbb{N}$ such that there exist some continuous functions $\mathcal{H}_{i,1}, \ldots, \mathcal{H}_{i,M_i} : \mathscr{Z}_i^{\mathscr{N}_i} \to \mathbb{R}$ and $\chi_i : \mathscr{Z}_i \times \mathbb{R}^{M_i} \to$

[72]See Appendix C.4 for the CES case, and see Matsuyama and Ushchev (2017) and Kasahara and Sugita (2020) for other examples.

$\mathbb{R}_+$ such that

$$q_{ik}^* = \chi_i(z_{ik}; \mathcal{H}_{i,1}(\mathbf{z}_i), \dots, \mathcal{H}_{i,M_i}(\mathbf{z}_i)), \tag{22}$$

where $\mathcal{H}_{i,m}(\mathbf{z}_i)$ is exchangeable in $(z_{i1}, \dots, z_{iN_i})$ for all $m \in \{1, \dots, M_i\}$.[73] This result suggests that the firm's equilibrium quantity depends on other firms' productivities only through M_i aggregates, all of which are common to all firms. These aggregates admit an interpretation analogous to the quantity index $A_i(\cdot)$ in (21); that is, the aggregate productivities $\{\mathcal{H}_{i,m}(\mathbf{z}_i)\}_{m=1}^{M_i}$ collectively constitute summary statistics for the competitors' productivities.[74] An intuition is that instead of interacting with one another, each firm only needs to interact with these aggregate productivities, as they act as a "translator" of the strategic interaction in the market. These aggregates can most naturally be understood as measures of the overall competitiveness of the market and can be viewed as versions of the conventional measure of market concentration, such as the Herfindahl-Hirschman Index (HHI).[75]

Remark 4.1. *(i) Assumption 4.4 is slightly stronger than the original definition by Matsuyama and Ushchev (2017), and abstracts from unobservable demand-side heterogeneity in the sectoral aggregator $F_i(\cdot)$. This assumption is adopted only to simplify identification and estimation, and can be relaxed at the cost of an additional technicality. See Kasahara and Sugita (2023). (ii) In the production function context, Blum et al. (2023), Ackerberg and De Loecker (2024) and Doraszelski and Jaumandreu (2024) consider demand functions similar in spirit to* (21). *The identification results of Ackerberg and De Loecker (2024) and Doraszelski and Jaumandreu (2024) require that their terms corresponding to $A_i(\mathbf{q}_i)$ be observable, while this paper, as well as Blum et al. (2023), do not. My approach differs from Blum et al. (2023) in that it does not rely on the observability of firm-level output prices and quantities.*

The last set of assumptions, together with Assumption 4.3, guarantees that the equilibrium quantity function $\chi_i(\cdot)$ is "invertible" with respect to the firm's own productivity.

Assumption 4.5. *For each $i \in \mathcal{N}$, the function $\chi_i(\cdot)$ in* (22) *satisfies the following properties. (i) $\chi_i(z_{ik}; \cdot)/z_{ik} \neq \chi_i(z_{ik'}; \cdot)/z_{ik'}$ for all $k, k' \in \mathcal{N}_i$. (ii) $\chi_i(\cdot)$ is strictly monotone in its first argument.*

[73] See Appendix C.1 for the proof.

[74] The idea is similar to sufficient statistics, but distinct in that these aggregate productivities do not even need to be observed by the econometrician. The only thing that needs to be known is the fact that the competitor's productivity is summarized by some sector-specific aggregates.

[75] They are, though, distinct in that the latter is usually observed in data, while the former is by definition not known to the econometrician. Note that owing to the completeness of the information structure, the values of these aggregate productivities are known to all firms in the same sector at the time of decision making.

Part (i), coupled with Assumption 4.3, ensures that variation in the firms' productivities is reflected in the difference in their input choices. Part (ii) pertains to the partial derivative of $\chi_i(\cdot)$ with respect to the firm's own productivity, keeping the aggregate productivities fixed. Note that Assumption 4.5 directly refers to the equilibrium configuration. Formally examining this requires detailed knowledge of the sectoral aggregator and the firm-level production function, which runs counter to the goal of this paper — an analysis with minimal assumptions. Nevertheless, there is reason to believe that part (i) is plausible because $\chi_i(\cdot)$ is given as a solution to a system of (possibly) highly nonlinear equations. Part (ii) is also likely to be the case with a strictly increasing $\chi_i(\cdot)$ because with the market competitiveness being constant, more productive firms tend to have higher market shares, producing more goods.

Taken together with (6), it follows from Assumptions 4.3 – 4.5 that there exists a continuous function $\mathcal{M}_i : \mathscr{L}_i \times \mathscr{M}_i \times \mathbb{R}^{M_i} \to \mathscr{Z}_i$ such that

$$z_{ik} = \mathcal{M}_i(\ell^*_{ik}, m^*_{ik}; \mathcal{H}_{i,1}(\mathbf{z}_i), \ldots, \mathcal{H}_{i,M_i}(\mathbf{z}_i)) \tag{23}$$

for all $k \in \mathscr{N}_i$. In light of this, the expression (22) and Assumption 4.5 jointly correspond to the scalar unobservability assumption and the strict monotonicity assumption of the existing literature.[76] The expression (23) accounts for unobservable productivity in terms of observable labor and material inputs, in the presence of firms' strategic interactions — a feature missing in the existing literature.[77] Coupled with Assumption 4.2, this expression allows the policymaker to recover both sector- and firm-level responses through variation in firms' input variables.

Assumptions 4.3 – 4.5 permit a variety of specifications for both sector- and firm-level production functions. The following example demonstrates that these assumptions are satisfied in a model widely used in the macroeconomics and international trade literature.

Example 4.1 (CES Sectoral Aggregator and Cobb-Douglas Production Function)**.** *Assume that for each* $i \in \mathscr{N}$, $F_i(\{q_{ik'}\}_{k'=1}^{N_i}) := (\sum_{k'=1}^{N_i} \delta_i^{\sigma} q_{ik'}^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}}$ *and* $f_i(\ell_{ik}, m_{ik}; z_{ik}) := z_{ik}\ell_{ik}^{\alpha} m_{ik}^{1-\alpha}$. *To make my claim as transparent as possible, I focus on the case of three firms* ($N_i = 3$) *and* $\sigma = 2$. *In this case, the Cournot-Nash equilibrium quantity is given by* $q^*_{ik} = \{(A^*_i\Phi_i)/(2A^{*2}_i z_{ik}^{-1} mc_i + \Phi_i)\}^2 =: \chi_i(z_{ik}; \mathcal{H}_{i,1}(\{z_{ik}\}_{k=1}^3), \mathcal{H}_{i,2}(\{z_{ik}\}_{k=1}^3))$*, where* $\mathcal{H}_{i,1}(\{z_{ik}\}_{k=1}^3) := z_{i1}^{-1} + z_{i2}^{-1} + z_{i3}^{-1}$ *and* $\mathcal{H}_{i,2}(\{z_{ik}\}_{k=1}^3) := z_{i1}z_{i2}z_{i3}$. *Here,* mc_i *stands for the part of the firm's marginal cost common to all firms. This conforms to the expression* (22)*, and satisfies Assumption 4.5.*

[76]See, for example, Ackerberg et al. (2015).

[77]Note that (23) does not involve a stochastic process for the firm's productivity because of a static nature of my model, while being in line with the repeated sample paradigm (see Section 3). See also Appendix C.11.

Taking this expression as given, the input decision is constrained by the production possibility frontier at output level q_{ik}^*: $z_{ik}{\ell_{ik}}^{\alpha}{m_{ik}}^{1-\alpha} = \left\{(A_i^*\Phi_i)/(2A_i^{*2}z_{ik}^{-1}mc_i + \Phi_i)\right\}^2$. *Upon solving this for* z_{ik}, *it is immediate that in equilibrium there exists a function* $\mathcal{M}_i(\cdot)$ *such that* $z_{ik} = \mathcal{M}_i(\ell_{ik}^*, m_{ik}^*, \mathcal{H}_{i,1}(\{z_{ik}\}_{k=1}^3), \mathcal{H}_{i,2}(\{z_{ik}\}_{k=1}^3)$, *yielding the expression* (23).[78]

With Assumptions 4.3 – 4.5, my approach extends Kasahara and Sugita (2020) to account for firms' strategic interactions, thereby identifying the equilibrium values of the firm-level quantities and prices, and those of the derivatives of the residual inverse demand functions. Moreover, under the CRS property (Assumption 2.4) and the Hicks-neutral productivity (Assumption 4.3), I can apply the method developed in Gandhi et al. (2019) to recover the equilibrium values of the first- and second-order derivatives of the production functions.

With additional regularity conditions,[79] I therefore obtain the following theorem.

Theorem 4.1 (Identification of the Object of Interest)**.** *Suppose that Assumptions 4.1 – 4.5, C.2 and C.3 hold. Then, the object of interest* (14) *is identified from the observables.*

Proof. See Appendix C.8. □

Remark 4.2. *(i) Under the same set of assumptions as Theorem 4.1, various other (both aggregate and distributional) causal parameters and the effects of changing subsidies to multiple sectors (e.g., universal treatments) can also be identified (Appendix D). (ii) A version of Theorem 4.1 remains valid for profit-maximizing monopolistic firms provided that the solution concept is appropriately modified (Appendix C.8).*

The identification result established in Theorem 4.1 exploits variation in firms' input variables instead of variation in policy variables. My framework can thus be used to study a large policy intervention that has never previously been implemented, as long as the new aspect of the policy can be recast in the historical variation of firms' input variables. This feature becomes particularly relevant when it comes to a drastic policy reform, such as the CHIPS and Science Act — the motivating example of this paper.

4.3 Estimation

Since the identification results demonstrated above are constructive, I build on the analogy principle to obtain a nonparametric estimator for the policy effect (14).[80] I first nonpara-

[78]See Appendix C.1.1 for details.

[79]These regularity conditions consist of three parts, namely, *a*) the strict exogeneity of the measurement error on firm-level revenues, *b*) continuous differentiability of the firm-level revenue function with respect to labor and material inputs, *c*) monotonicity and invertibility of the firm-level revenue function in terms of the firm's output quantity, and *d*) normalization of both the firm's production function and the sectoral aggregator.

[80]My approach takes a stance on econometric estimation rather than calibration. See Hansen and Heckman (1996) and Dawkins et al. (2001) for an extensive discussion about the methodological difference between

metrically estimate the equilibrium values of the firm-level quantities and prices, and those of the first- and second-order derivatives of the firms' production and inverse demand functions. Guided by the theory, I then combine these to derive the nonparametric estimator for (14). Given that the target effect is continuous with respect to the recovered variables and responses, the resulting estimator is consistent. The accuracy of my estimator is verified through a numerical simulation in Appendix F.

As stated in Section 3, I acknowledge the possibility that the data on firm-level revenues are contaminated by measurement errors. To purge the measurement errors, my estimation of the firm-level quantity and price follows the convention of the industrial organization literature in applying a polynomial regression. In estimating the firms' production elasticities, I follow the specification suggested in Gandhi et al. (2019). See Appendix E for the details.

Compared to the calibration-type approach, my estimation procedure has four practical advantages. First, it only needs to recover a limited number of comparative statics, rendering the empirical analysis more robust against misspecification and computationally less cumbersome. Second, my approach does not require any external information (e.g., parameter estimates from the preceding research) and thus can be performed in a self-contained fashion. This feature obviates the need for conducting a "robustness check" with respect to the pre-specified values of some parameters (see Section 5.1.1).[81] Third, the loss functions in my estimation are motivated by the preceding identification argument, eliminating the arbitrariness in the choice of the "targeted moments." Lastly, while the canonical calibration method is merely a benchmarking exercise, my approach prepares the ground for statistical hypothesis testing about the causal effects, thereby allowing for the accumulation of knowledge in the hypothetico-deductive way.[82,83]

these two approaches. See Matzkin (2013) for nonparametric estimation.

[81] The benefit of this property becomes acute when there are no existing works that align closely with the setup being studied by the researcher, as there is no hope of "borrowing" estimates from other research. This is actually the case with the present paper.

[82] See Dawkins et al. (2001) for a further discussion about these two methodologies. Deaton and Cartwright (2018) compare the econometric policy analysis and statistical causal inference methods (such as randomized control trials) from a philosophical viewpoint.

[83] Formally conducting inference, however, requires *i*) deriving the asymptotic distribution, and *ii*) deriving an analytical expression for standard errors, or alternatively, developing a new bootstrap method appropriate for my setup. These are far beyond the scope of this paper and left for future work.

5 Empirical Application: CHIPS and Science Act of 2022

In this section, I study the empirical relevance of the joint existence of a production network and firms' strategic interactions by taking my model to the real-world data described in Section 3. As a policy narrative, I investigate the recent episode of the CHIPS and Science Act (CHIPS), which was passed into law in 2022 and aims to invest nearly $53 billion in the U.S. semiconductor manufacturing, research and development, and workforce. This policy also includes a 25% tax credit for manufacturing investment, which is projected to provide up to $24.25 billion for the next 10 years. My framework serves as a plausible policy-evaluation tool for this policy episode because the U.S. government at the time of enactment acknowledged the prevalence of market concentration and the importance of supply chains.[84] In my model, the tax credit can be analyzed as an additional subsidy targeted at the computer and electronic products industry, which is indexed by n.[85] In my dataset, the historically observed support for the subsidy to this industry is between 6.84% and 16.70%.[86] To render the analysis as close to reality as possible, I set the current policy regime to the one at the time of the decision-making, which is 2021.

The exercise of this section focuses on a part of the CHIPS subsidy. Specifically, I consider a hypothetical policy scenario of increasing the subsidy on the semiconductor industry from the 2021 level of 15.43% to an alternative ratio of 18.43% — equivalent to $2.02 billion.[87,88] In general, analyzing this policy scenario requires extrapolating the target parameter with respect to the policy instrument because the counterfactual under consideration sends the policy variable outside the observed support. The standard empirical approach that relies on policy instruments for the source of identifying variation is thus incapable of analyzing the current policy scenario unless a parametric functional-form assumption is imposed. In contrast, my approach exploits variation in input variables, thereby allowing the policymaker to analyze a counterfactual policy as long as the new aspect of the policy can be recast in the historical variation of input variables. Throughout the empirical illustration, Assumption 4.2 is verified to be true.

[84]See Appendix G.1 for the details of the CHIPS and Science Act of 2022.

[85]See Appendix B.2.2.

[86]In the dataset, the semiconductor subsidy ranged from 6.84% in 2010 to 16.70% in 2019.

[87]Simply dividing the estimated $24.25 billion by 10 years implies $2.43 billion per year. Thus, the policy counterfactual under consideration roughly corresponds to a one-year portion of the entire tax credit. One way to interpret this policy scenario is that it takes time to put the whole part of the CHIPS Act into effect, and what can be realized in the short run — the time horizon of this paper — is only a part of it.

[88]The total amount of value-added tax in 2021 is $8.44 billion, and the total value of material input (before tax and subsidy are applied) is $46.28 billion. Hence, $(8.44 + 2.02)/(46.28 + 8.44 + 2.02) \times 100 = 18.43\%$. See Appendix B.2.2.

The goal of this section is to discuss the empirical relevance of the joint existence of a production network and firms' strategic interactions by first estimating the change in GDP due to this counterfactual industrial policy and then analyzing the mechanism behind the estimated policy effect. In Section 5.1, I calculate the estimate of the policy effect (14). To put things into perspective, I carry out the estimation for both monopolistic and oligopolistic cases.[89] In Section 5.2, I take advantage of the structural construction of my framework to provide a breakdown of the gains and losses of the policy reform into sector-level price and quantity effects. To understand the determination of these effects, I further explore the comovement of (or pass-through between) sectoral price and material cost indices.

5.1 The Policy Effect: Change in GDP

Based on (15), I estimate the change in GDP due to the policy reform from $\tau_n^0 = 0.1543$ to $\tau_n^1 = 0.1843$. An advantage of my approach is that the responsiveness of GDP can be traced out as a (possibly nonlinear) function of the subsidy over $[\tau_n^0, \tau_n^1]$. For computation, I divide this interval evenly into a fixed number of segments and calculate the estimate according to

$$\widehat{\Delta Y}(\tau_n^0, \tau_n^1) \approx \sum_{v=0}^{\bar{v}-1} \sum_{i=1}^{N} \widehat{\frac{dY_i(s)}{ds}\bigg|_{s=\tau_n^0+v\Delta\tau_n}} \times \Delta\tau_n, \tag{24a}$$

where the symbol $\widehat{\ }$ is used to denote an estimator or estimate, and $\Delta\tau_n := (\tau_n^1 - \tau_n^0)/\bar{v}$ with $\bar{v}$ being the number of bins equally segmenting the interval $[\tau_n^0, \tau_n^1]$.[90] To highlight the consequence of ignoring the nonlinearity, I also estimate the policy effect using the following approximation:

$$\widehat{\Delta Y}(\tau_n^0, \tau_n^1) \approx \sum_{i=1}^{N} \widehat{\frac{dY_i(s)}{ds}\bigg|_{s=\tau_n^0}} \times (\tau_n^1 - \tau_n^0). \tag{24b}$$

That is, the estimate is computed by assuming that the responsiveness of GDP remains constant at the level of the current policy regime over the course of the policy reform.

Table 1 compares the estimates for the policy effect based on (24a) and (24b) across monopolistic and oligopolistic competition. Two things stand out about this table. First, the estimate (24a) under oligopolistic competition is markedly different from that under monopolistic competition: The former is about 107 percent lower relative to the latter, flipping the sign from positive to negative. This considerable discrepancy reflects the policy effects mediated by firms' strategic interactions (Section 2.7), highlighting their empirical

[89] In view of Corollary C.5, these two cases can be analyzed by the same procedure.

[90] In this analysis, I set $\bar{v} = 100$.

relevance. Second, the estimates based on (24b) are noticeably different from those based on (24a).[91] This underlines the substantial degree of nonlinearity in the responsiveness of GDP as a function of the subsidy, which is visualized in Figure 2. The nonlinearity essentially arises from the fact that a firm's reaction depends on all firms' quantity and price levels, as well as their production and demand elasticities, all of which in turn depend on the value of the underlying subsidy.

Three caveats in interpreting the implications of Table 1 should be clarified before proceeding. First, the primary focus of this section is not on accurately gauging the size of the actual policy effect, but on empirically assessing the significance of the presumed economic mechanism in policy effects. Second, the dataset used in this paper is by no means representative of the universe of U.S. firms. Third, the estimates are obtained by ignoring a part of the demand-side heterogeneity (Assumption 4.4). With these caveats firmly in mind, it is important not to misconstrue Table 1 as a generic endorsement of the (in)effectiveness of industrial policy; rather, it should be understood as empirical evidence in support of the policy relevance of the firms' strategic forces accruing through the production network, a property illuminated in Section 2.7. The appropriate choice of a model (as well as a methodology) depends on the economic question being investigated.

Table 1: The estimated policy effect under different market structures

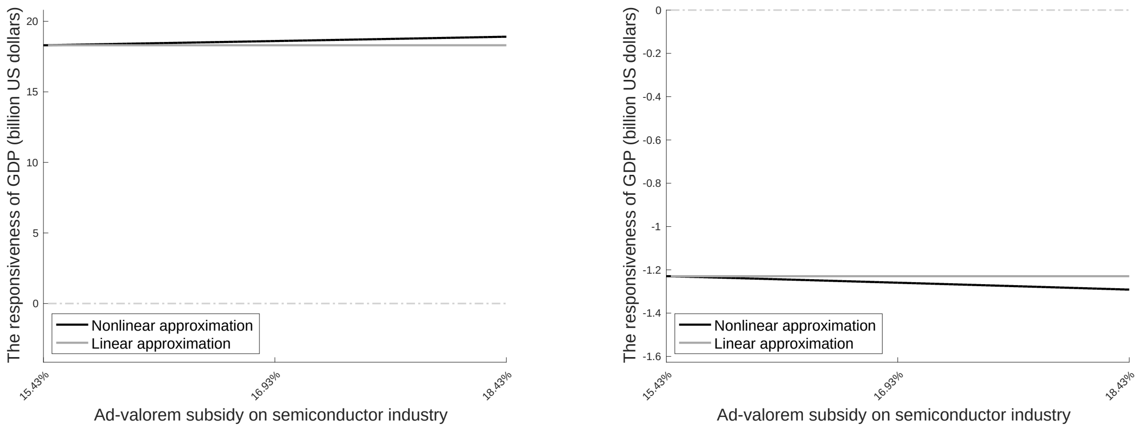
(billion U.S. dollars)	Monopolistic competition	Oligopolistic competition
Estimates based on (24a)	0.5581	-0.0378
Estimates based on (24b)	0.5491	-0.0369

Note: This table compares the estimates for the object of interest (14) based on the benchmark and my method. The estimates are measured in billions of U.S. dollars.

5.1.1 Robustness

In general, there are three types of "robustnesses" that require some care, namely, *i*) robustness with respect to the choices of pre-specified parameter values, *ii*) robustness with respect to the criteria for data construction and cleaning, and *iii*) robustness with respect to the choices of truncation and turning parameters in the estimators. For the first case, as discussed in Section 4.3, my approach does not presuppose any external information, thereby being free from any concern of this type. Second, the dataset used in my analysis

[91] The nonlinear estimate (24a) under oligopolistic competition is 2.44% lower in magnitude compared to the linear estimate (24b). When firms are monopolistic, the estimate based on (24a) is 1.64% higher than the one based on (24b).



(a) Monopolistic Competition (b) Oligopolistic Competition

Note: This figure illustrates the estimates of the total derivative of (economy-wide) GDP with respect to the semiconductor subsidy between 15.43% and 18.43%. Panel (a) shows the result for the case of monopolistic competition, and panel (b) displays the result for the case of oligopolistic competition. The solid black line represents the estimates based on the nonlinear approximation (24a). The solid dark grey line indicates the estimates based on the linear approximation (24b). The dash-dotted light grey line stands for the horizontal axis at zero.

Figure 2: The total derivative of Y with respect to τ_n

goes through several steps of outlier and missing data elimination. These manipulations are rationalized by the assumptions imposed on the model (see Appendix B). Relaxing the criteria for these steps runs the risk of misspecification, which is of great interest in its own right and exceeds the scope of this paper. The third type, in my case, pertains to *iii-a*) the choice of the degree of a polynomial in estimating the firm-level revenue function and share regressions, and *iii-b*) the choice of the number of bins (i.e., $\bar{v}$ in (24a)). In my estimation algorithm, the former is chosen adaptively,[92] leaving the latter as the only computation parameter that needs to be given before the implementation. In calculating the main results, it is set to be 100. Robustness checks with respect to this choice are conducted and illustrated in Appendix G.2.1. The results are both quantitatively and qualitatively unaffected.

5.2 Mechanism

To study the mechanism behind the results obtained in Section 5.1, I investigate the determination of the integrand of (15).

[92] I use the root-mean-squared error as a criterion for the adaptive degree selection. Investigating the performance of criteria per se is of independent interest and is left to be explored.

5.2.1 Responsiveness of sectoral GDP

Design.— I anchor my interpretation of the responsiveness of sectoral GDP around (16):

$$\left.\frac{dY_i(s)}{ds}\right|_{s=\tau_n} = \underbrace{\sum_{k=1}^{N_i} \frac{dp_{ik}^*}{d\tau_n} q_{ik}^*}_{\text{price effect}} + \underbrace{\sum_{k=1}^{N_i} p_{ik}^* \frac{dq_{ik}^*}{d\tau_n}}_{\text{quantity effect}} + \left\{ \underbrace{\left(- \sum_{k=1}^{N_i} \sum_{j=1}^{N} \frac{dP_j^*}{d\tau_n} m_{ik,j}^* \right)}_{\text{wealth effect}} + \underbrace{\left(- \sum_{k=1}^{N_i} \sum_{j=1}^{N} P_j^* \frac{dm_{ik,j}^*}{d\tau_n} \right)}_{\text{switching effect}} \right\}, \tag{25}$$

which states that the marginal effect of a policy change consists of changes in revenue and expenditure on material input net of subsidies. The former is broken down into price and quantity effects. When a firm produces more of its output, the price effect dictates the loss due to the increased supply in light of the law of demand. Under oligopolistic competition, this downward pressure depends not only on the increase in a firm's own quantity, but also on a change in every other firm's output quantity through the cross-price elasticities of demand. The other component of (25) can similarly be decomposed into two parts: the wealth and switching effects. The wealth effects are changes in a firm's "budget" as a result of changes in sectoral price indices. The switching effects are changes in the sectoral composition of the firm's input purchase, holding the price level constant.

Result.— The empirical estimates for (25) at $\tau_n = \tau_n^0$ are displayed in Tables 7 and 8. From these tables, it can be seen that the sectoral distributional consequence — which sectors gain and lose — depends on the tension between the two types of price and quantity effects illustrated in (25). For example, take the computer and electronic products industry — the targeted industry. In an oligopolistic environment, this industry is the least benefited: The negative quantity effects are exactly offset by the positive price effects,[93] while positive wealth effects are surpassed by negative switching effects, leaving the firms with higher input costs. An intuition is that the semiconductor firms choose to produce more of their output in the hope of lower input costs; however, the material input cost ends up being much higher than expected because other industries turn out to decrease their production, thereby increasing their output prices. When firms are monopolistic, on the other hand, the semiconductor industry is one of the most benefited industries: The quantity effect is large enough to cover both the price and switching effects. These observations underline the fact that firms' strategic interactions play a critical role in determining the industry's effective location on the production network, as foreshadowed in Section 2.7.

To further explore this intuition, I next focus on the comovements between sectoral price

[93] When the market is oligopolistic, this is a systematic pattern induced by the identification assumptions. See Appendix C.9 for the details.

indices.

5.2.2 Macro and Micro Complementarities

Key equations.— Here, I derive the general-equilibrium version of the reduced-form relationships (i) and (ii) in Section 2.7.1: (i)' $dP_i^{M*}/d\tau_n = -h_{i,n}^M P_n^{M*}/(1-\tau_n) + h_i^L(dW^*/d\tau_n)$ and (ii)' $dP_i^*/d\tau_n = \bar{\lambda}_{i\cdot}^M(dP_i^{M*}/d\tau_n) + \bar{\lambda}_{i\cdot}^L(dW^*/d\tau_n)$, where $h_{i,n}^M$ and $\bar{\lambda}_{i\cdot}^M$ are, respectively, macro and micro complementarities with respect to material input; and h_i^L and $\bar{\lambda}_{i\cdot}^L$ are analogously defined with respect to labor input.[94] These two equations jointly envision the comovement of the sectoral price and material cost indices. Note that $-P_n^{M*}/(1-\tau_n)$ can be interpreted as the "initial" impact of the policy change.

The firms' strategic forces within each sector are captured by the micro complementarities, which in turn are aggregated through a production network to yield the macro complementarities, as described in Section 2.7. Both complementarities, combined with the firms' production elasticities, determine the general equilibrium feedback effect through the change in wage. In a special case of my setup, in which oligopolistic competition takes place with Cobb-Douglas firm-level production functions and CES sectoral aggregators, the incremental and decremental responses of wage are exactly offset, shutting down the general equilibrium feedback (Corollary C.11 in Appendix C.10).

Result.— In my estimation, $dW^*/d\tau_n$ evaluated at $\tau_n = \tau_n^0$ is -0.0338 for monopolistic competition, and 0.0408 for oligopolistic competition. In the monopolistic case, the macro complementarities with respect to both labor and material inputs are positive. The magnitude of the former is larger than one for all industries, which means the impact of the lower wage is amplified in the material cost index. This, in conjunction with the initial impact of the policy change, implies that the material cost index substantially decreases for all industries. Associated with this are higher firm-level output quantities and lower output prices. In contrast, when firms engage in strategic competition, most industries exhibit only modest degrees of macro complementarities. Consequently, the decrease in the material cost index is nuanced compared to the monopolistic environment.[95] This result is mirrored by firms' behavior in several industries that reduces output quantity to raise output price.

This observation highlights two channels through which the macro and micro complementarities manifest themselves in the causal policy effect. First, in view of Corollary C.11, my estimate of non-zero wage response can be viewed as an indication that the commonly used specification is not supported on empirical grounds.[96] This underscores the importance

[94]See Appendix A.1 for the precise definition.

[95]Three out of twenty-six industries are even confronted with a rise in material input costs.

[96]Although this statement has not been examined by a rigorous statistical hypothesis testing, a back-of-

of a nonparametric policy evaluation framework, such as the one put forth in the present paper. Second, the configurations of the equilibrium responses of both sector- and firm-level variables are, in principle — even in the absence of general equilibrium feedback — different between monopolistic and oligopolistic competition. This fact warrants a careful specification of market competition as well as a production network.

6 Conclusions

Industrial policies have been and will continue to be an important policy tool for policymakers to achieve a range of policy goals. This paper studies the causal impact of an industrial policy on an aggregate outcome in the presence of sectoral production networks and firm-level strategic interactions by making two contributions.

First, to define the causal effect as a *ceteris paribus* difference in outcome variables across different policy regimes, I develop a general equilibrium multisector model of heterogeneous oligopolistic firms with a production network. My causal estimand is inclusive of firms' strategic interactions, network spillovers, and general equilibrium feedback, all of which are typically assumed away in the treatment effect literature. A key mechanism of my model is that when firm-level production functions exhibit constant returns to scale, policy effects are mediated by the production network that compounds changes in firms' marginal profits not only through adjustments of their own actions but also via those of competitors' actions (i.e., strategic complementarities), with the latter being absent in monopolistic competition.

Second, I establish a new nonparametric identification methodology that accounts for firms' strategic interactions and production networks. My approach recovers firm-level comparative statics by restricting the class of firm-level and sectoral production functions. Yet, the identification analysis i) is general enough to encompass many specifications commonly used in the macroeconomics and international trade literature, ii) can handle an unprecedently large policy reform as long as firms' input variables stay in the historically observed support, and iii) is constructive, so that a nonparametric estimator for the policy effect can be obtained without adopting any external information.

In my empirical estimation, the policy effect under oligopolistic competition is approximately 107 percent lower than under monopolistic competition. This observation echoes the policy relevance of jointly accounting for firms' strategic interactions and a production network. It should, though, be clarified that the goal of this paper is neither i) to argue that one

the-envelope calculation indicates $(dW^*/d\tau_n)/W^* \times 100 = 0.0408/21.2506 \times 100 = 0.1920\%$, suggesting a noticeable size of the wage response. Developing a formal statistical test goes well beyond the scope of this paper and is reserved for future work.

type of competition should always be preferred over the other, nor *ii*) to criticize the policy analyses based on quantitative macroeconomic models and the empirical treatment effect approach. Rather, I wish *i*)' to caution that the policy conclusion may vary depending on how the policymaker models the economy, and *ii*)' to supplement the existing methodologies for causal policy analysis. The appropriate choice of model specification and methodology depends on the application at hand.

Interpreting the results displayed in this paper requires some care because they are susceptible to errors to the extent that the Compustat data are incomplete and non-representative. Besides the data limitation, there are three directions for future work. First, since my framework is fairly general, it can straightforwardly be extended to embrace other types of policies, such as fiscal and monetary policies and trade policies. Second, this paper focuses on short-run policy effects and abstracts away from the firm's endogenous entry and exit over the course of policy reform. Accommodating a long-run perspective inserts an additional layer into my framework, namely, the free-entry condition. Third, the identification analysis of this paper assumes that the economy features a single equilibrium, the same equilibrium is played over the course of a policy reform, and the policy reform does not send firms' input variables outside the historically observed support. These limitations can be simultaneously addressed at the cost of additional assumptions concerning the equilibrium selection probability, as studied in Canen and Song (2022). Lastly, my model is static and thus silent about the policy implications for capital accumulation, which is usually at the center of policy debate. An extension to a dynamic environment requires an explicit consideration of not only the firm's own future choices but also competitors' future choices. This convoluted, forward-looking nature opens up another source of multiplicity of equilibria.

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Supplementary Material for: "Industrial Policies, Production Networks, and Oligopolistic Competition: Econometric Evaluation of the U.S. Semiconductor Subsidy"

Ko Sugiura

Abstract

This appendix consists of seven parts. In Appendix A, I derive theoretical properties of my model. In the subsequent section, I provide a detailed description of the data used in the main text. Appendix C develops the identification analysis, while its extensions are delegated to Appendix D. The following section presents the estimation strategy. Appendix F shows the results of Monte Carlo experiments, followed by Appendix G, where the details of the empirical illustration are provided.

A Comparative Statics

In this section, theoretical results displayed in Section 2 are derived. The goal of this section is to solve for comparative statics — the responsiveness of sector- and firm-level variables with respect to a change in the policy variable (i.e., the sector-specific subsidy). The results of this section express the comparative statics in terms of the endogenous variables in the current equilibrium, the exogenous variables, and the policy-invariant functions, each of which is delineated in Section 2. The exposition is organized along the firm's decision process.

Remark A.1. *For the sake of econometric analysis, the main text assumes that the quantity of labor input is determined prior to material input, as described in* (6)*. As far as its theoretical implications are concerned, however, this "sequential decision" problem can equally be rewritten as a standard simultaneous decision problem (Ackerberg et al., 2015). For ease of exposition, I thus consider the simultaneous decision formulation throughout this section.*

A.1 Profit Maximization

In each sector $i \in \mathscr{N}$, for the equilibrium wage W^*, the material price index $P_i^{M^*}$ and for each firm's optimal quantity q_{ik}^*, there exists a pair of labor and material inputs that satisfies the following one-step profit maximization problem:

$$(\bar{\ell}_{ik}^*, \bar{m}_{ik}^*) \quad \in \quad \operatorname*{arg\,max}_{\ell_{ik}, m_{ik}} \left\{ p_{ik}^* q_{ik}^* - (W^* \ell_{ik} + P_i^{M^*} m_{ik}) \right\} \qquad s.t. \qquad q_{ik}^* = f_i(\ell_{ik}, m_{ik}; z_{ik}).$$

The first order conditions with respect to labor and material inputs, respectively, are given by

$$[\ell_{ik}] : mr_{ik}(\cdot)^* \frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}} = W^* \tag{26}$$

$$[m_{ik}] : mr_{ik}(\cdot)^* \frac{\partial f_i(\cdot)^*}{\partial m_{ik}} = P_i^{M*}, \tag{27}$$

where $mr_{ik}(\mathbf{q}_i)$ is firm k's marginal revenue function at $\mathbf{q}_i := \{q_{ik'}\}_{k'=1}^{N_i}$, and I denote $mr_{ik}(\cdot)^* := mr_{ik}(\mathbf{q}_i^*)$. Moreover, define $\frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}} := \frac{\partial f_i(\cdot)}{\partial \ell_{ik}}\Big|_{(\ell_{ik}, m_{ik})=(\bar{\ell}_{ik}^*, \bar{m}_{ik}^*)}$, and $\frac{\partial f_i(\cdot)^*}{\partial m_{ik}} := \frac{\partial f_i(\cdot)}{\partial m_{ik}}\Big|_{(\ell_{ik}, m_{ik})=(\bar{\ell}_{ik}^*, \bar{m}_{ik}^*)}$. Likewise, $\frac{\partial mr_{ik}(\cdot)^*}{\partial q_{ik}} := \frac{\partial mr_{ik}(\cdot)}{\partial q_{ik'}}\Big|_{\mathbf{q}_i=\mathbf{q}_i^*}$. Taking total derivatives of the both hand sides of (26) and (27) in terms of τ_n, respectively, yields

$$\left(\sum_{k'=1}^{N_i} \frac{\partial mr_{ik}(\cdot)^*}{\partial q_{ik'}} \frac{dq_{ik'}^*}{d\tau_n}\right) \frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}} + mr_{ik}(\cdot)^* \left(\frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik}^2} \frac{d\bar{\ell}_{ik}^*}{d\tau_n} + \frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik} \partial m_{ik}} \frac{d\bar{m}_{ik}^*}{d\tau_n}\right) = \frac{dW^*}{d\tau_n} \tag{28}$$

$$\left(\sum_{k'=1}^{N_i} \frac{\partial mr_{ik}(\cdot)^*}{\partial q_{ik'}} \frac{dq_{ik'}^*}{d\tau_n}\right) \frac{\partial f_i(\cdot)^*}{\partial m_{ik}} + mr_{ik}(\cdot)^* \left(\frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik} m_{ik}} \frac{d\bar{\ell}_{ik}^*}{d\tau_n} + \frac{\partial^2 f_i(\cdot)^*}{\partial m_{ik}^2} \frac{d\bar{m}_{ik}^*}{d\tau_n}\right) = \frac{dP_i^{M*}}{d\tau_n}, \tag{29}$$

where

$$\frac{dq_{ik}^*}{d\tau_n} = \frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}} \frac{d\bar{\ell}_{ik}^*}{d\tau_n} + \frac{\partial f_i(\cdot)^*}{\partial m_{ik}} \frac{d\bar{m}_{ik}^*}{d\tau_n}.$$

Remark A.2. *Here, remember that firms only choose their output quantities through profit maximization, while input decisions are made in a way that minimizes total costs. Thus, the "optimal" quantities of labor $\bar{\ell}_{ik}^*$ and material $\bar{m}_{ik}^*$ inputs chosen above are not necessarily the same as the ones that are actually chosen by the firms. Rather, $\bar{\ell}_{ik}^*$ and $\bar{m}_{ik}^*$ should be understood as a combination of input variables that only pins down the change in the firm's output quantity, whose corresponding production possibility frontier is in turn used to determine the optimal input choice in the subsequent cost minimization problem (see also Remark A.7 in Appendix A.2).*

From (28) and (29), it follows that, in equilibrium,

$$
\begin{aligned}
&\left(\sum_{k'=1}^{N_i} \frac{\partial mr_{ik}(\cdot)^*}{\partial q_{ik'}} \frac{dq^*_{ik'}}{d\tau_n}\right)\left(\frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}}\bar{\ell}^*_{ik} + \frac{\partial f_i(\cdot)^*}{\partial m_{ik}}\bar{m}^*_{ik}\right) \\
&+ mr_{ik}(\cdot)^*\left(\frac{\partial^2 f_i(\cdot)^*}{\partial \ell^2_{ik}}\bar{\ell}^*_{ik} + \frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik}\partial m_{ik}}\bar{m}^*_{ik}\right)\frac{d\bar{\ell}^*_{ik}}{d\tau_n} + mr_{ik}(\cdot)^*\left(\frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik}\partial m_{ik}}\bar{\ell}^*_{ik} + \frac{\partial^2 f_i(\cdot)^*}{\partial m^2_{ik}}\bar{m}^*_{ik}\right)\frac{d\bar{m}^*_{ik}}{d\tau_n} \\
&= \frac{dW^*}{d\tau_n}\bar{\ell}^*_{ik} + \frac{dP^{M*}_i}{d\tau_n}\bar{m}^*_{ik} \\
&\therefore \sum_{k'=1}^{N_i} \frac{\partial mr_{ik}(\cdot)^*}{\partial q_{ik'}} \frac{dq^*_{ik'}}{d\tau_n} = \frac{1}{q^*_{ik}}\left(\frac{dW^*}{d\tau_n}\bar{\ell}^*_{ik} + \frac{dP^{M*}_i}{d\tau_n}\bar{m}^*_{ik}\right), \qquad (30)
\end{aligned}
$$

where the implication is a consequence of Assumption 2.4 (i). The expression (30) is true for each firm $k \in \mathscr{N}_i$ in the same sector i, and thus constitutes a system of N_i equations. This system of equations can be summarized in the following matrix form:

$$
\underbrace{\begin{bmatrix}
\frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i1}} & \frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i2}} & \cdots & \frac{\partial mr_{i1}(\cdot)^*}{\partial q_{iN_i}} \\
\frac{\partial mr_{i2}(\cdot)^*}{\partial q_{i1}} & \frac{\partial mr_{i2}(\cdot)^*}{\partial q_{i2}} & \cdots & \frac{\partial mr_{i2}(\cdot)^*}{\partial q_{iN_i}} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\partial mr_{iN_i}(\cdot)^*}{\partial q_{i1}} & \frac{\partial mr_{iN_i}(\cdot)^*}{\partial q_{i2}} & \cdots & \frac{\partial mr_{iN_i}(\cdot)^*}{\partial q_{iN_i}}
\end{bmatrix}}_{=:\Lambda_{i,1}}
\begin{bmatrix}
\frac{dq^*_{i1}}{d\tau_n} \\ \frac{dq^*_{i2}}{d\tau_n} \\ \vdots \\ \frac{dq^*_{iN_i}}{d\tau_n}
\end{bmatrix}
=
\underbrace{\begin{bmatrix}
\frac{\bar{\ell}^*_{i1}}{q^*_{i1}} & \frac{\bar{m}^*_{i1}}{q^*_{i1}} \\
\frac{\bar{\ell}^*_{i2}}{q^*_{i2}} & \frac{\bar{m}^*_{i2}}{q^*_{i2}} \\
\vdots & \vdots \\
\frac{\bar{\ell}^*_{iN_i}}{q^*_{iN_i}} & \frac{\bar{m}^*_{iN_i}}{q^*_{iN_i}}
\end{bmatrix}}_{=:\Lambda_{i,2}}
\begin{bmatrix}
\frac{dW^*}{d\tau_n} \\ \frac{dP^{M*}_i}{d\tau_n}
\end{bmatrix}. \qquad (31)
$$

In order to ensure that this system generates a unique set of firms' quantity changes in response to a shift in subsidy, I impose the following regularity condition.

Assumption A.1 (Regularity Condition 1)**.** *For each sector $i \in \mathscr{N}$, the matrix*

$$
\Lambda_{i,1} := \begin{bmatrix}
\frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i1}} & \frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i2}} & \cdots & \frac{\partial mr_{i1}(\cdot)^*}{\partial q_{iN_i}} \\
\frac{\partial mr_{i2}(\cdot)^*}{\partial q_{i1}} & \frac{\partial mr_{i2}(\cdot)^*}{\partial q_{i2}} & \cdots & \frac{\partial mr_{i2}(\cdot)^*}{\partial q_{iN_i}} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\partial mr_{iN_i}(\cdot)^*}{\partial q_{i1}} & \frac{\partial mr_{iN_i}(\cdot)^*}{\partial q_{i2}} & \cdots & \frac{\partial mr_{iN_i}(\cdot)^*}{\partial q_{iN_i}}
\end{bmatrix}
$$

is nonsingular.

Remark A.3. *Assumption A.1 requires that the column vectors of $\Lambda_{i,1}$ are linearly independent and guarantees the premultiplying term on the left-hand side of* (31) *is invertible. This assumption trivially holds in monopolistic competition, wherein the matrix $\Lambda_{i,1}$ simplifies to a diagonal matrix.*

Remark A.4. *Combined with the set of identifying assumptions proposed in this paper,* *Assumption A.1 is testable. Developing a formal statistical inference, however, is outside the scope of this paper. See Appendix F for a discussion.*

Note here that under the setup in Section 2, the firms' marginal costs are constant, and thus it holds

$$\frac{\partial mr_{ik}(\cdot)}{\partial q_{ik'}} = \frac{\partial \frac{\partial \pi_{ik}(\cdot)}{\partial q_{ik}}}{\partial q_{ik'}}.$$

In light of this, the economic content of Assumption A.1 can be envisioned in terms of firms' strategic complementarities.

Example A.1 (Duopoly). *For simplicity, consider a case of duopoly, in which firm* 1 *and* 2 *engage in quantity competition. It generally holds that* $\left|\frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i1}}\right| \geq \left|\frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i2}}\right|$. *But, it is also true that* $\left|\frac{\partial mr_{i2}(\cdot)^*}{\partial q_{i1}}\right| \leq \left|\frac{\partial mr_{i2}(\cdot)^*}{\partial q_{i2}}\right|$. *Hence, there is no such a constant that makes the column vectors* $\Lambda_{i,1}$ *linearly dependent. In this sense, Assumption A.1 excludes the situation where the firm's own strategic complementarity is exactly the same as the competitor's. See also Appendix A.4.2.*

Under Assumption A.1, the system of equations (31) can be solved for $\{\frac{dq_{ik}^*}{d\tau_n}\}_{k=1}^{N_i}$:

$$\begin{bmatrix} \frac{dq_{i1}^*}{d\tau_n} \\ \frac{dq_{i2}^*}{d\tau_n} \\ \vdots \\ \frac{dq_{iN_i}^*}{d\tau_n} \end{bmatrix} = \Lambda_{i,1}^{-1}\Lambda_{i,2} \begin{bmatrix} \frac{dW^*}{d\tau_n} \\ \frac{dP_i^{M*}}{d\tau_n} \end{bmatrix}.$$

In this expression, $\Lambda_{i,1}^{-1}$ captures firms' strategic interactions in terms of their strategic complementarities. Moreover, it can also be seen that $\{\frac{dq_{ik}^*}{d\tau_n}\}_{k=1}^{N_i}$ depend on the levels of firm's current input and output variables through $\Lambda_{i,2}$ as well as the responsivenesses of the wage and material cost index with respect to a subsidy change.

Letting $\lambda_{ik,k'}^{-1}$ be the (k, k') entry of the matrix $\Lambda_{i,1}^{-1}$, I obtain

$$\begin{aligned} \frac{dq_{ik}^*}{d\tau_n} &= \left(\sum_{k'=1}^{N_i} \lambda_{ik,k'}^{-1} \frac{\bar{\ell}_{ik'}^*}{q_{ik'}^*}\right)\frac{dW^*}{d\tau_n} + \left(\sum_{k'=1}^{N_i} \lambda_{ik,k'}^{-1} \frac{\bar{m}_{ik'}^*}{q_{ik'}^*}\right)\frac{dP_i^{M*}}{d\tau_n} \\ &= \bar{\lambda}_{ik}^L \frac{dW^*}{d\tau_n} + \bar{\lambda}_{ik}^M \frac{dP_i^{M*}}{d\tau_n}, \end{aligned} \tag{32}$$

where $\bar{\lambda}_{ik}^L := \sum_{k'=1}^{N_i} \lambda_{ik,k'}^{-1} \frac{\bar{\ell}_{ik'}^*}{q_{ik'}^*}$ and $\bar{\lambda}_{ik}^M := \sum_{k'=1}^{N_i} \lambda_{ik,k'}^{-1} \frac{\bar{m}_{ik'}^*}{q_{ik'}^*}$ correspond to the kth element of the first and second column of the matrix $\Lambda_{i,1}^{-1}\Lambda_{i,2}$, respectively. In (32), the weighted sums

$\bar{\lambda}^L_{ik}$ and $\bar{\lambda}^M_{ik}$ dictate the comovements, respectively, between the change in firm-level output quantity and the change in wage, and between the change in firm-level output quantity and the change in the sectoral material cost index.[97]

Notice that the denominator of $\bar{\lambda}^L_{ik}$ includes all of $\{\frac{\partial mr_{ik}(\cdot)}{\partial q_{ik'}}\}_{k,k' \in \mathcal{N}_i}$ and thus can be viewed as a sectoral measure of strategic complementarity. Note also that the numerator does not contain the terms $\{\frac{\partial mr_{ik'}(\cdot)}{\partial q_{ik}}\}_{k \in \mathcal{N}_i}$. Hence, the ratio $\bar{\lambda}^L_{ik}$ backs out the contribution of changes in q_{ik}.[98] The same is true for $\bar{\lambda}^M_{ik}$. These indices are informative about the extent to which the market competition is affected by the change in the firm k's output quantity and are similar in spirit to the index of competitor price changes of Amiti et al. (2019).[99] This observation can clearly be seen in the following examples of duopoly and monopolistic competition.

Example A.2 (Duopoly)**.** *Continuing the same setup as in Example A.1, the inverse matrix* $\Lambda^{-1}_{i,1}$ *is given by:*

$$\Lambda^{-1}_{i,1} = \frac{1}{det(\Lambda_{i,1})} \begin{bmatrix} \frac{\partial mr_{i2}(\cdot)^*}{\partial q_{i2}} & -\frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i2}} \\ -\frac{\partial mr_{i2}(\cdot)^*}{\partial q_{i1}} & \frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i1}} \end{bmatrix}$$

where $det(\Lambda_{i,1}) = \frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i1}} \frac{\partial mr_{i2}(\cdot)^*}{\partial q_{i2}} - \frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i2}} \frac{\partial mr_{i2}(\cdot)^*}{\partial q_{i1}}$. *Note first that the denominator on the right-hand side (i.e.,* $det(\Lambda_{i,1})$*) involves every element of* $\Lambda_{i,1}$*, and thus can be viewed as a measure of the sector's overall strategic complementarity.*[100] *Next, each of the first row of the numerators (i.e.,* $\frac{\partial mr_{i2}(\cdot)^*}{\partial q_{i2}}$ *and* $-\frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i2}}$*) represents the strategic complementarity with respect to the firm 2's quantity adjustment. Divided by* $det(\Lambda_{i,1})$ *and summed over columns with the weights, the indices* $\bar{\lambda}^L_{i1}$ *and* $\bar{\lambda}^M_{i1}$ *back out the contribution of the firm 1's quantity change to the sector's overall strategic complementarity. See also Appendix A.4.2.*

[97]The weights $\frac{\bar{\ell}^*_{ik}}{q^*_{ik}}$ and $\frac{\bar{m}^*_{ik}}{q^*_{ik}}$ can be interpreted as crude measures of the firm k's labor and material productivity, respectively. Note that these weights are not normalized to equal one.

[98]To see this, observe that for a square matrix $\mathcal{O}$, the inverse matrix $\mathcal{O}^{-1}$ is given by $\mathcal{O}^{-1} = \frac{\text{adj}(\mathcal{O})}{|\mathcal{O}|}$, where $\text{adj}(\mathcal{O})$ is the adjoint matrix of $\mathcal{O}$, i.e., the transpose of the cofactor matrix. The cofactor matrix C of $\mathcal{O}$ is defined as $C := [c_{a,b}]_{a,b}$, where $c_{a,b} := (-1)^{a+b}|M_{a,b}|$, with $M_{a,b}$ representing the minor matrix of $\mathcal{O}$ that can be created by eliminating the ath row and bth column from the matrix $\mathcal{O}$. In my context, the k'th column of the cofactor matrix of $\Lambda_{i,1}$ excludes $\{\frac{\partial mr_{ik}(\cdot)^*}{\partial q_{ik'}}\}_{k=1}^{N_i}$, all of which are in turn ruled out from the k'th row of the adjoint matrix. Since the determinant involves the impact of all firms' quantity responses, the weighted sum along each row of $\Lambda^{-1}_{i,1}$ reflects the contribution of the changes in firm k''s output quantity.

[99]While their index compares the firm's contribution to the rest of the market, my indices $\bar{\lambda}^L_{ik}$ and $\bar{\lambda}^M_{ik}$ compare the rest of the market to the entire market, backing out the firm's contribution.

[100]In general, the determinant of a 2×2 matrix gives the (signed) area of a parallelogram spanned by its column vectors. In the case of $\Lambda_{i,1}$, the column vectors consist of the partial derivatives of the firm's marginal revenues with respect to each firm. Thus, $det(\Lambda_{i,1})$ is a natural measure that summarizes firms' contributions to the overall strategic complementarity. (Without loss of generality, the sign of the determinant can be assumed to be positive, as it can be reversed through swapping some of the column vectors.) Moreover, it is a mapping of the overall strategic substitutability/complementarity from $(-\infty, \infty)$ to $[0, \infty)$, acting as a normalization constant.

Example A.3 (Monopolistic Competition)**.** *I consider the same setup as in Example A.1, but depart by assuming that both firms are monopolistic. In this case,*

$$\Lambda_{i,1}^{-1} = \begin{bmatrix} (\frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i1}})^{-1} & 0 \\ 0 & (\frac{\partial mr_{i2}(\cdot)^*}{\partial q_{i2}})^{-1} \end{bmatrix}.$$

Then, the two measures of the firm 1's contribution to the overall sectoral strategic complementarity are given by $\bar{\lambda}_{i1}^L = (\frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i1}})^{-1}\frac{\bar{\ell}_{i1}^*}{q_{i1}^*}$ *and* $\bar{\lambda}_{i1}^M = (\frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i1}})^{-1}\frac{\bar{m}_{i1}^*}{q_{i1}^*}$*, both of which are typically negative.*[101] *Provided that both* $\bar{\lambda}_{i1}^L$ *and* $\bar{\lambda}_{i1}^M$ *are negative,* (32) *implies that when the wage and material cost index become higher in reaction to a policy change, firm 1 decreases its output quantity. An analogous argument applies to firm 2. When the firms are oligopolistic as in Example A.2, the signs of* $\bar{\lambda}_{i1}^L$ *and* $\bar{\lambda}_{i1}^M$ *are ambiguous because they involve strategic complementarities. See Section 2.7 and Appendix A.4*

In equilibrium, the sectoral price index associated with the sectoral aggregator (3) satisfies the following unit cost condition: for each $i = 1, \ldots, N$,

$$P_i^* \quad = \quad \min_{\{e_{ik}\}_{i=1}^{N}} \sum_{k=1}^{N_i} p_{ik}^* e_{ik} \qquad s.t. \qquad F_i(\{e_{ik}\}_{k=1}^{N_i}) \geq 1, \tag{33}$$

where p_{ik}^* is the equilibrium price of a product set by firm k in sector i. By solving this, it follows that there exists a mapping $\mathcal{P}_i : \mathscr{S}_i^{N_i} \to \mathbb{R}_+$ such that

$$P_i^* = \mathcal{P}_i(\mathbf{q}_i^*). \tag{34}$$

Totally differentiating (34) yields

$$\frac{dP_i^*}{d\tau_n} = \sum_{k'=1}^{N_i} \frac{\partial \mathcal{P}_i(\cdot)^*}{\partial q_{ik'}} \frac{dq_{ik'}^*}{d\tau_n}, \tag{35}$$

where $\frac{\partial \mathcal{P}_i(\cdot)^*}{\partial q_{ik'}} := \frac{\partial \mathcal{P}_i(\cdot)}{\partial q_{ik'}}\Big|_{\mathbf{q}_i=\mathbf{q}_i^*}$

Remark A.5. *(i) Associated with* (33) *is the (residual) inverse demand function* $\wp_{ik}(\cdot)$ *such that* $p_{ik} = \wp_{ik}(\mathbf{q}_i^*)$*. By the chain rule, it holds that*

$$\frac{dp_{ik}^*}{d\tau_n} = \sum_{k'=1}^{N_i} \frac{\partial \wp_{ik}(\cdot)^*}{\partial q_{ik'}} \frac{dq_{ik'}^*}{d\tau_n}, \tag{36}$$

[101]Precisely speaking, the sign depends on the demand-side parameters. For instance, when the sectoral aggregator takes the form of a constant-elasticity-of-substitution (CES) production function as in Example C.1, these indices are negative as long as $\sigma_i > 2$.

where $\frac{\partial \wp_{ik}(\cdot)^*}{\partial q_{ik'}} := \frac{\partial \wp_{ik}(\cdot)}{\partial q_{ik'}}\Big|_{\mathbf{q}_i=\mathbf{q}_i^*}$. *Substituting* (32) *for* $\frac{dq^*_{ik'}}{d\tau_n}$ *leads to*

$$\frac{dp^*_{ik}}{d\tau_n} = \left(\sum_{k'=1}^{N_i} \frac{\partial \wp_{ik}(\cdot)^*}{\partial q_{ik'}} \bar{\lambda}^L_{ik'}\right) \frac{dW^*}{d\tau_n} + \left(\sum_{k'=1}^{N_i} \frac{\partial \wp_{ik}(\cdot)^*}{\partial q_{ik'}} \bar{\lambda}^M_{ik'}\right) \frac{dP^{M*}_i}{d\tau_n}. \tag{37}$$

(ii) An expression analogous to (35) *can be derived with respect to the firms' prices* $\{p_{ik'}\}_{k'=1}^{N_i}$. *With a slight abuse of notation, let* $\mathcal{P}_i(\mathbf{p}_i)$ *be a function such that* $P_i = \mathcal{P}_i(\mathbf{p}_i)$. *Then, a version of* (35) *is given by*

$$\frac{dP^*_i}{d\tau_n} = \sum_{k'=1}^{N_i} \frac{\partial \mathcal{P}_i(\cdot)^*}{\partial p_{ik'}} \frac{dp_{ik'}}{d\tau_n}. \tag{38}$$

Upon substituting (32) into (35), it holds that

$$\begin{aligned} \frac{dP^*_i}{d\tau_n} &= \sum_{k'=1}^{N_i} \frac{\partial \mathcal{P}_i(\cdot)^*}{\partial q_{ik'}} \left(\bar{\lambda}^L_{ik'} \frac{dW^*}{d\tau_n} + \bar{\lambda}^M_{ik'} \frac{dP^{M*}_i}{d\tau_n}\right) \\ &= \bar{\lambda}^L_{i\cdot} \frac{dW^*}{d\tau_n} + \bar{\lambda}^M_{i\cdot} \frac{dP^{M*}_i}{d\tau_n}, \end{aligned} \tag{39}$$

where $\bar{\lambda}^L_{i\cdot} := \sum_{k'=1}^{N_i} \frac{\partial \mathcal{P}_i(\cdot)^*}{\partial q_{ik'}} \bar{\lambda}^L_{ik'}$ and $\bar{\lambda}^M_{i\cdot} := \sum_{k'=1}^{N_i} \frac{\partial \mathcal{P}_i(\cdot)^*}{\partial q_{ik'}} \bar{\lambda}^M_{ik'}$. These are a weighted sum of the elasticities of the sectoral price index with respect to firms' quantities, with weights assigned to firms' contributions to the overall measure of sectoral strategic complementarity.[102] From the expression (39), $\bar{\lambda}^L_{i\cdot}$ and $\bar{\lambda}^M_{i\cdot}$ can be interpreted as representing a pass-through of a change in the wage and material input cost to the sectoral price index, respectively.

Example A.4 (Monopolistic Competition)**.** *Continuing Example A.3 and assuming that* $\bar{\lambda}^L_{i1}$, $\bar{\lambda}^L_{i2}$, $\bar{\lambda}^M_{i1}$ *and* $\bar{\lambda}^M_{i2}$ *have all turned out to be negative, I can proceed to calculate* $\bar{\lambda}^L_{i\cdot}$ *and* $\bar{\lambda}^M_{i\cdot}$. *Due to the law of demand (i.e.,* $\frac{\partial \mathcal{P}_i(\cdot)^*}{\partial q_{ik'}} < 0$ *for all* $k' \in \mathcal{N}_i$*), these are both positive. In light of* (39)*, this, in turn, implies a higher sectoral price index in response to increases in wage and material cost indices, which accord with lower output quantities, as seen in Example A.3.*

Meanwhile, the equilibrium material cost index P^{M*}_i satisfies the following unit cost condition:

$$P^{M*}_i \quad = \quad \min_{\{m_{ik,j}\}_{j\in\mathcal{N}}} \sum_{j=1}^{N} (1-\tau_i) P^*_j m_{ik,j} \qquad s.t. \qquad \mathcal{G}_i(\{m_{ik,j}\}_{j=1}^N) \geq 1,$$

[102] Alternatively, it can be viewed as a weighted sum of the firms' contributions to the overall measure of sectoral strategic complementarity, with weights assigned to the elasticities of the sectoral price index with respect to firms' quantities.

from which I can write P_i^{M*} as a function of the sectoral price indices and the sector-specific subsidy, i.e.,

$$P_i^{M*} = \mathcal{P}_i^M(\{P_j^*\}_{j=1}^N, \tau_i). \tag{40}$$

Note that the function $\mathcal{P}_i^M(\cdot)$ encodes the information about the production network that is carried over from the aggregator $\mathcal{G}_i(\cdot)$.

Taking total derivatives of (40), it holds that

$$\frac{dP_i^{M*}}{d\tau_n} = \sum_{j=1}^N \frac{\partial \mathcal{P}_i^M(\cdot)^*}{\partial P_j} \frac{dP_j^*}{d\tau_n} + \frac{\partial \mathcal{P}_i^M(\cdot)}{\partial \tau_n} \mathbb{1}_{\{n=i\}}, \tag{41}$$

where $\mathbb{1}_{\{n=i\}}$ takes one if $n = i$, and zero otherwise. Substituting (39) for $\{\frac{dP_j^*}{d\tau_n}\}_{j=1}^N$ into (41), I obtain

$$\frac{dP_i^{M*}}{d\tau_n} = \left(\sum_{j=1}^N \frac{\partial \mathcal{P}_i^M(\cdot)^*}{\partial P_j} \bar{\lambda}_{j\cdot}^L\right) \frac{dW^*}{d\tau_n} + \sum_{j=1}^N \frac{\partial \mathcal{P}_i^M(\cdot)^*}{\partial P_j} \bar{\lambda}_{j\cdot}^M \frac{dP_j^{M*}}{d\tau_n} + \frac{\partial \mathcal{P}_i^M(\cdot)^*}{\partial \tau_n} \mathbb{1}_{\{n=i\}}. \tag{42}$$

The equation (42) holds true for all sectors, constituting a system of equations (i.e., simultaneous/structural equations). The next step is to solve these equations for comparative statics, or to derive "reduced-form" equations. Denoting $\Gamma_1 := \left[\frac{\partial \mathcal{P}_i^M(\cdot)^*}{\partial P_j} \bar{\lambda}_{j\cdot}^L\right]_{i,j=1}^N$ and $\Gamma_2 := \left[\frac{\partial \mathcal{P}_i^M(\cdot)^*}{\partial P_j} \bar{\lambda}_{j\cdot}^M\right]_{i,j=1}^N$, and letting $\iota := [1, 1, \ldots, 1]'$ be a $N \times 1$ vector of ones, I stack (42) over sectors to obtain the following system of equations:

$$\begin{aligned} \begin{bmatrix} \frac{dP_1^{M*}}{d\tau_n} \\ \vdots \\ \frac{dP_N^{M*}}{d\tau_n} \end{bmatrix} &= \Gamma_1 \iota \frac{dW^*}{d\tau_n} + \Gamma_2 \begin{bmatrix} \frac{dP_1^{M*}}{d\tau_n} \\ \vdots \\ \frac{dP_N^{M*}}{d\tau_n} \end{bmatrix} + \begin{bmatrix} \frac{\partial \mathcal{P}_1^M(\cdot)^*}{\partial \tau_n} \mathbb{1}_{\{n=1\}} \\ \vdots \\ \frac{\partial \mathcal{P}_N^M(\cdot)^*}{\partial \tau_n} \mathbb{1}_{\{n=N\}} \end{bmatrix} \\ \therefore (I - \Gamma_2) \begin{bmatrix} \frac{dP_1^{M*}}{d\tau_n} \\ \vdots \\ \frac{dP_N^{M*}}{d\tau_n} \end{bmatrix} &= \Gamma_1 \iota \frac{dW^*}{d\tau_n} + \begin{bmatrix} \frac{\partial \mathcal{P}_1^M(\cdot)^*}{\partial \tau_n} \mathbb{1}_{\{n=1\}} \\ \vdots \\ \frac{\partial \mathcal{P}_N^M(\cdot)^*}{\partial \tau_n} \mathbb{1}_{\{n=N\}} \end{bmatrix} \end{aligned} \tag{43}$$

where I represents an $N \times N$ identity matrix.

To ensure a unique solution, I impose the following regularity condition.

Assumption A.2 (Regularity Condition 2)**.** *The matrix* $(I - \Gamma_2)$ *is nonsingular.*

Remark A.6. *Combined with the set of identifying assumptions proposed in this paper, Assumption A.2 is testable. Developing a formal statistical inference, however, is outside the* *scope of this paper. See Appendix F for a discussion.*

This assumption guarantees that the premultiplying term in (43) is invertible. Under Assumption A.2, it thus follows that

$$\begin{bmatrix} \frac{dP_1^{M*}}{d\tau_n} \\ \vdots \\ \frac{dP_N^{M*}}{d\tau_n} \end{bmatrix} = (I - \Gamma_2)^{-1}\Gamma_1 \iota \frac{dW^*}{d\tau_n} + (I - \Gamma_2)^{-1} \begin{bmatrix} \frac{\partial \mathcal{P}_1^M(\cdot)^*}{\partial \tau_n} \mathbb{1}_{\{n=1\}} \\ \vdots \\ \frac{\partial \mathcal{P}_N^M(\cdot)^*}{\partial \tau_n} \mathbb{1}_{\{n=N\}} \end{bmatrix}. \tag{44}$$

Observe here that Γ_2 is a version of the adjacency matrix Ω. Hence, $(I - \Gamma_2)^{-1}$ can be conceived as a type of the Leontief inverse matrix, augmented by measures of strategic competition in the source sectors $\bar{\lambda}_{j\cdot}^M$ (i.e., market distortion). The (i, n) entry of this strategic-complementarity-adjusted Leontief inverse, denoted by $h_{i,n}^M$, can be written as a geometric sum:[103] if $n \neq i$,

$$\bar{\lambda}_{n\cdot}^M \frac{\partial \mathcal{P}_i^M(\cdot)^*}{\partial P_n} + \sum_{j=1}^N \bar{\lambda}_{n\cdot}^M \frac{\partial \mathcal{P}_i^M(\cdot)^*}{\partial P_j} \bar{\lambda}_{j\cdot}^M \frac{\partial \mathcal{P}_j^M(\cdot)^*}{\partial P_n} + \sum_{j=1}^N \sum_{j'=1}^N \bar{\lambda}_{n\cdot}^M \frac{\partial \mathcal{P}_j^M(\cdot)^*}{\partial P_n} \bar{\lambda}_{j\cdot}^M \frac{\partial \mathcal{P}_{j'}^M(\cdot)^*}{\partial P_j} \bar{\lambda}_{j'\cdot}^M \frac{\partial \mathcal{P}_i^M(\cdot)^*}{\partial P_{j'}} + \dots, \tag{45}$$

and if $n = i$,

$$1 + \bar{\lambda}_{n\cdot}^M \frac{\partial \mathcal{P}_i^M(\cdot)^*}{\partial P_n} + \sum_{j=1}^N \bar{\lambda}_{n\cdot}^M \frac{\partial \mathcal{P}_i^M(\cdot)^*}{\partial P_j} \bar{\lambda}_{j\cdot}^M \frac{\partial \mathcal{P}_j^M(\cdot)^*}{\partial P_n} + \sum_{j=1}^N \sum_{j'=1}^N \bar{\lambda}_{n\cdot}^M \frac{\partial \mathcal{P}_j^M(\cdot)^*}{\partial P_n} \bar{\lambda}_{j\cdot}^M \frac{\partial \mathcal{P}_{j'}^M(\cdot)^*}{\partial P_j} \bar{\lambda}_{j'\cdot}^M \frac{\partial \mathcal{P}_i^M(\cdot)^*}{\partial P_{j'}} + \dots. \tag{46}$$

To gain some intuition for this infinite-sum expression, suppose that sector i purchases intermediate good from sector n ($n \neq i$), both directly and indirectly, along the production network. For the sake of simplicity, assume in addition that $\bar{\lambda}_{j\cdot}^M > 0$ for all $j \in \mathcal{N}$. When sector n is subsidized, the reduced input cost stimulates production in that sector, leading to a lower sectoral output price index of sector n according to (39), with the pass-through ratio being given by $\bar{\lambda}_{n\cdot}^M$. This change in the sector n's output price index affects the sector i's cost index through multiple channels. The first term in (45) represents the first-order spillover effect: A decrease in the sector n's output price index directly reduces the sector i's material input cost. The second term captures the second-order spillover effect coming via a third sector j. The output price index of sector j decreases as the firms in sector j can produce more of their goods by taking advantage of cheaper input costs. This effect

[103] For any square matrix A, the corresponding Leontief inverse matrix, if exists, can be written as $(I - A)^{-1} = \sum_{m=0}^{\infty} A^m$, where I define $A^0 = I$.

is captured by $\bar{\lambda}^M_{j\cdot}$. This chain of reductions in input cost takes place along the network. I refer to this comovement of sectoral variables reflected in $h^M_{i,n}$ as *macro complementarity* with respect to material input. In general, however, the sign and magnitude of the macro complementarity are ambiguous, because they are mediated by the source sector's overall strategic complementarities, encoded in $\bar{\lambda}^M_{j\cdot}$, which I call *micro complementarity* with respect to material input. Similarly, $\bar{\lambda}^L_{i\cdot}$ is referred to as the sector i's micro complementarity with respect to labor input. Also, h^L_i is defined as the ith entry of $(I-\Gamma_2)^{-1}\Gamma_1\iota$ in (44) and called the sector i's macro complemetarity with respect to labor input.

A.2 Cost Minimization 1: Input Decision

In equilibrium, firm k in sector i chooses labor and material inputs according to the following constrained cost-minimization problem:[104]

$$(\ell^*_{ik}, m^*_{ik}) \in \underset{\{\ell_{ik}, m_{ik}\}}{\arg\min}\, W^*\ell_{ik} + P_i^{M^*} m_{ik} \quad s.t. \quad f_i(\ell_{ik}, m_{ik}; z_{ik}) \geq q^*_{ik}. \tag{47}$$

The associated Lagrange function is

$$\mathcal{L}_i(\ell_{ik}, m_{ik}, \xi_{ik}) \coloneqq W^*\ell_{ik} + P_i^{M^*} m_{ik} - \xi_{ik}\Big(f_i(\ell_{ik}, m_{ik}; z_{ik}) - q^*_{ik}\Big).$$

In equilibrium, the following first-order conditions are satisfied at $(\ell_{ik}, m_{ik}) = (\ell^*_{ik}, m^*_{ik})$:

$$[\ell_{ik}] : W^* = \xi^*_{ik}\frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}}$$
$$[m_{ik}] : P_i^{M^*} = \xi^*_{ik}\frac{\partial f_i(\cdot)^*}{\partial m_{ik}}$$
$$[\xi_{ik}] : f_i(\ell^*_{ik}, m^*_{ik}; z_{ik}) = q^*_{ik},$$

where ξ^*_{ik} is the marginal cost of production at the given quantity q^*_{ik}. Note that under Assumption 2.4 (i), ξ^*_{ik} equals the average cost, i.e., $\xi^*_{ik} = \frac{TC^*_{ik}}{q^*_{ik}}$ where $TC^*_{ik} \coloneqq TC_{ik}(W, P_i^M, q_{ik})\big|_{(W,P_i^M,q_{ik})=(W^*,P}$ with $TC_{ik}(\cdot)$ denoting firm k's total cost function (see Fact C.3).

Remark A.7. *Two sets of "optimal" labor and material inputs* $(\bar{\ell}^*_{ik}, \bar{m}^*_{ik})$ *and* (ℓ^*_{ik}, m^*_{ik}) *need to be distinguished. They reside on the same production possibility frontier, but do not necessarily coincide. It is the latter that minimizes the total cost of producing* q^*_{ik}.

In addition to Assumption 2.4, the firm-level production function $f_i(\cdot)$ satisfies the following assumption:

[104]See Remark A.1.

Assumption A.3. *For each sector $i \in \mathscr{N}$ each firm $k \in \mathscr{N}_i$, it holds that* $\left(\frac{\partial f_i(\cdot)}{\partial \ell_{ik}}\right)^2 \frac{\partial^2 f_i(\cdot)}{\partial m_{ik}^2} + \left(\frac{\partial f_i(\cdot)}{\partial m_{ik}}\right)^2 \frac{\partial^2 f_i(\cdot)}{\partial \ell_{ik}^2} - 2\frac{\partial f_i(\cdot)}{\partial \ell_{ik}} \frac{\partial f_i(\cdot)}{\partial m_{ik}} \frac{\partial^2 f_i(\cdot)}{\partial \ell_{ik}\partial m_{ik}} < 0$ *for all* $(\ell_{ik}, m_{ik}) \in \mathbb{R}^2_+$.

This assumption guarantees that the cost-minimization problem (47) has an interior solution. In what follows, I refer to Assumption A.3 as Assumption 2.4 (v).

Totally differentiating the first-order conditions yields

$$\frac{dW^*}{d\tau_n} = \frac{d\xi^*_{ik}}{d\tau_n}\frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}} + \xi^*_{ik}\left(\frac{\partial^2 f_i(\cdot)^*}{\partial \ell^2_{ik}}\frac{d\ell^*_{ik}}{d\tau_n} + \frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik}\partial m_{ik}}\frac{dm^*_{ik}}{d\tau_n}\right) \tag{48}$$

$$\frac{dP^{M*}_i}{d\tau_n} = \frac{d\xi^*_{ik}}{d\tau_n}\frac{\partial f_i(\cdot)^*}{\partial m_{ik}} + \xi^*_{ik}\left(\frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik} m_{ik}}\frac{d\ell^*_{ik}}{d\tau_n} + \frac{\partial^2 f_i(\cdot)^*}{\partial m^2_{ik}}\frac{dm^*_{ik}}{d\tau_n}\right) \tag{49}$$

$$\frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}}\frac{d\ell^*_{ik}}{d\tau_n} + \frac{\partial f_i(\cdot)^*}{\partial m_{ik}}\frac{dm^*_{ik}}{d\tau_n} = \frac{dq^*_{ik}}{d\tau_n}. \tag{50}$$

Notice that (50) dictates the changes of labor and material input along the new production possibility frontier induced by the change in output quantity.

Observe here that

$$\begin{aligned}\frac{d\xi^*_{ik}}{d\tau_n} &= \frac{1}{q^*_{ik}}\left(\frac{\partial TC_{ik}(\cdot)^*}{\partial W}\frac{dW^*}{d\tau_n} + \frac{\partial TC_{ik}(\cdot)^*}{\partial P^M_i}\frac{dP^{M*}_i}{d\tau_n} + \frac{\partial TC_{ik}(\cdot)^*}{\partial q_{ik}}\frac{dq^*_{ik}}{d\tau_n}\right) - \frac{1}{q^*_{ik}}\frac{TC^*_{ik}}{q^*_{ik}}\frac{dq^*_{ik}}{d\tau_n}\\ &= \frac{1}{q^*_{ik}}\left(\ell^*_{ik}\frac{dW^*}{d\tau_n} + m^*_{ik}\frac{dP^{M*}_i}{d\tau_n} + \xi^*_{ik}\frac{dq^*_{ik}}{d\tau_n}\right) - \frac{1}{q^*_{ik}}\xi^*_{ik}\frac{dq^*_{ik}}{d\tau_n}\\ &= \frac{\ell^*_{ik}}{q^*_{ik}}\frac{dW^*}{d\tau_n} + \frac{m^*_{ik}}{q^*_{ik}}\frac{dP^{M*}_i}{d\tau_n}.\end{aligned} \tag{51}$$

where the second equality is a consequence of Shephard's lemma and the fact that the marginal cost equals average cost under Assumption 2.4 (i). It then follows from (48) and (51) that

$$\xi^*_{ik}\frac{\partial^2 f_i(\cdot)^*}{\partial \ell^2_{ik}}\frac{d\ell^*_{ik}}{d\tau_n} + \xi^*_{ik}\frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik}\partial m_{ik}}\frac{dm^*_{ik}}{d\tau_n} = \left(1 - \frac{\ell^*_{ik}}{q^*_{ik}}\frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}}\right)\frac{dW^*}{d\tau_n} - \frac{m^*_{ik}}{q^*_{ik}}\frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}}\frac{dP^{M*}_i}{d\tau_n}. \tag{52}$$

Likewise, combining (49) and (51) leads to

$$\xi^*_{ik}\frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik}\partial m_{ik}}\frac{d\ell^*_{ik}}{d\tau_n} + \xi^*_{ik}\frac{\partial^2 f_i(\cdot)^*}{\partial m^2_{ik}}\frac{dm^*_{ik}}{d\tau_n} = -\frac{\ell^*_{ik}}{q^*_{ik}}\frac{\partial f_i(\cdot)^*}{\partial m_{ik}}\frac{dW^*}{d\tau_n} + \left(1 - \frac{m^*_{ik}}{q^*_{ik}}\frac{\partial f_i(\cdot)^*}{\partial m_{ik}}\right)\frac{dP^{M*}_i}{d\tau_n}. \tag{53}$$

Notice that under Assumption 2.4 (i), (52) and (53) are essentially identical. Hence, the first order conditions (48) – (50) can be summarized by (50) and (52) (or equivalently (50) and (53)), and thus can be compactly expressed as the following single equation:

$$\begin{bmatrix} \xi_{ik}^* \frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik}^2} & \xi_{ik}^* \frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik} \partial m_{ik}} \\ \frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}} & \frac{\partial f_i(\cdot)^*}{\partial m_{ik}} \end{bmatrix} \begin{bmatrix} \frac{d\ell_{ik}^*}{d\tau_n} \\ \frac{dm_{ik}^*}{d\tau_n} \end{bmatrix} = \begin{bmatrix} 1 - \frac{\ell_{ik}^*}{q_{ik}^*} \frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}} & -\frac{m_{ik}^*}{q_{ik}^*} \frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}} \\ \bar{\lambda}_{ik}^L & \bar{\lambda}_{ik}^M \end{bmatrix} \begin{bmatrix} \frac{dW^*}{d\tau_n} \\ \frac{dP_i^{M*}}{d\tau_n} \end{bmatrix}. \tag{54}$$

It is immediate to show that (54) can be inverted for $\frac{d\ell_{ik}^*}{d\tau_n}$ and $\frac{dm_{ik}^*}{d\tau_n}$ as soon as acknowledging the following fact.

Fact A.1. *Suppose that Assumption 2.4 holds. Then, the matrix*

$$\begin{bmatrix} \xi_{ik}^* \frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik}^2} & \xi_{ik}^* \frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik} \partial m_{ik}} \\ \frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}} & \frac{\partial f_i(\cdot)^*}{\partial m_{ik}} \end{bmatrix}$$

is nonsingular, i.e., invertible.

Proof. By Assumption 2.4 (i), it holds by Euler's theorem for homogeneous functions that for each firm k,

$$\frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}} \ell_{ik}^* + \frac{\partial f_i(\cdot)^*}{\partial m_{ik}} m_{ik}^* = q_{ik}^*$$

and

$$\frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik}^2} \ell_{ik}^* + \frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik} \partial m_{ik}} m_{ik}^* = 0. \tag{55}$$

Then the determinant of the matrix in question is given by

$$\begin{aligned} \begin{vmatrix} \xi_{ik}^* \frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik}^2} & -\xi_{ik}^* \frac{\partial f_i^2(\cdot)^*}{\partial \ell_{ik} \partial m_{ik}} \\ \frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}} & \frac{\partial f_i(\cdot)^*}{\partial m_{ik}} \end{vmatrix} &= \begin{vmatrix} -\xi_{ik}^* \frac{m_{ik}^*}{\ell_{ik}^*} \frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik} \partial \ell_{ik}} & \xi_{ik}^* \frac{\partial f_i^2(\cdot)^*}{\partial \ell_{ik} \partial m_{ik}} \\ \frac{q_{ik}^*}{\ell_{ik}^*} - \frac{m_{ik}^*}{\ell_{ik}^*} \frac{\partial f_i(\cdot)^*}{\partial m_{ik}} & \frac{\partial f_i(\cdot)^*}{\partial m_{ik}} \end{vmatrix} \\ &= -\xi_{ik}^* \frac{q_{ik}^*}{\ell_{ik}^*} \frac{\partial f_i^2(\cdot)^*}{\partial \ell_{ik} \partial m_{ik}} \\ &< 0, \end{aligned}$$

where the last strict inequality is due to Assumptions 2.4 (v).[105] This means that the matrix is nonsingular, as claimed. □

In light of Fact A.1, the system of equations (54) can be uniquely solved for $\frac{d\ell_{ik}^*}{d\tau_n}$ and $\frac{dm_{ik}^*}{d\tau_n}$:

[105] Assumption 2.4 (v) is meant to be Assumption A.3, as defined in Appendix A.2.

$$\begin{bmatrix} \frac{d\ell_{ik}^*}{d\tau_n} \\ \frac{dm_{ik}^*}{d\tau_n} \end{bmatrix} = \underbrace{-\left(\xi_{ik}^* \frac{q_{ik}^*}{\ell_{ik}^*} \frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik} \partial m_{ik}}\right)^{-1} \begin{bmatrix} \frac{\partial f_i(\cdot)^*}{\partial m_{ik}} & -\xi_{ik}^* \frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik} \partial m_{ik}} \\ -\frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}} & \xi_{ik}^* \frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik}^2} \end{bmatrix} \begin{bmatrix} 1 - \frac{\ell_{ik}^*}{q_{ik}^*} \frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}} & -\frac{m_{ik}^*}{q_{ik}^*} \frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}} \\ \bar{\lambda}_{ik}^L & \bar{\lambda}_{ik}^M \end{bmatrix}}_{\text{firm } k\text{'s input elasticities}}$$

$$\times \underbrace{\begin{bmatrix} \frac{dW^*}{d\tau_n} \\ \frac{dP_i^{M*}}{d\tau_n} \end{bmatrix}}_{\text{responses to a policy shock}}. \tag{56}$$

The leading three terms on the right-hand side of (56) jointly account for the responsiveness of the firm's labor and material inputs to the changes in wage and the material cost index due to a policy shift, with the latter being given by the last term.[106]

Now, notice from (32), (36), (39) and (56) that $\frac{dq_{ik}^*}{d\tau_n}$, $\frac{dp_{ik}^*}{d\tau_n}$, $\frac{d\ell_{ik}^*}{d\tau_n}$, $\frac{dm_{ik}^*}{d\tau_n}$ and $\frac{dP_i^*}{d\tau_n}$ are expressed in terms of $\frac{dW^*}{d\tau_n}$ and $\frac{dP_i^{M*}}{d\tau_n}$. Moreover, it follows from (44) that $\frac{dP_i^{M*}}{d\tau_n}$ can be written by $\frac{dW^*}{d\tau_n}$. Hence, it remains to "solve" for $\frac{dW^*}{d\tau_n}$. This is accomplished by making use of the labor market clearing condition (11).

First, let

$$D_{ik} = \begin{bmatrix} d_{ik,11} & d_{ik,12} \\ d_{ik,21} & d_{ik,22} \end{bmatrix}$$

be the 2×2 matrix expressing the firm's input elasticity in (56), i.e.,

$$D_{ik} := -\left(\xi_{ik}^* \frac{q_{ik}^*}{\ell_{ik}^*} \frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik} \partial m_{ik}}\right)^{-1} \begin{bmatrix} \frac{\partial f_i(\cdot)^*}{\partial m_{ik}} & -\xi_{ik}^* \frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik} \partial m_{ik}} \\ -\frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}} & \xi_{ik}^* \frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik}^2} \end{bmatrix} \begin{bmatrix} 1 - \frac{\ell_{ik}^*}{q_{ik}^*} \frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}} & -\frac{m_{ik}^*}{q_{ik}^*} \frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}} \\ \bar{\lambda}_{ik}^L & \bar{\lambda}_{ik}^M \end{bmatrix}. \tag{57}$$

Then, (56) can be written as

$$\frac{d\ell_{ik}^*}{d\tau_n} = d_{ik,11} \frac{dW^*}{d\tau_n} + d_{ik,12} \frac{dP_i^{M*}}{d\tau_n}, \tag{58}$$

$$\frac{dm_{ik}^*}{d\tau_n} = d_{ik,21} \frac{dW^*}{d\tau_n} + d_{ik,22} \frac{dP_i^{M*}}{d\tau_n}. \tag{59}$$

Next, let $\vartheta_{1,i}$ and $\vartheta_{2,i}$ be, respectively, the ith elements of $(I - \Gamma_2)^{-1}\Gamma_1 \iota$ and $(I -$

[106] The former is determined independently of the latter. Because of this, once the former is obtained, (56) can be viewed as a "reduced-form" relationship between the changes of labor and material inputs and those of wage and material cost index.

$\Gamma_2)^{-1}\left[\frac{\partial \mathcal{P}_1^M(\cdot)}{\partial \tau_n}\mathbb{1}_{\{n=1\}},\ldots,\frac{\partial \mathcal{P}_N^M(\cdot)}{\partial \tau_n}\mathbb{1}_{\{n=N\}}\right]'$. Then, the ith element of (44) can be written as

$$\frac{dP_i^{M*}}{d\tau_n} = \vartheta_{1,i}\frac{dW^*}{d\tau_n} + \vartheta_{2,i}. \tag{60}$$

Therefore, upon substituting (60) into (58), I have

$$\begin{aligned}\frac{d\ell_{ik}^*}{d\tau_n} &= d_{ik,11}\frac{dW^*}{d\tau_n} + d_{ik,12}\left(\vartheta_{1,i}\frac{dW^*}{d\tau_n} + \vartheta_{2,i}\right)\\ &= (d_{ik,11} + \vartheta_{1,i}d_{ik,12})\frac{dW^*}{d\tau_n} + \vartheta_{2,i}d_{ik,12}.\end{aligned} \tag{61}$$

To ensure the unique solution, I maintain the following regularity condition.

Assumption A.4 (Regularity Condition 3)**.** $\sum_{i=1}^N\sum_{k=1}^{N_i}(d_{ik,11} + \vartheta_{1,i}d_{ik,12}) \neq 0$.

Remark A.8. *Combined with the set of identifying assumptions proposed in this paper, Assumption A.4 is testable. Developing a formal statistical inference, however, is outside the scope of this paper. See Appendix F for a discussion.*

Totally differentiating the labor market clearing condition (11) delivers

$$\frac{dL}{d\tau_n} = \sum_{i=1}^N\sum_{k=1}^{N_i}\frac{d\ell_{ik}^*}{d\tau_n}.$$

Since labor supply is inelastic, it then must be $\frac{dL}{d\tau_n} = 0$, so that

$$0 = \sum_{i=1}^N\sum_{k=1}^{N_i}\frac{d\ell_{ik}^*}{d\tau_n}. \tag{62}$$

Substituting (61) for $\frac{d\ell_{ik}^*}{d\tau_n}$ into (62) leads to

$$0 = \sum_{i=1}^N\sum_{k=1}^{N_i}\left\{(d_{ik,11} + \vartheta_{1,i}d_{ik,12})\frac{dW^*}{d\tau_n} + \vartheta_{2,i}d_{ik,12}\right\}, \tag{63}$$

which, under Assumption A.4, can be rearranged to

$$\frac{dW^*}{d\tau_n} = -\frac{\sum_{i=1}^N\sum_{k=1}^{N_i}\vartheta_{2,i}d_{ik,12}}{\sum_{i=1}^N\sum_{k=1}^{N_i}(d_{ik,11} + \vartheta_{1,i}d_{ik,12})}. \tag{64}$$

Combining (64) with (32), (36), (39), (44) and (56), I can "solve" for $\frac{dq_{ik}^*}{d\tau_n}$, $\frac{dp_{ik}^*}{d\tau_n}$, $\frac{d\ell_{ik}^*}{d\tau_n}$,

$\frac{dm^*_{ik}}{d\tau_n}$, $\frac{dP^*_i}{d\tau_n}$, $\frac{dP^{M*}_i}{d\tau_n}$ and $\frac{dW^*}{d\tau_n}$ in terms of the endogenous variables in the current equilibrium, exogenous variables and the policy-invariant functions.

Now, it remains to study the responsiveness of the derived demand for sectoral goods with respect to a marginal change in the subsidy $\frac{dm^*_{ik,j}}{d\tau_n}$.

A.3 Cost Minimization 2: Derived Demand for Sectoral Goods

In equilibrium, firm k in sector i purchases sectoral intermediate goods according to the following cost minimization problem:

$$\{m^*_{ik,j}\}_{j=1}^N \in \operatorname*{arg\,min}_{\{m_{ik,j}\}_{j\in\mathcal{N}}} \sum_{j=1}^N (1-\tau_i)P^*_j m_{ik,j} \quad s.t. \quad \mathcal{G}_i(\{m_{ik,j}\}_{j=1}^N) \geq m^*_{ik}.$$

leading to the derived demand for sectoral goods:

$$m^*_{ik,j} = m_{ik,j}(\{P^*_j\}_{j=1}^N, \tau_i, m^*_{ik}), \tag{65}$$

where $m_{ik,j}(\cdot)$ is a mapping from a combination $(\{P_j\}_{j=1}^N, \tau_i, m_{ik})$ to a real value representing the demand for sector j's intermediate good $m_{ik,j}$.

Totally differentiating (65) (and evaluating at the equilibrium values of its arguments) delivers

$$\frac{dm^*_{ik,j}}{d\tau_n} = \sum_{j'=1}^N \frac{\partial m_{ik,j}(\cdot)^*}{\partial P_{j'}} \frac{dP^*_{j'}}{d\tau_n} + \frac{\partial m_{ik,j}(\cdot)^*}{\partial \tau_n} \mathbb{1}_{\{n=i\}} + \frac{\partial m_{ik,j}(\cdot)^*}{\partial m_{ik}} \frac{dm^*_{ik}}{d\tau_n}, \tag{66}$$

where $\mathbb{1}_{\{n=i\}}$ is an indicator function that takes one if $n = i$, and zero otherwise. Since both $\frac{dP^*_{j'}}{d\tau_n}$ and $\frac{dm^*_{ik}}{d\tau_n}$ are already solved above, (66) in turn gives $\frac{dm^*_{ik,j}}{d\tau_n}$.

A.4 An Illustrative Example

To gain a clear view of how the macro and micro complementarities work, this subsection considers a special case of the general setup described in Section 2. The model of this subsection posits a constant elasticity of substitution (CES) production function for the sectoral aggregators, and a Cobb-Douglas production function for both individual firm-level technology and the economy-wide aggregator.[107]

[107] A version of this parametric setup is widely used in the macroeconomics and international trade literature (e.g., Atkeson and Burstein, 2008; Gaubert and Itskhoki, 2020; Gaubert et al., 2021; Bigio and La'O, 2020; La'O and Tahbaz-Salehi, 2022).

A.4.1 Setup

The economy-wide aggregator $\mathcal{F}(\cdot)$ in (2) is given by a Cobb-Douglas production function:

$$\mathcal{F}(\{X_j\}_{j=1}^{N}) \coloneqq \prod_{j=1}^{N} X_j^{\beta_j},$$

where β_j is the elasticity parameter with respect to the sector j's good. The sectoral aggregator $F_i(\cdot)$ in (3) takes the form of a constant elasticity of substitution (CES) production function:

$$F_i(\{q_{ik}\}_{k=1}^{N_i}) \coloneqq \left(\sum_{k=1}^{N_i} \delta_i q_{ik}^{\frac{\sigma_i - 1}{\sigma_i}}\right)^{\frac{\sigma_i}{\sigma_i - 1}},$$

where δ_i is a sector-specific demand shifter and $\sigma_i > 0$ represents elasticity of substitution. The associated sectoral price index is

$$P_i = \left(\sum_{k=1}^{N_i} \delta_i^{\sigma_i} p_{ik}^{1-\sigma_i}\right)^{\frac{1}{1-\sigma_i}}. \tag{67}$$

The firm-level production function $f_i(\cdot)$ in (4) is a Cobb-Douglas aggregator with productivity being Hicks-neutral:

$$f_i(\ell_{ik}, m_{ik}; z_{ik}) \coloneqq z_{ik} \ell_{ik}^{\alpha_i} m_{ik}^{1-\alpha_i},$$

where α_i is a sector-specific parameter indicating the output-labor ratio. The material aggregator $\mathcal{G}_i(\cdot)$ in (5) is again given by a Cobb-Douglas production:

$$\mathcal{G}(\{m_{ik,j}\}_{j=1}^{N}) \coloneqq \prod_{j=1}^{N} m_{ik,j}^{\gamma_{i,j}},$$

where $\gamma_{i,j}$ corresponds to the input share of sector j's intermediate good, reflecting the production network Ω. The associated unit cost condition yields the material cost index:

$$P_i^M = \prod_{j=1}^{N} \frac{1}{\gamma_{i,j}^{\gamma_{i,j}}} \Big\{(1-\tau_i) P_j\Big\}^{\gamma_{i,j}}. \tag{68}$$

The firm's profit-maximization problem (7) can be formulated as

$$q_{ik}^* \in \arg\max_{q_{ik}} \left\{ \frac{\delta_i q_{ik}^{\frac{\sigma_i - 1}{\sigma_i}}}{\sum_{k'=1}^{N_i} \delta_i q_{ik'}^{\frac{\sigma_i - 1}{\sigma_i}}} R_i - mc_{ik} q_{ik} \right\},$$

where R_i is the total income of the sectoral aggregator. The equilibrium prices and quantities are given by the following system of firms' pricing equations:

$$p_{ik}^* = \frac{\sigma_i}{(1-\sigma_i)(1-s_{ik}^*)} mc_{ik}$$
$$s_{ik}^* = \delta_i^{\sigma_i} \left(\frac{p_{ik}^*}{P_i^*} \right)^{1-\sigma_i},$$

where s_{ik} is the equilibrium value of firm k's market share. Note that the firm k's marginal revenue function $mr_{ik}(\cdot)$ is given by

$$mr_{ik}(\{q_{ik'}\}_{k'=}^{N}) = \frac{\sigma_i - 1}{\sigma_i} p_{ik}(1 - s_{ik}).$$

Moreover, it is immediate to verify that

$$\frac{\partial p_{ik}(\cdot)}{\partial q_{ik}} = \begin{cases} \frac{p_{ik}}{q_{ik}} \left\{ \frac{\sigma_i - 1}{\sigma_i}(1 - s_{ik}) - 1 \right\} & \text{if } k' = k \\ -\frac{\sigma_i - 1}{\sigma_i} \frac{p_{ik}}{q_{ik'}} s_{ik'} & \text{if } k' \neq k, \end{cases}$$

and

$$\frac{\partial (1 - s_{ik}(\cdot))}{\partial q_{ik}} = \begin{cases} -\frac{\sigma_i - 1}{\sigma_i} \frac{1}{q_{ik}} s_{ik}(1 - s_{ik}) & \text{if } k' = k \\ -\frac{\sigma_i - 1}{\sigma_i} \frac{1}{q_{ik'}} s_{ik} s_{ik'} & \text{if } k' \neq k. \end{cases}$$

In equilibrium, it follows from (67) that

$$\frac{\partial \mathcal{P}_i(\cdot)}{\partial q_{ik}^*} = -\frac{s_{ik}^*}{q_{ik}^*} P_i^* \qquad \forall k \in \mathscr{N}_i,$$

and from (68) that

$$\frac{\partial \mathcal{P}_i^M(\cdot)}{\partial P_j^*} = \gamma_{i,j} \frac{P_i^{M*}}{P_j^*} \qquad \forall j \in \mathscr{N}$$
$$\frac{\partial \mathcal{P}_i^M(\cdot)}{\partial \tau_n} = -\frac{P_i^{M*}}{1 - \tau_i} \mathbb{1}_{\{n=i\}}.$$

The relationship between strategic complementarities and the market share is studied in the following proposition.

Proposition A.1. *Consider the economy defined in Appendix A.4.1. For each sector $i \in \mathscr{N}$, the following statements hold:*

(i) If $\sigma_i > 1$, then (i-a) for each $k \in \mathscr{N}_i$, $\frac{\partial mr_{ik}(\cdot)^*}{\partial q_{ik}} < 0$; and (i-b) for each $k \in \mathscr{N}_i$ and $k' \in \mathscr{N}_i \backslash \{k\}$, $\frac{\partial mr_{ik}(\cdot)^*}{\partial q_{ik'}} < 0$ if $s_{ik} < \frac{1}{2}$, $\frac{\partial mr_{ik}(\cdot)^*}{\partial q_{ik'}} = 0$ if $s_{ik} = \frac{1}{2}$ and $\frac{\partial mr_{ik}(\cdot)^*}{\partial q_{ik'}} > 0$ otherwise.

(ii) If $\sigma_i < 1$, then (ii-a) for each $k \in \mathscr{N}_i$, $\frac{\partial mr_{ik}(\cdot)^*}{\partial q_{ik}} < 0$ if $s_{ik} > -\frac{1}{2(\sigma_1 - 1)}$, $\frac{\partial mr_{ik}(\cdot)^*}{\partial q_{ik}} = 0$ if $s_{ik} = -\frac{1}{2(\sigma_1 - 1)}$ and $\frac{\partial mr_{ik}(\cdot)^*}{\partial q_{ik}} < 0$ otherwise; and (ii-b) for each $k \in \mathscr{N}_i$ and $k' \in \mathscr{N}_i \backslash \{k\}$, $\frac{\partial mr_{ik}(\cdot)^*}{\partial q_{ik'}} < 0$ if $s_{ik} < \frac{1}{2}$, $\frac{\partial mr_{ik}(\cdot)^*}{\partial q_{ik'}} = 0$ if $s_{ik} = \frac{1}{2}$ and $\frac{\partial mr_{ik}(\cdot)^*}{\partial q_{ik'}} > 0$ otherwise.

Proof. *(i)* Suppose $\sigma_i > 1$.

(i-a) It is straightforward that

$$\frac{\partial mr_{ik}(\cdot)}{\partial q_{ik}} \gtreqless 0 \iff -\frac{1}{2(\sigma_i - 1)} \gtreqless s_{ik}. \tag{69}$$

Given the hypothesis (i.e., $\sigma_i > 1$), the left-hand side of (69) is negative, while s_{ik} is by definition positive. Hence, it is always true that $\frac{\partial mr_{ik}(\cdot)}{\partial q_{ik}} < s_{ik}$, from which it follows that $\frac{\partial mr_{ik}(\cdot)}{\partial q_{ik}} < 0$.

(i-b) It is straightforward that

$$\frac{\partial mr_{ik}(\cdot)}{\partial q_{ik'}} \gtreqless 0 \iff \frac{1}{2} \lesseqgtr s_{ik}.$$

This proves the statement.

(ii) Suppose $\sigma_i < 1$.

(ii-a) It is straightforward that

$$\frac{\partial mr_{ik}(\cdot)}{\partial q_{ik}} \gtreqless 0 \iff -\frac{1}{2(\sigma_i - 1)} \gtreqless s_{ik}. \tag{70}$$

According to the hypothesis (i.e., $\sigma_i < 1$), the left-hand side of (70) is positive. Then there can be three configurations depending on the value of s_{ik}. This observation directly leads to the statement.

(ii-b) It is straightforward that

$$\frac{\partial mr_{ik}(\cdot)}{\partial q_{ik'}} \gtreqless 0 \iff \frac{1}{2} \lesseqgtr s_{ik}.$$

This proves the statement. □

Notice that in Proposition A.1, part *(i-b)* is identical to part *(ii-b)*, i.e., they do not depend on the value of σ_i. This observation immediately leads to the following corollaries.

Corollary A.1. *Consider the economy defined in Appendix A.4.1.*

(i) If there exists a firm $\bar{k} \in \mathscr{N}_i$ *such that* $s_{i\bar{k}} > \frac{1}{2}$*, then* $\frac{\partial mr_{i\bar{k}}(\cdot)^*}{\partial q_{ik'}} > 0$ *for all* $k' \in \mathscr{N}_i \backslash \{\bar{k}\}$*; and* $\frac{\partial mr_{ik}(\cdot)^*}{\partial q_{ik'}} < 0$ *for all* $k, k' \in \mathscr{N}_i \backslash \{\bar{k}\}$ *such that* $k \neq k'$*, regardless of the value of* σ_i*.*

(ii) If $s_{ik} < \frac{1}{2}$ *for all* $k \in \mathscr{N}_i$*, then for each* $k \in \mathscr{N}_i$*,* $\frac{\partial mr_{ik}(\cdot)^*}{\partial q_{ik'}} < 0$ *for all* $k' \in \mathscr{N}_i \backslash \{k\}$*, regardless of the value of* σ_i*.*

Proof. The proof is omitted. □

These corollaries can yield further implications for the case of a duopoly, as studied below.

A.4.2 Duopoly

Consider the same setup as described above. But, suppose that each sector is populated by two firms, i.e., $N_i = \{1, 2\}$ for all $i \in \mathscr{N}$. Without loss of generality, I assume $s_{i1} > \frac{1}{2}$, which in turn means that $s_{i2} < \frac{1}{2}$, i.e., firm 1 has a larger market share than firm 2. In this setup, Proposition A.1 delivers the following corollaries.

Corollary A.2. *In duopoly, wherein* $s_{i1} > \frac{1}{2}$*, it holds that* $\frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i2}} > 0$ *and* $\frac{\partial mr_{i2}(\cdot)^*}{\partial q_{i1}} < 0$*.*

Proof. The proof is omitted. □

Corollary A.3. *In duopoly, wherein* $s_{i1} > \frac{1}{2}$ *and* $\sigma_i > 1$*, it holds that (i)* $\frac{\partial mr_{ik}(\cdot)^*}{\partial q_{ik}} < 0$ *for all* $k \in \{1, 2\}$*; (ii)* $\frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i2}} > 0$*; and (iii)* $\frac{\partial mr_{i2}(\cdot)^*}{\partial q_{i1}} < 0$*, so that* $det(\Lambda_{i,1}) > 0$*.*

Proof. The proof is omitted. □

Noticing that the firm's marginal cost is constant in the firm's profit-maximization problem, the following corollary is straightforward.

Corollary A.4. *(i) Firm 1's quantity decision is a strategic complement to firm 2's quantity decision. (ii) Firm 2's quantity decision is a strategic substitute to firm 1's quantity decision.*

Proof. It is immediate to see that

$$0 < \frac{\partial mr_{i1}(\cdot)}{\partial q_{i2}} = \frac{\partial (mr_{i1}(\cdot) - mc_{i1})}{\partial q_{i2}} = \frac{\partial \frac{\partial \pi_{i1}(\cdot)}{\partial q_{i1}}}{\partial q_{i2}}.$$

An analogous argument applies to firm 2, completing the proof. □

To study micro complementarities, I focus on $\bar{\lambda}^M_{i\cdot}$ in the subsequent analysis. A parallel argument holds for $\bar{\lambda}^L_{i\cdot}$ as well. In what follows, I assume that $\sigma_i > 1$. First,

$$\bar{\lambda}^M_{i1} = \frac{1}{det(\Lambda_{i,1})}\left(\frac{m^*_{i1}}{q^*_{i1}}\frac{\partial mr_{i2}(\cdot)^*}{\partial q_{i2}} - \frac{m^*_{i2}}{q^*_{i2}}\frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i2}}\right)$$
$$\bar{\lambda}^M_{i2} = \frac{1}{det(\Lambda_{i,1})}\left(-\frac{m^*_{i1}}{q^*_{i1}}\frac{\partial mr_{i2}(\cdot)^*}{\partial q_{i1}} + \frac{m^*_{i2}}{q^*_{i2}}\frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i1}}\right),$$

where $det(\Lambda_{i,1}) = \frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i1}}\frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i1}} - \frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i2}}\frac{\partial mr_{i2}(\cdot)^*}{\partial q_{i1}}$. From Corollary A.3, it follows that $\bar{\lambda}^M_{i1} < 0$ as well as $det(\Lambda_{i,1}) > 0$.

The following lemma characterizes the sign of $\bar{\lambda}^M_{i2}$ in terms of the partial derivatives of the marginal revenue functions and firms' productivities.

Lemma A.1. $\bar{\lambda}^M_{i2} \lesseqgtr 0 \Longleftrightarrow \frac{z_{i1}}{z_{i2}}\frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i1}} \lesseqgtr \frac{\partial mr_{i2}(\cdot)^*}{\partial q_{i1}}$.

Proof. First, observe that

$$\bar{\lambda}^M_{i2} \lesseqgtr 0 \Longleftrightarrow \frac{\frac{m_{i2}}{q_{i2}}}{\frac{m_{i1}}{q_{i1}}}\frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i1}} \lesseqgtr \frac{\partial mr_{i2}(\cdot)^*}{\partial q_{i2}}.$$

Here, under the Cobb-Douglas production function, the material productivity is proportional to the inverse of the firm's productivity: for each $k \in \mathscr{N}_i$,

$$\frac{m^*_{ik}}{q^*_{ik}} = z^{-1}_{ik}\left(\frac{\alpha_i}{1-\alpha_i}\right)^{-\alpha_i}\left(\frac{P^{M*}_i}{W^*}\right)^{\alpha_i}.$$

Substituting this into the above equivalence proves the claim. □

Remark A.9. *Due to the presumption (i.e.,* $s_{i1} > s_{i2}$*), it holds that* $\frac{z_{i1}}{z_2} > 1$.

The following proposition gives a sufficient condition for $\bar{\lambda}^M_{i\cdot}$ to be positive, and states that if firm 2 is a "relatively strong" strategic substitute, then the sectoral measure of strategic complementarity is positive.

Proposition A.2. *Suppose* $\frac{z_{i1}}{z_{i2}}\frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i1}} < \frac{\partial mr_{i2}(\cdot)^*}{\partial q_{i1}}$*. Then,* $\bar{\lambda}^M_{i\cdot} > 0$.

Proof. First, by construction, $P_iQ_i = R_i$. Differentiation with respect to q_{ik} leads to

$$\frac{\partial \mathcal{P}_i(\cdot)}{\partial q_{ik}} = -\frac{s_{ik}}{q_{ik}}P_i.$$

Next, by definition,

$$\bar{\lambda}^M_{i\cdot} = \frac{\partial \mathcal{P}_i(\cdot)}{\partial q^*_{i1}}\bar{\lambda}^M_{i1} + \frac{\partial \mathcal{P}_i(\cdot)}{\partial q^*_{i2}}\bar{\lambda}^M_{i2} = -\left(\frac{s^*_{i1}}{q^*_{i1}}\bar{\lambda}^M_{i1} + \frac{s^*_{i2}}{q^*_{i2}}\bar{\lambda}^M_{i2}\right)P^*_i.$$

Acknowledging that $\bar{\lambda}_{i1}^M < 0$ due to Corollary A.3, and $\bar{\lambda}_{i2}^M < 0$ because of Lemma A.1, it follows that $\bar{\lambda}_{i\cdot}^M > 0$. □

Notice that the hypothesis of this proposition reads

$$\frac{\partial \frac{\partial \pi_{i2}(\cdot)^*}{\partial q_{i2}}}{\partial q_{i1}} \Big/ \frac{\partial \frac{\partial \pi_{i1}(\cdot)^*}{\partial q_{i1}}}{\partial q_{i1}} \in \left(\frac{z_{i1}}{z_{i2}}, \infty \right).$$

This requires that firm 2's output is a "relatively strong" strategic substitute in the sense that the proportion of the sensitivity of firm 2's marginal profit to firm 1's quantity adjustment relative to that of firm 1's marginal profit to its own quantity change is at least as large as the productivity ratio between the two firms.[108] Note that the converse of Proposition A.2 is not true.[109] Nevertheless, a positive micro complementarity can be viewed as an indication that firm 2 might be a "relatively strong" strategic substitute. Moreover, the contrapositive suggests that negative micro complementarity is evidence of firm 2's being a "relatively modest" strategic substitute.

Remark A.10. *The converse is not true. A necessary and sufficient condition for the sign of* $\bar{\lambda}_{i\cdot}^M$ *reads*

$$\bar{\lambda}_{i\cdot}^M \gtreqless 0 \Longleftrightarrow \bar{\lambda}_{i2}^M \lesseqgtr -\frac{p_{i1}^*}{p_{i2}^*} \bar{\lambda}_{i1}^M .$$

While it is possible to further rewrite this in terms of partial derivatives of the marginal revenue functions, its economic content is not easy to interpret.

[108] By setup, $\frac{z_{i1}}{z_{i2}} > 1$.

[109] Although it is possible to characterize the necessary and sufficient condition in terms of firms' strategic complementarities, its economic content is not clear. See Remark A.10.

B Detail of Data

This section provides a detailed account of the data source used in the main text and explains how I construct the empirical counterparts of the variables set out in Section 2.

B.1 Aggregate Data

Data on the wage-related concepts are obtained from the U.S. Bureau of Labor Statistics (BLS) through the Federal Reserve Bank of St. Louis (FRED) at an annual frequency. In my model, labor is assumed to be frictionlessly mobile across sectors so that the wage is common across all sectors. Thus, I use *Average hourly earnings of all employees, total private* as the empirical analogue of the wage W in my model. In addition, I also obtain a measurement of the average hours worked per employee per year (*Average weekly hours of all employees, total private*). It should be remarked that these data exclude agricultural workers mainly due to the peculiarities of the structure of the agricultural industry and characteristics of its workers — for example, various definitions of agriculture, farms, farmers, and farmworkers; and considerable seasonal fluctuation in employment (Daberkow and Whitener, 1986). Note also that these data do not include information on government employees, either. Data on sectoral price indices are available at the Bureau of Economic Analysis (BEA). I use *U.Chain-Type Price Indexes for Gross Output by Industry — Detail Level (A)* as the empirical counterparts of $\{P_i^*\}_{i=1}^N$.

These are summarized in the following fact.

Fact B.1 (Wage and Sectional Price Index). *The wage W^* and sectoral price indices $\{P_i^*\}_{i=1}^N$ are directly observed in the data.*

B.2 Sector-Level Data: Industry Economic Accounts (IEA)

My analysis utilizes two types of sector-level data, namely, the input-output table and sector-specific tax/subsidy, both of which come from the input-output accounts data of the Bureau of Economic Analysis (BEA). In line with the global economic accounting standards, such as the System of National Accounts 2008 (UN, 2008), the BEA input-output table consists of two tables: the use and supply tables.

The use table shows the uses of commodities (goods and services) by industries as intermediate inputs and by final users, with columns indicating the industries and final users and rows representing commodities. This table reports three pieces of information: intermediate inputs, final demand, and value added. Each cell in the intermediate input section records the amount of a commodity purchased by each industry as an intermediate input, valued at producers' or purchasers' prices.[110] The final demand section accounts for expenditure-side components of GDP. The value-added part bridges the difference between an industry's total output and its total cost for intermediate inputs. This part will further be expanded in the upcoming section (Appendix B.2.2).

The supply table shows the total supply of commodities by industries, with columns indicating the industries and rows representing commodities. This table comprises domestic output and imports. Each cell in the domestic output section presents the total amount of each commodity supplied domestically by each industry, valued at the basic prices. The import section records the total amount of each commodity imported from foreign countries, valued at the importer's customs frontier price (i.e., the c.i.f. valuation).[111]

Segmentation. My analysis is based on the BEA's industry classification at the summary level, which is roughly equivalent to the three-digit NAICS (North American Industry Classification System). I make four major modifications in accordance with other aggregate and firm-level data as well as my model (Section 2). First, I omit several industries and products from my analysis. Following Bigio and La'O (2020), I exclude the finance, insurance, real estate, rental and leasing (FIRE) sectors from my analysis. In the BEA's input-output table, these sectors are indexed by 521CI, 523, 524, 525, HS, ORE, and 532RL. I also follow Baqaee and Farhi (2020) in dropping the scrap, used and secondhand goods industry/commodity, and the noncomparable imports and rest-of-the-world adjustment industry/commodity. In the original data, the former is indexed by Used and the latter by Others. I again follow Baqaee and Farhi (2020) in removing the government sectors, which are reported with the indices 81, GFGD, GFGN, GFE, GSLG, and GSLE. This aligns with both my model (Section 2) and aggregate data (Appendix B.1). Second, drawing on Gutiérrez and Philippon (2017), I merge several BEA's industries. This manipulation ensures that each industry has a good coverage of the Compustat firms (Gutiérrez and Philippon, 2017).[112] Third, I elim-

[110] Typically, the IEA is valued at either of the producers', basic, or purchasers' prices. The producers' prices are the total amount of monetary units received from the purchasers for a unit of a good or service that is sold. The basic prices mean the total amount retained by the producer for a unit of a good or service. This price plays a pivotal role in the producer's decision-making about production and sales. The purchasers' prices refer to the total amount paid by the purchasers for a unit of a good or service that they purchase. This is the key for the purchasers to make their purchasing decisions. By definition, the basic prices are equal to the producers' prices minus taxes payable for a unit of a good and service, plus any subsidy receivable for a unit of a good and service; and the purchasers' prices are equivalent to the sum of the producers' prices and any wholesale, retail, or transportation markups charged by intermediaries between producers and purchasers. See BEA (2009) and Young et al. (2015) for the detail.

[111] The importers' customs frontier price is calculated as the cost of the product at the foreign port value plus insurance and freight charges to move the product to the domestic port. See Young et al. (2015) for details.

[112] For example, the nonparametric estimation of the share regression using the polynomials of degree 2 requires at least 6 observations in the same sector. See Appendix E.2.

inate the farm industry (BEA code 111CA) and the forestry, fishing, and related activities (BEA code 113FF), in view of the construction of the aggregate employment data (Appendix B.1). Fourth, I drop the health care industries (BEA code 621, 622, 623 and 624) because my model may not capture several key aspects of the industry's competition nature.[113] I also omit the performing arts, spectator sports, museums, and related activities industry (BEA code 711A) and the amusements, gambling, and recreation industries industry (BEA code 713) for the same reason. Lastly, the management of companies and enterprises industry (BEA code 55) and the rail transportation industry (BEA code 482) are eliminated because there are no corresponding firms in the Compustat data. After all, I am left with 26 industries listed in Table B.2.

[113]Recent works model the health care industry as a mix-oligopoly, in which public and private providers compete to maximize, respectively, the consumer surplus and profits (e.g., Jofre-Bonet, 2000; Bisceglia et al., 2023).

Table 2: Mapping of BEA Industry Codes to Segments

BEA code	Industry	Mapped segment
111CA	Farms	Omitted
113FF	Forestry, fishing, and related activities	Omitted
211	Oil and gas extraction	Oil and gas extraction and mining
212	Mining, except oil and gas	Oil and gas extraction and mining
213	Support activities for mining	Oil and gas extraction and mining
22	Utilities	Omitted
23	Construction	Construction
321	Wood products	Wood and nonmetallic mineral products
327	Nonmetallic mineral products	Wood and nonmetallic mineral products
331	Primary metals	Primary metals
332	Fabricated metal products	Fabricated metal products
333	Machinery	Machinery
334	Computer and electronic products	Computer and electronic products
335	Electrical equipment, appliances, and components	Electrical equipment, appliances, and components
3361MV	Motor vehicles, bodies and trailers, and parts	Motor vehicles, bodies and trailers, and parts
3364OT	Other transportation equipment	Other transportation equipment
337	Furniture and related products	Furniture and related products
339	Miscellaneous manufacturing	Miscellaneous manufacturing
311FT	Food and beverage and tobacco products	Food and beverage and tobacco products
313TT	Textile mills and textile product mills	Textile mills and apparel products
315AL	Apparel and leather and allied products	Textile mills and apparel products
322	Paper products	Paper products and related services
323	Printing and related support activities	Paper products and related services
324	Petroleum and coal products	Petroleum and coal products

BEA code	Industry	Mapped segment
325	Chemical products	Chemical products
326	Plastics and rubber products	Plastics and rubber products
42	Wholesale trade	Wholesale trade
441	Motor vehicle and parts dealers	Retail Trade
445	Food and beverage stores	Retail Trade
452	General merchandise stores	Retail Trade
4A0	Other retail	Retail Trade
481	Air transportation	Transportation
482	Rail transportation	Omitted
483	Water transportation	Transportation
484	Truck transportation	Transportation
485	Transit and ground passenger transportation	Transportation
486	Pipeline transportation	Transportation
487OS	Other transportation and support activities	Transportation
493	Warehousing and storage	Omitted
511	Publishing industries, except internet (includes software)	Information
512	Motion picture and sound recording industries	Information
513	Broadcasting and telecommunications	Information
514	Data processing, internet publishing, and other information services	Information
521CI	Federal Reserve banks, credit intermediation, and related activities	Omitted
523	Securities, commodity contracts, and investments	Omitted
524	Insurance carriers and related activities	Omitted
525	Funds, trusts, and other financial vehicles	Omitted
HS	Housing	Omitted
ORE	Other real estate	Omitted
532RL	Rental and leasing services and lessors of intangible assets	Omitted

BEA code	Industry	Mapped segment
5411	Legal services	Professional services
5412OP	Miscellaneous professional, scientific, and technical services	Professional services
5415	Computer systems design and related services	Professional services
55	Management of companies and enterprises	Omitted
561	Administrative and support services	Administrative and waste management
562	Waste management and remediation services	Administrative and waste management
61	Educational services	Educational services
621	Ambulatory health care services	Omitted
622	Hospitals	Omitted
623	Nursing and residential care facilities	Omitted
624	Social assistance	Omitted
711AS	Performing arts, spectator sports, museums, and related activities	Omitted
713	Amusements, gambling, and recreation industries	Omitted
721	Accommodation	Accommodation and food services
722	Food services and drinking places	Accommodation and food services
81	Other services, except government	Omitted
GFGD	Federal general government (defense)	Omitted
GFGN	Federal general government (nondefense)	Omitted
GFE	Federal government enterprises	Omitted
GSLG	State and local general government	Omitted
GSLE	State and local government enterprises	Omitted
Used	Scrap, used and secondhand goods	Omitted
Other	Noncomparable imports and rest-of-the-world adjustment	Omitted

Note: This table shows the correspondence between the BEA's industry classification (at summary level) and my segmentation, which draws heavily on Gutiérrez and Philippon (2017). The first two columns (*BEA code* and *Industry*) list the BEA codes and the corresponding industries as used in the BEA's input-output table. The third column (*Mapped segment*) indicates the names of the segments I define.

B.2.1 Transformation to Symmetric Input-Output Tables

The use table cannot be directly adopted in my empirical analysis as it only shows the uses of each commodity by each industry, not the uses of each industrial product by each industry. This is because the BEA's accounting system allows each industry to produce multiple commodities (e.g., secondary production), and thus is incompatible with my conceptualization. Hence, I first need to convert the use table into a symmetric industry-by-industry input-output table by transferring input and output over the rows in the use and supply tables, respectively.[114] To this end, I impose an assumption about how each commodity is used.

Assumption B.1 (Fixed Product Sales Structures, (Eurostat, 2008))**.** *Each product has its own specific sales structure, irrespective of the industry where it is produced.*

The term sales structure here refers to the shares of the respective intermediate and final users in the sales of a commodity. Under Assumption B.1, each commodity is used at constant rates regardless of in which industry it is produced. For example, a unit of a manufacturing product supplied by the agriculture industry will be transferred from the use of manufacturing products to that of agricultural products in the use table in the same proportion to the use of manufacturing products.[115] Note that the value-added part remains intact throughout this manipulation. Recorded in each cell of the intermediate inputs section of the resulting industry-by-industry table is the empirical counterpart of the sectoral material input cost in my model (i.e., $(1-\tau_i)\sum_{k=1}^{N_i} P_i^* m_{ik,j}^*$). Moreover, each cell of the compensation of employees corresponds to the sectoral labor input cost, given by $\sum_{k=1}^{N_i} W^* \ell_{ik}^*$. These are the data used to construct the production network in my empirical analysis, as shown in the following fact.

Fact B.2. *Under Assumption B.1, the input-output linkages $\boldsymbol{\omega}_L$ and Ω are recovered from the observables.*

Proof. By Shephard's lemma, it holds that for each $i, j \in \mathscr{N}$, the cost-based intermediate expenditure shares $\omega_{i,j}$ satisfies

$$\omega_{i,j} = \frac{(1-\tau_i)\sum_{k=1}^{N_i} P_j^* m_{ik,j}^*}{\sum_{j'=1}^{N}(1-\tau_i)\sum_{k=1}^{N_i} P_{j'}^* m_{ik,j'}^* + \sum_{k=1}^{N_i} W^* \ell_{ik}^*}. \tag{71}$$

[114]For example, if there is a non-zero entry in the cell of the supply table whose column is agriculture and whose row is manufacturing products, it is recorded in the use table as the supply of manufacturing products, the largest component of which should be accounted for by the supply from manufacturing industry. Now my goal is to modify this attribution in a way that the supply of manufacturing products by agriculture industry is treated as agricultural products. To this end, I need to subtract the contributions of agriculture industry from the use of manufacturing products and transfer them to the agricultural commodities, thereby changing the classification of the row from commodity to industry.

[115]There are variants of Assumption B.1. However, it is this assumption that is widely used by statistical offices for various reasons. See Eurostat (2008) for details.

Also, for each $i \in \mathscr{N}$, cost-based equilibrium factor expenditure shares $\omega_{i,L}$ satisfies:

$$\omega_{i,L} = \frac{\sum_{k=1}^{N_i} W^* \ell_{ik}^*}{\sum_{j'=1}^{N}(1-\tau_i)\sum_{k=1}^{N_i} P_{j'}^* m_{ik,j'}^* + \sum_{k=1}^{N_i} W^* \ell_{ik}^*}.$$

Since $\left\{(1-\tau_i)\sum_{k=1}^{N_i} P_j^* m_{ik,j}^*\right\}_{i,j=1}^{N}$ and $\{\sum_{k=1}^{N_i} W^* \ell_{ik}^*\}_{i=1}^{N}$ are directly observed in the transformed industry-by-industry input-output table, I can immediately recover $\boldsymbol{\omega}_L$ and $\boldsymbol{\Omega}$, as desired. □

Figure 3 compares the input-output table based on the use table with the transformed industry-by-industry input-output table.

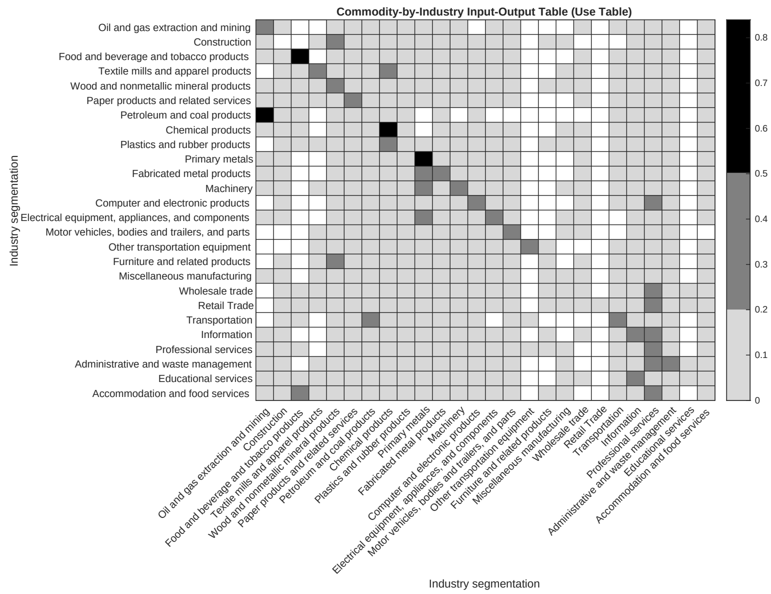
B.2.2 Sectoral Tax/Subsidy

Given that the use table has been transformed into a symmetric industry-by-industry input-output table, I can proceed to back out the tax/subsidy from the transformed table. In this step, I exploit the feature of the use table that reports value added at basic and purchasers' prices. The value added measured at basic prices is composed of (*i*) compensation of employees (V001), (*ii*) gross operating surplus (V003), and (*iii*) other taxes on production (T00OTOP) less subsidies (T00OSUB). The value added at producers' prices further entails (*iv*) taxes on products (T00TOP) and imports less subsidies (T00SUB).[116] According to BEA (2009), the tax-related components of (*iii*) and (*iv*) jointly include, among many others, sales and excise taxes, customs duties, property taxes, motor vehicle licenses, severance taxes, other taxes and special assessments as well as commodity taxes, while the subsidy-related components refer to monetary grants paid by government agencies to private businesses and to government enterprises at another level of government.

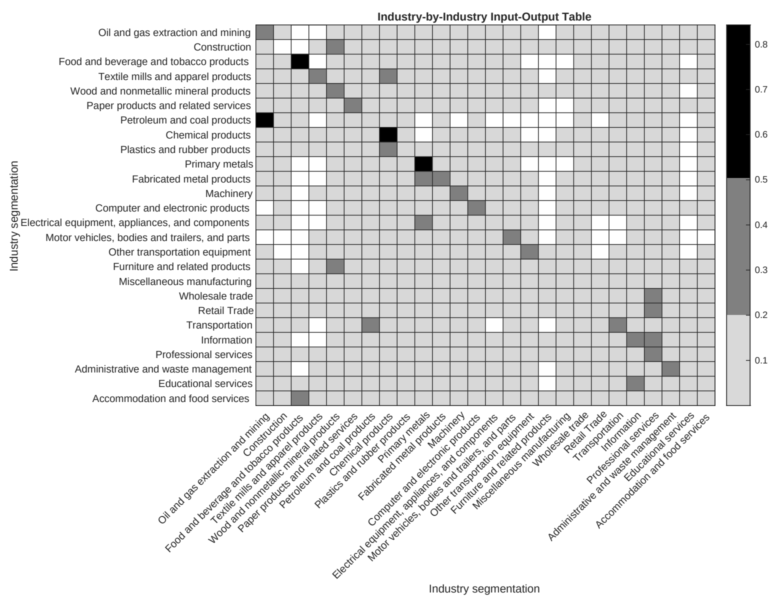
I consider the sum of (*iii*) and (*iv*) to be the empirical counterpart of the policy expenditure in my model. This choice is motivated by the mapping between the BEA's data construction and my conceptualization. To see this, observe that the construction of the data reads

$$\begin{aligned} Profit_i &= (Revenue_i + TaxSubsidy1_i) - (LaborCost_i + MaterialCost_i + TaxSubsidy2_i) \\ \therefore \underbrace{Revenue - MaterialCost_i}_{\text{Value-added}} &= \underbrace{Profit_i}_{\text{Gross operating surplus}} + \underbrace{LaborCost_i}_{\text{Compensation of employees}} \\ &\quad - \underbrace{(TaxSubsidy1_i - TaxSubsidy2_i)}_{\text{Value-added taxes less subsidies}}, \end{aligned} \tag{72}$$

[116]By construction, the sum of the latter across all industries has to coincide with GDP for the economy.



(a) Use table



(b) Transformed industry-by-industry table

Note: This figure illustrates the input-output table in terms of the cost share of sectoral goods. Panel (a) shows the use table that is provided by BEA, while panel (b) reports the transformed industry-by-industry table. White cells indicate zero, while light, medium and dark grey cells represent the low ($0 \sim 0.2$), medium ($0.2 \sim 0.5$) and high ($0.5 \sim 1.0$) cost shares, respectively.

Figure 3: Comparison of Input-Output Tables

where $TaxSubsidy1_i$ represents taxes less subsidies on revenues, and $TaxSubsidy2_i$ corresponds to those on input costs. Notice that the value-added taxes less subsidies ($TaxSubsidy1_i - TaxSubsidy2_i$) are available in the data. The model counterpart of the data construction (72) stems from the definition of the sector-level profit:

$$
\begin{aligned}
\sum_{k=1}^{N_i} \pi_{ik}^* &= \sum_{k=1}^{N_i} p_{ik}^* q_{ik}^* - \left\{ W^* \ell_{ik}^* + (1-\tau_i) \sum_{j=1}^{N} P_i^{M*} m_{ik,j}^* \right\} \\
\therefore \underbrace{\sum_{k=1}^{N_i} p_{ik}^* q_{ik}^* - \sum_{j=1}^{N} P_i^{M*} m_{ik,j}^*}_{\text{Value-added}} &= \underbrace{\sum_{k=1}^{N_i} \pi_{ik}^*}_{\text{Gross operating surplus}} + \underbrace{W^* \ell_{ik}^*}_{\text{Compensation of employees}} \\
&\quad - \underbrace{\tau_i \sum_{j=1}^{N} P_i^{M*} m_{ik,j}^*}_{\text{Value-added taxes less subsidies}} .
\end{aligned} \tag{73}
$$

Comparing (72) and (73), the data on ad-valorem taxes/subsidy can be backed out from the constructed input-output table, as summarized in the following fact.

Fact B.3. *Under Assumption B.1, sector-specific subsidies* $\boldsymbol{\tau} := \{\tau_i\}_{i=1}^N$ *are recovered from the observables.*

Proof. For each sector (industry) $i \in \mathscr{N}$, I have

$$
(1-\tau_i) \sum_{j=1}^{N} \sum_{k=1}^{N_i} P_j^* m_{ik,j}^* = \sum_{j=1}^{N} IntermExpend_{i,j}, \tag{74}
$$

where $IntermExpend_{i,j}$ means the sector i's total expenditure on sector j, which is observed in the (i,j) entry of the industry-by-industry input-output table constructed in Appendix B.2.1. Meanwhile, comparing (72) to (73), I obtain

$$
\tau_i \sum_{j=1}^{N} \sum_{k=1}^{N_i} P_j^* m_{ik,j}^* = VAT_i, \tag{75}
$$

where VAT_i stands for the sector i's value-added taxes less subsidies, reported in the BEA use table.

Rearranging (74) and (75), I can recover the data for sector-specific taxes/subsidies as

$$
\tau_i = \frac{VAT_i}{VAT_i + \sum_{j=1}^{N} IntermExpend_{i,j}}.
$$

□

Remark B.1. *Operationalizing the ad-valorem taxes/subsidies in this way, their empirical content should be understood as an overall extent of wedges that encourage or discourage the purchase of input goods.*

To anchor the scale of firm-level data around aggregate data, I compare the sector-level total revenue calculated from the firm-level data with sectoral gross output.

Definition B.1 (Scale Constant). *For each sector $i \in \mathcal{N}$, the scaling constant $\backepsilon_i$ is defined as*

$$\backepsilon_i := \frac{GrossOutput_i}{\sum_{k=1}^{N_i} p_{ik}^* q_{ik}^*}, \tag{76}$$

where $GrossOutput_i$ represents gross output reported in the BEA data, and $p_{ik}^* q_{ik}^*$ is the firm-level revenue available in the firm-level data (see Appendix B.3).

This scaling constant is used in compiling the firm-level data, as illustrated in Appendix B.3.3.

B.3 Firm-Level Data: Compustat Data

The data source for firm-level data is the Compustat data provided by the Wharton Research Data Services (WRDS). This database provides detailed information about firms' fundamentals, based on financial accounts. For the analysis of this paper, I use the following items: Sales (SALE), Costs of Goods Sold (COGS), Selling, General & Administrative Expense (XSGA), and Number of Employees (EMP). Though the coverage is limited to publicly traded firms, they tend to be much larger than private firms and thus account for the dominant portion of the industry dynamics (Grullon et al., 2019). The construction of the empirical counterparts of the variables in my model follows the existing literature in dropping outliers, as summarized in Appendix B.3.3.

In line with De Loecker et al. (2020) and De Loecker et al. (2021), I consider SALE corresponding to the firm's revenue, COGS to the firm's variable costs, and XSGA to the firm's fixed costs. Since my model abstracts away from fixed entry costs, I need to apportion labor and material input costs between variable and fixed costs to recover labor input and material input. To this end, De Loecker et al. (2020) rely on a parametric assumption, whereas my framework avoids imposing a specific functional-form restriction on the firm-level production. Thus, I instead use a direct measure of the number of employees (EMP) and assume that the cost shares of labor and material inputs are the same across fixed and variable costs.

Assumption B.2 (Constant Cost Share)**.** *For each sector $i \in \mathcal{N}$ and each firm $k \in \mathcal{N}_i$, $VariableLaborCost_{ik} : VariableMaterialCost_{ik} = FixedLaborCost_{ik} : FixedMaterialCost_{ik} = \delta_{ik} : 1 - \delta_{ik}$, where $\delta_{ik} \in [0, 1]$ is a constant specific to firm k.*

This assumption states that $COGS_{ik}$ and $XSGA_{ik}$ are made up of the same proportion of labor and material inputs.

B.3.1 Labor and Material Inputs

As in De Loecker et al. (2021), my construction starts from combining $COGS_{ik}$ and $XSGA_{ik}$ to compute the firm k's total cost:

$$\begin{aligned} TotalCost_{ik} &= TotalLaborCost_{ik} + TotalMaterialCost_{ik} \\ &= \underbrace{VariableLaborCost_{ik} + VariableMaterialCost_{ik}}_{COGS_{ik}} \\ &\quad + \underbrace{FixedLaborCost_{ik} + FixedMaterialCost_{ik}}_{XSGA_{ik}} \qquad (77) \\ &= COGS_{ik} + XSGA_{ik}. \qquad (78) \end{aligned}$$

Since both $COGS_{ik}$ and $XSGA_{ik}$ are observed in the data, I can compute the firm k's total expense ($TotalCost_{ik}$).

Next, the total expenditure on labor input is

$$\begin{aligned} TotalLaborCost_{ik} &= VariableLaborCost_{ik} + FixedLaborCost_{ik} \\ &= W^* \times AverageHoursWorked \times \underbrace{Employees_{ik}}_{EMP_{ik}} \\ &= W^* \times \frac{TotalHours}{TotalEmployees} \times EMP_{ik}. \qquad (79) \end{aligned}$$

From Appendix B.1, both W^* and $TotalHours/TotalEmployees$ are directly observed in the data. Moreover, the Compustat data provide information about the number of employees (EMP_{ik}). Hence, I can calculate the firm k's total labor expense ($TotalLaborCost_{ik}$). Then, the total expenditure on material input is obtained as

$$TotalMaterialCost_{ik} = TotalCost_{ik} - TotalLaborCost_{ik}. \qquad (80)$$

Now, I invoke Assumption B.2 to derive,

$$\delta_{ik} = \frac{TotalMaterialCost_{ik}}{TotalLaborCost_{ik} + TotalMaterialCost_{ik}}, \tag{81}$$

where $TotalLaborCost_{ik}$ and $TotalMaterialCost_{ik}$ can be calculated according to (79) and (80), respectively. Since δ_{ik} is given by (81), I can recover $VariableLaborCost_{ik}$ (the empirical counterpart of $W^*\ell_{ik}^*$) and $VariableMaterialCost_{ik}$ (the empirical counterpart of $P_i^{M*}m_{ik}^*$) according to

$$VariableLaborCost_{ik} = \delta_{ik}COGS_{ik}$$
$$VariableMaterialCost_{ik} = (1 - \delta_{ik})COGS_{ik}.$$

In view of Fact B.1, once outlier eliminations are done (explained in Appendix B.3.3), I can divide the former by the wage W^* to obtain labor input ℓ_{ik}^*, and the latter by the sectoral cost index P_i^{M*} to derive material input m_{ik}^*. These are summarized in the following fact.

Fact B.4 (Labor & Material Inputs)**.** *Under Assumption B.2, the firm-level labor input ℓ_{ik}^* and material input m_{ik}^* are recovered from the data.*

Remark B.2. *In deriving the firm-level input variables ℓ_{ik}^* and m_{ik}^*, the firm's revenue and total cost are scaled up/down by $\ni_i$ (see Definition B.1), so that sectoral revenues computed from the firm-level data coincide with those reported directly in the input-output table.*

B.3.2 Derived Demand for Sectoral Intermediate Goods

Since I lack separate data on firm-level input demand for sectoral intermediate goods, I have to apportion the firm's material expenditure in a way that is consistent with the input-output linkage. To this end, I make an additional assumption on the form of the aggregator function $\mathcal{G}_i(\cdot)$ in (4). Specifically, I assume that material input m_{ik} aggregates sectoral intermediate goods according to the Cobb-Douglas production function.[117]

Assumption B.3. *The material input m_{ik} comprises sectoral intermediate goods according*

[117] In principle, this assumption is necessitated in order to compensate for the limitation of the dataset at hand. This assumption could be relaxed to the extent that allows the researcher to recover the material input and demand for sectoral intermediate goods. Also, this assumption could even be omitted if detailed data on firm-to-firm trade are available, as studied for the Belgium data (Dhyne et al., 2021), the Chilean data (Huneeus, 2020), and the Japanese data (Bernard et al., 2019).

to the Cobb-Douglas production function:

$$m_{ik} = \prod_{j=1}^{N} m_{ik,j}^{\gamma_{i,j}},$$

where $m_{ik,j}$ is sector j's intermediate good demanded by firm k in sector i and $\gamma_{i,j}$ denotes the elasticity of output with respect to sector j's intermediate good, with $\sum_{j=1}^{N} \gamma_{i,j} = 1$.

In view of the structure of the input markets, it is implicit that the input share is the same within sector i. The producer price index for material input P_i^{M*} satisfies the following cost minimization problem:

$$P_i^{M*} = \min_{\{m^{\circ}_{ik,j}\}_{j=1}^{N}} \sum_{j=1}^{N} (1-\tau_i) P_j^* m^{\circ}_{ik,j} \quad s.t. \quad \prod_{j=1}^{N} (m^{\circ}_{ik,j})^{\gamma_{i,j}} \geq 1. \tag{82}$$

Under Assumption B.3, together with (82), I can recover both the material cost index and the input demand for sectoral intermediate goods.

Fact B.5 (Identification of $\gamma_{i,j}$, P_i^{M*} and $m^*_{ik,j}$)**.** *Suppose that Assumption B.3 holds. Then, (i) for each sector $i \in \mathscr{N}$, the input shares $\{\gamma_{i,j}\}_{j=1}^{N}$, and the cost index for material input P_i^{M*} are identified from the observables; and ii) for each sector $i \in \mathscr{N}$ and for each firm $k \in \mathscr{N}_i$, the input demand for composite intermediate goods $\{m^*_{ik,j}\}_{j=1}^{N}$ are identified from the observables.*

Proof. (i) From the first order conditions for the cost minimization, I have

$$(1-\tau_i) P_{j'}^* m^*_{ik,j'} = \frac{\gamma_{i,j'}}{\gamma_{i,j}} (1-\tau_i) P_j^* m^*_{ik,j},$$

Substituting this into (71) leads to

$$\omega_{i,j} = \frac{\sum_{k=1}^{N_i} (1-\tau_i) P_j^* m^*_{ik,j}}{\frac{1}{\gamma_{i,j}} \sum_{k=1}^{N_i} (1-\tau_i) P_j^* m^*_{ik,j} + \sum_{k=1}^{N_i} W^* \ell^*_{ik}},$$

where I note $\sum_{j'=1}^{N} \gamma_{i,j'} = 1$ by assumption. Rearranging this yields

$$\gamma_{i,j} = \frac{\sum_{k=1}^{N_i} (1-\tau_i) P_j^* m^*_{ik,j}}{\frac{1}{\omega_{i,j}} \sum_{k=1}^{N_i} (1-\tau_i) P_j^* m^*_{ik,j} - \sum_{k=1}^{N_i} W^* \ell^*_{ik}} = \frac{\omega_{i,j}}{\sum_{j'=1}^{N} \omega_{i,j'}}.$$

Since the terms in the rightmost expression $\{\omega_{i,j'}\}_{j'=1}^{N}$ are available in the data (see Appendix B.2.1), the parameter $\gamma_{i,j}$ can then be identified for all $i \in \mathscr{N}$.

From (82), the equilibrium value of the cost index for material input P_i^{M*} is given by

$$P_i^{M*} = \prod_{j=1}^{N} \frac{1}{\gamma_{i,j}^{\gamma_{i,j}}} \{(1-\tau_i)P_j^*\}^{\gamma_{i,j}}. \tag{83}$$

Given that $\{\gamma_{i,j}\}_{j=1}^N$ are identified above, (83) recovers P_i^{M*}.

(ii) Now, using the first order condition for the cost minimization problem again, I have

$$(1-\tau_i)P_j^* = \nu_{ik}\gamma_{i,j}\frac{m_{ik}^*}{m_{ik,j}^*},$$

where ν_{ik} is the firm k's marginal cost of constructing an additional unit of material input (De Loecker and Warzynski, 2012; De Loecker et al., 2016, 2020), which equals P_i^{M*}. Hence, it follows

$$m_{ik,j}^* = \gamma_{i,j}\frac{P_i^{M*}}{(1-\tau_i)P_j^*}m_{ik}^*, \tag{84}$$

from which $m_{ik,j}^*$, the input demand for sector j's composite intermediate good from sector i, is identified. This completes the poof. □

B.3.3 Data Construction

My dataset spans from 2010 to 2021. I do not exploit the time-series variation; rather, I regard it as a collection of snapshots of the same economy, where each year constitutes an individual snapshot. In this way, I can construct "repeated samples." This setup is plausible in view of the model (Section 2) and the identifying assumptions (Section 4), and is in line with the approach adopted in Ackerberg and De Loecker (2024). In constructing individual snapshots, I pick up those firms that operate in the previous year, as well as the current year. This means that my dataset only collects incumbent firms in line with the setup of my model. I follow the existing literature (e.g., Baqaee and Farhi, 2020; De Loecker et al., 2021) in eliminating entries with missing data or zeros, and in dropping firms in the top and bottom 1% percentiles.

Furthermore, the proportional relationship between labor and material input suggested by Lemma C.11 gives a rationale for an additional outlier elimination. I first run a linear regression with labor input being the dependent variable and material input (together with an intercept term) being the independent variable.[118] Next, for each data point, I calculate the Cook's distance (Cook, 1977, 1979). Then, data points with Cook's distance higher than

[118] The roles of labor and material inputs could be switched.

a certain threshold are removed as influential points. In light of the difference in sample size between sectors, I use an adaptive criterion for influential points: A data point with Cook's distance no less than $(K+1)/(N_i - K - 1)F_{K+1,N_i-K-1}(0.99)$ is considered to be influential, where K indicates the number of independent variable other than the intercept (i.e., $K = 1$); N_i represents the number of firms in sector i; and $F_{a,b}(c)$ stands for the critical value of the F distribution with degrees of freedom a and b at the nominal size c.

The procedure for constructing the firm-level dataset is summarized as follows:

Part A: For each year (i.e., each snapshot), the data goes through the following procedures:

Step 1: Eliminate entries with missing data or zeros in either SALE, COGS, XSGA, or EMP.

Step 2: Drop firms that are not operative in the previous year.

Step 2: Drop firms with negative profits, which is calculated as SALE minus COGS minus XSGA.

Step 3: Omit firms with SALE-to-COGS and SALE-to-XSGA ratios in the top and bottom 1%.

Step 4: Apply the results developed in Appendices B.3.1 and B.3.2 to construct the dataset for firm-level variables.

Part B: The individual snapshots are concatenated according to the following procedures:

Step 1: Drop firms that are not operative in any of the individual snapshots.

Step 2: For each sector, regress material input on labor input and determine influential points that are to be removed as outliers.

Step 2: Drop firms whose labor or material inputs in 2021 do not fall into the historical support between 2010 and 2020.

Step 3: For each sector, scale up/down firm-level variables so that the sectoral revenue computed from the firm-level data equals the gross output observed in the aggregate data.

B.3.4 Market Concentration

To study the degree of oligopolistic competition in each industry, I calculate the concentration ratio, a measure defined as the sum of market shares of the largest firms in an industry. In light of the difference in the number of firms, I consider both the four- and eight-firm concentration ratios, denoted by CR4 and CR8, respectively. Table 3 displays the CR4 and CR8 of each industry, along with the number of firms. According to the classification proposed in Shepherd (2018), it is fair to say that all the industries in my dataset fall into the categories of either loose oligopoly or tight oligopoly.[119]

[119]It should be remarked that the industry definition of my analysis is closer to the three-digit NAICS classification, a broader categorization than the four- or five-digit classifications, which are commonly used in the analysis of market concentration, such as antitrust and meager analyses. Due to this coarser definition, the CRs in this paper may well appear to be lower compared to other studies based on much finer industry codes. Moreover, it is this construction that motivates the use of the CRs over the Herfindahl-Hirschman Index (HHI).

Table 3: Number of Firms and Concentration Ratio

Industry	Number of firms	CR4	CR8
Oil and gas extraction and mining	23	0.66	0.83
Construction	13	0.65	0.87
Food and beverage and tobacco products	48	0.34	0.53
Textile mills and apparel products	28	0.41	0.61
Wood and nonmetallic mineral products	13	0.65	0.91
Paper products and related services	15	0.63	0.87
Petroleum and coal products	11	0.75	0.98
Chemical products	80	0.27	0.43
Plastics and rubber products	8	0.86	1.00
Primary metals	17	0.60	0.84
Fabricated metal products	34	0.42	0.61
Machinery	55	0.32	0.54
Computer and electronic products	119	0.25	0.40
Electrical equipment, appliances, and components	18	0.71	0.86
Motor vehicles, bodies and trailers, and parts	37	0.58	0.78
Other transportation equipment	19	0.54	0.80
Furniture and related products	8	0.74	1.00
Miscellaneous manufacturing	30	0.39	0.66
Wholesale trade	53	0.37	0.56
Retail Trade	61	0.40	0.61
Transportation	24	0.52	0.78
Information	101	0.39	0.51
Professional services	42	0.43	0.61
Administrative and waste management	22	0.49	0.73
Educational services	7	0.82	1.00
Accommodation and food services	11	0.70	0.96

Note: This table displays the number of firms and the four- and eight-firm concentration ratio (CR4 and CR8) of each industry. The definition of industry is based on the segmentation shown in Table B.2.

C Identification

The goal of this section is to prove Theorem 4.1. The proof requires recovering firm-level quantities and prices, and comparative statics of both sector- and firm-level variables. The latter, moreover, calls for the identification of the derivatives of firm-level production and inverse demand functions. To this end, I exploit the identifying assumptions detailed in Section 4 in conjunction with the model defined in Section 2, and the data described in Section 3. In what follows, I first derive (22), before proceeding to the identification of firm-level price and quantity, and the identification of the derivatives of the production and inverse demand functions.

C.1 Derivation of (22)

The derivation of (22) draws on the exchangeability inherent to the quantity index $A_i(\cdot)$ in (21),[120] and the characterization result concerning exchangeable functions, with the latter being derived in the recent literature on computer science.

First of all, I show that under Assumption 4.4, the quantity index $A_i(\cdot)$ is exchangeable in $\{q_{ik'}\}_{k'=1}^{N_i}$.[121]

Fact C.1. *Suppose that Assumption 4.4 holds. Then, for each $i \in \mathscr{N}$, the quantity index $A_i(\cdot)$ is exchangeable in $(q_{i1}, q_{i2}, \cdots, q_{iN_i})$.*

Proof. Assume for the sake of contradiction that the quantity index $A_i(\cdot)$ in (21) is not exchangeable in $(q_{i1}, q_{i2}, \cdots, q_{iN_i})$. Then, there exists a permutation $\hat{\varsigma}_i \coloneqq (\hat{\varsigma}(1), \hat{\varsigma}(2), \ldots, \hat{\varsigma}(N_i))$ such that

$$A_i(\mathbf{q}_i) \coloneqq A_i(q_{i1}, q_{i2}, \cdots, q_{iN_i}) \neq A_i(q_{i\hat{\varsigma}(1)}, q_{i\hat{\varsigma}(2)}, \cdots, q_{i\hat{\varsigma}(N_i)}) =: A_i(\mathbf{q}_{i\hat{\varsigma}_i}).$$

Without loss of generality, I can concentrate on the case of

$$A_i(q_{i1}, q_{i2}, \cdots, q_{iN_i}) > A_i(q_{i\hat{\varsigma}(1)}, q_{i\hat{\varsigma}(2)}, \cdots, q_{i\hat{\varsigma}(N_i)}),$$

which can compactly be written as $A_i(\mathbf{q}_i) > A_i(\mathbf{q}_{i\hat{\varsigma}_i})$. Here, since $\Psi_i(\cdot)$ in (21) is increasing

[120] A function $h(x_1, \ldots, x_n)$ is said to be exchangeable (or permutation invariant) in $(x_1, \ldots, x_n)$ if $h(x_1, \ldots, x_n) = h(x_{\varsigma(1)}, \ldots, x_{\varsigma(n)})$ for all ς, where $\varsigma \coloneqq (\varsigma(1), \ldots, \varsigma(n))$ is a permutation of $(1, \ldots, n)$. See Kallenberg (2005) and de Finetti (2017) for the concept of exchangeability.

[121] I thank Yoichi Sugita for providing the proof.

in $\frac{q_{ik}}{A_i}$ for all $k \in \mathscr{N}_i$, it thus holds that

$$\sum_{k'=1}^{N_i} \Psi_i\Big(\frac{q_{ik'}}{A_i(\mathbf{q}_i)}\Big) < \sum_{k'=1}^{N_i} \Psi_i\Big(\frac{q_{ik'}}{A_i(\mathbf{q}_{i\hat{\varsigma}_i})}\Big). \tag{85}$$

However, each of $A_i(\mathbf{q}_i)$ and $A_i(\mathbf{q}_{i\hat{\varsigma}_i})$ satisfies (21), implying

$$\sum_{k'=1}^{N_i} \Psi_i\Big(\frac{q_{ik'}}{A_i(\mathbf{q}_i)}\Big) = 1 = \sum_{k'=1}^{N_i} \Psi_i\Big(\frac{q_{ik'}}{A_i(\mathbf{q}_{i\hat{\varsigma}_i})}\Big),$$

which contradicts (85). This proves by way of contradiction that the quantity index $A_i(\cdot)$ is exchangeable in $(q_{i1}, q_{i2}, \cdots, q_{iN_i})$. □

Next, I take advantage of the recently developed characterization result concerning exchangeable functions. For the sake of exposition, the main result is summarized as a lemma below.

Lemma C.1 (Subdecomposition (Zaheer et al., 2018; Wagstaff et al., 2019))**.** *Let $J \in \mathbb{N}$, and let $h : [0,1]^J \to \mathbb{R}$ be a continuous function. Then, $h(x_1, \ldots, x_J)$ is exchangeable in $(x_1, \ldots, x_J)$ if and only if it can be expressed as $h(x_1, \ldots, x_J) = \upsilon(\sum_{j=1}^{J} \rho(x_j))$ for some outer function $\upsilon : \mathbb{R}^{J+1} \to \mathbb{R}$ and some inner function $\rho : \mathbb{R} \to \mathbb{R}^{J+1}$.*

Proof. See Zaheer et al. (2018) and Wagstaff et al. (2019). □

Now, the expression (22) can be proved by the multiple application of this lemma.

Proposition C.1. *Suppose that Assumption 4.4 holds. Then, for each $i \in \mathscr{N}$, there exists a constant $M_i \in \mathbb{N}$ such that there exist some continuous functions $\mathcal{H}_{i,1}, \ldots, \mathcal{H}_{i,M_i} : \mathscr{Z}_i^{\mathscr{N}_i} \to \mathbb{R}$ and $\chi_i : \mathscr{Z}_i \times \mathbb{R}^{M_i} \to \mathbb{R}_+$ such that*

$$q_{ik}^* = \chi_i(z_{ik}; \mathcal{H}_{i,1}(\mathbf{z}_i), \ldots, \mathcal{H}_{i,M_i}(\mathbf{z}_i)),$$

where $\mathcal{H}_{i,m}(\mathbf{z}_i)$ is exchangeable in $(z_{i1}, \ldots, z_{iN_i})$ for all $m \in \{1, \ldots, M_i\}$.

Proof. First of all, it follows from Fact C.1 and Lemma C.1 that there exist continuous functions $\upsilon_0 : \mathbb{R}^{N_i+1} \to \mathbb{R}$ and $\rho_0 : \mathbb{R} \to \mathbb{R}^{N_i+1}$ such that

$$A_i(\{q_{ik'}\}_{k'=1}^{N_i}) = \upsilon_0(\sum_{k'=1}^{N_i} \rho_0(q_{ik'})).$$

In consequence, the partial derivative of $A_i(\cdot)$ with respect to q_{ik} is given by

$$\frac{\partial A_i(\cdot)}{\partial q_{ik}} = (\upsilon_0'(\sum_{k'=1}^{N_i} \rho_0(q_{ik'})))^T \rho_0'(q_{ik}),$$

where $\upsilon_0'(\cdot)$ and $\rho_0'(\cdot)$ are $(N_i + 1) \times 1$ vectors whose kth entry indicates the derivatives of $\upsilon_i(\cdot)$ and $\rho_0(\cdot)$ with respect to the kth argument, respectively; and T denotes the transpose of a vector.

Next, let $mc_{ik} = mc_i(z_{ik})$ be the firm k's marginal cost. Due to Assumption 2.4 *(i)*, mc_{ik} is independent of the firm's output quantity q_{ik}. Under Assumption 4.4, the Cournot-Nash equilibrium quantities satisfy the following system of first-order conditions:

$$\Phi_i \Psi_i'\left(\frac{q_{ik}}{A_i(\{q_{ik'}\}_{k'=1}^{N_i})}\right) \frac{A_i(\{q_{ik'}\}_{k'=1}^{N_i}) - \frac{\partial A_i(\cdot)}{\partial q_{ik}}}{A_i(\{q_{ik'}\}_{k'=1}^{N_i})^2} = mc_{ik},$$

for all $k \in \mathscr{N}_i$. Note that the firm k's identity can alternatively be traced via the marginal costs mc_{ik}. Thus, it holds by symmetry that there exists a constant $M_i \in \mathbb{N}$ such that $H_{i,1}, \ldots, H_{i,M_i} : \mathbb{R}_+^{N_i} \to \mathbb{R}$ and $\chi_i^a : \mathscr{Z} \times \mathbb{R}^{M_i} \to \mathbb{R}$ such that

$$q_{ik}^* = \chi_i^a\big(mc_{ik}; H_{i,1}(\{mc_{ik'}\}_{k' \neq k}), \ldots, H_{i,M_i}(\{mc_{ik'}\}_{k' \neq k})\big),$$

where each of $H_{i,1}(\cdot), \ldots, H_{i,M_i}(\cdot)$ is exchangeable in $(mc_{i1}, \ldots, mc_{i(k-1)}, mc_{i(k+1)}, \ldots, mc_{iN_i})$. Again, by Lemma C.1, this can further be rewritten as

$$\begin{aligned} q_{ik}^* &= \chi_i^a\Bigg(mc_{ik}; \upsilon_1^a\Big(\sum_{k' \neq k} \rho_1(mc_{ik'})\Big), \ldots, \upsilon_{M_i}^a\Big(\sum_{k' \neq k} \rho_{M_i}(mc_{ik'})\Big)\Bigg) \\ &= \chi_i^b\Bigg(mc_{ik}; \sum_{k' \neq k} \rho_1(mc_{ik'}), \ldots, \sum_{k' \neq k} \rho_{M_i}(mc_{ik'})\Bigg) \\ &= \chi_i^b\Bigg(mc_{ik}; \sum_{k'=1}^{N_i} \rho_1(mc_{ik'}) - \rho_1(mc_{ik}), \ldots, \sum_{k'=1}^{N_i} \rho_{M_i}(mc_{ik'}) - \rho_{M_i}(mc_{ik})\Bigg) \\ &= \chi_i^c\Bigg(mc_{ik}; \sum_{k'=1}^{N_i} \rho_1(mc_{ik'}), \ldots, \sum_{k'=1}^{N_i} \rho_{M_i}(mc_{ik'})\Bigg) \\ &= \chi_i^d\Bigg(mc_{ik}; \upsilon_1^b\Big(\sum_{k'=1}^{N_i} \rho_1(mc_{ik'})\Big), \ldots, \upsilon_{M_i}^b\Big(\sum_{k'=1}^{N_i} \rho_{M_i}(mc_{ik'})\Big)\Bigg), \end{aligned}$$

for some functions $\{\rho_m(\cdot)\}_{m=1}^{M_i}$, $\{\upsilon_m^a(\cdot)\}_{m=1}^{M_i}$, $\{\upsilon_m^b(\cdot)\}_{m=1}^{M_i}$, $\chi_i^b(\cdot)$, $\chi_i^c(\cdot)$ and $\chi_i^d(\cdot)$, each of which is appropriately defined. Applying once again Lemma C.1, it follows that for each $m =$

$1, \ldots, M_i$,

$$\check{H}_{i,m}(\{mc_{ik'}\}_{k'=1}^{N_i}) := \upsilon_m^b\Big(\sum_{k'=1}^{N_i} \rho_m(mc_{ik'})\Big)$$

is exchangeable in $(mc_{i1}, \ldots, mc_{iN_i})$. Hence, the equilibrium quantity can be written as

$$q_{ik}^* = \chi_i^d\big(mc_{ik}; \check{H}_{i,1}(\{mc_{ik'}\}_{k'=1}^{N_i}), \ldots, \check{H}_{i,M_i}(\{mc_{ik'}\}_{k'=1}^{N_i})\big).$$

Since $mc_{ik} = mc_i(z_{ik})$, this can in turn be rearranged so that there exist some functions $\mathcal{H}_{i,1}, \ldots, \mathcal{H}_{i,M_i} : \mathscr{Z}^{N_i} \to \mathbb{R}$ and $\chi_i : \mathscr{Z} \times \mathbb{R}^{M_i} \to \mathbb{R}$ such that

$$q_{ik}^* = \chi_i\big(z_{ik}; \mathcal{H}_{i,1}(\{z_{ik'}\}_{k'=1}^{N_i}), \ldots, \mathcal{H}_{i,M_i}(\{z_{ik'}\}_{k'=1}^{N_i})\big),$$

where each of $\mathcal{H}_{i,1}(\cdot), \ldots, \mathcal{H}_{i,M_i}(\cdot)$ is, by construction, exchangeable in $(z_{i1}, \ldots, z_{iN_i})$. This proves the proposition. □

C.1.1 Detail of Example 4.1

As studied in Example C.1, suppose that the sectoral aggregator takes the form of a CES function: $F_i(\{q_{ik}\}_{k\in\mathscr{N}_i}) := \big(\sum_{k=1}^{N_i} \delta_i q_{ik}^{\frac{\sigma-1}{\sigma}}\big)^{\frac{\sigma}{\sigma-1}}$. As shown in Example C.1, the associated inverse demand function is given by $p_{ik} = \frac{\Phi_i}{q_{ik}} \frac{\delta_i q_{ik}^{\frac{\sigma-1}{\sigma}}}{\sum_{k'=1}^{N_i} \delta_i q_{ik'}^{\frac{\sigma-1}{\sigma}}}$, and the quantity index can be expressed as $A_i(\mathbf{q}_i) = \frac{1}{B_0}\sum_{k'=1}^{N_i} \delta_i q_{ik'}^{\frac{\sigma-1}{\sigma}}$, where B_0 is a normalization constnat. In the interest of clarity of exposition, assume that there are only three firms in each sector, i.e., $\mathscr{N}_i = \{1, 2, 3\}$, and consider the case of $\sigma = 2$, $\delta_i = 1$ and $B_0 = 1$. Assume in addition that the firm's production technology is given by a Cobb-Douglas function: $q_{ik} = z_{ik}\ell_{ik}^{\alpha} m_{ik}^{1-\alpha}$, while the material aggregator $\mathcal{G}_i(\cdot)$ is left unspecified.

Under this setup, the Cournot-Nash equilibrium quantities $\{q_{ik}^*\}_{k=1}^3$ satisfy the following system of equations:

$$\frac{\frac{\sigma-1}{\sigma} q_{i1}^{*\,-\frac{1}{\sigma}}(A_i^* - q_{i1}^{*\,\frac{\sigma-1}{\sigma}})}{A_i^{*2}}\Phi_i = mc_{i1}$$
$$\frac{\frac{\sigma-1}{\sigma} q_{i2}^{*\,-\frac{1}{\sigma}}(A_i^* - q_{i2}^{*\,\frac{\sigma-1}{\sigma}})}{A_i^{*2}}\Phi_i = mc_{i2}$$
$$\frac{\frac{\sigma-1}{\sigma} q_{i3}^{*\,-\frac{1}{\sigma}}(A_i^* - q_{i3}^{*\,\frac{\sigma-1}{\sigma}})}{A_i^{*2}}\Phi_i = mc_{i3},$$

where A_i^* represents the equilibrium value of the quantity index, and $mc_{ik} \coloneqq z_{ik}^{-1} mc_i$ is the firm k's marginal cost.[122] In particular, when $\sigma = 2$, this system can be solved for the equilibrium quantities, yielding

$$q_{ik}^* = \Big(\frac{A_i^* \Phi_i}{2 A_i^{*2} mc_{ik} + \Phi_i} \Big)^2 \tag{86}$$

for each $k \in \{1, 2, 3\}$. By construction, the equilibrium quantity index A_i^* satisfies

$$\begin{aligned} A_i^* &= q_{i1}^{*\frac{1}{2}} + q_{i2}^{*\frac{1}{2}} + q_{i3}^{*\frac{1}{2}} \\ &= \frac{A_i^* \Phi_i}{2 A_i^{*2} mc_{i1} + \Phi_i} + \frac{A_i^* \Phi_i}{2 A_i^{*2} mc_{i2} + \Phi_i} + \frac{A_i^* \Phi_i}{2 A_i^{*2} mc_{i3} + \Phi_i}. \end{aligned}$$

Rearranging this leads to

$$8 mc_{i1} mc_{i2} mc_{i3} A_i^{*6} - 2(mc_{i1} + mc_{i2} + mc_{i3}) \Phi_i^2 A_i^{*2} - 2\Phi_i^3 = 0.$$

Noticing that A_i^* has to be a real number, it follows from the general cubic formula (or the Cardano formula) that

$$A_i^{*2} = -\sqrt[3]{B} - \sqrt[3]{C}, \tag{87}$$

where $B = \frac{3\sqrt{3}t + \sqrt{27t^2 + s^3}}{6\sqrt{3}}$ and $C = \frac{3\sqrt{3}t - \sqrt{27t^2 + s^3}}{6\sqrt{3}}$ with $s = -\frac{mc_{i1} + mc_{i2} + mc_{i3}}{4 mc_{i1} mc_{i2} mc_{i3}} \Phi_i = -\frac{z_{i1}^{-1} + z_{i2}^{-1} + z_{i3}^{-1}}{4(z_{i1} z_{i2} z_{i3})^{-1} mc_i^2}$ and $t = -\frac{\Phi_i^3}{4 mc_{i1} mc_{i2} mc_{i3}} = -\frac{\Phi_i^3}{4(z_{i1} z_{i2} z_{i3})^{-1} mc_i^3}$.

Combining (86) and (87), one obtains

$$\begin{aligned} q_{ik}^* &= \frac{\Phi_i^2 A_i^{*2}}{(2 mc_{ik} A_i^{*2} + \Phi_i)^2} \\ &= \chi_i(z_{ik}; \mathcal{H}_{i,1}(\{z_{ik'}\}_{k'=1}^3), \mathcal{H}_{i,2}(\{z_{ik'}\}_{k'=1}^3)), \end{aligned}$$

for some continuous function $\chi_i(\cdot)$, where $\mathcal{H}_{i,1}(\{z_{ik'}\}_{k'=1}^3) \coloneqq z_{i1}^{-1} + z_{i2}^{-1} + z_{i3}^{-1}$ and $\mathcal{H}_{i,2}(\{z_{ik'}\}_{k'=1}^3) \coloneqq z_{i1} z_{i2} z_{i3}$. Note here that both $\mathcal{H}_{i,1}(\cdot)$ and $\mathcal{H}_{i,2}(\cdot)$ are clearly exchangeable in (z_{i1}, z_{i2}, z_{i3}).

Next, the subsequent input choice — specifically, the inner optimization of (6) — is constrained by the production possibility frontier

$$\chi_i(z_{ik}; \mathcal{H}_{i,1}(\{z_{ik'}\}_{k'=1}^3), \mathcal{H}_{i,2}(\{z_{ik'}\}_{k'=1}^3)) = q_{ik}^* = z_{ik} \ell_{ik}^{\alpha} m_{ik}^{1-\alpha}.$$

[122] Precisely speaking, mc_i represents the component of the marginal cost common to all firms, and it is given by $mc_i = \alpha^{-\alpha}(1-\alpha)^{1-\alpha} W^{*\alpha} (P_i^{M*})^{1-\alpha}$.

Since $\chi_i(\cdot)$ obviously satisfies Assumption 4.5, this equation can be solved for z_{ik}. By the quadratic formula, it holds in equilibrium that

$$
\begin{aligned}
z_{ik} = \frac{1}{2{\ell^*_{ik}}^{\alpha}{m^*_{ik}}^{1-\alpha}\Phi_i}\bigg\{ &- (4mc_i{\ell^*_{ik}}^{\alpha}{m^*_{ik}}^{1-\alpha}A_i^{*2}\Phi_i - A_i^{*2}\Phi_i^2) \\
&\pm \sqrt{(4mc_i{\ell^*_{ik}}^{\alpha}{m^*_{ik}}^{1-\alpha}A_i^{*2}\Phi_i - A_i^{*2}\Phi_i^2)^2 - 16mc_i^2({\ell^*_{ik}}^{\alpha}{m^*_{ik}}^{1-\alpha})^2A_i^{*2}\Phi_i}\bigg\} \\
=: \mathcal{M}_i(\ell^*_{ik}, m^*_{ik}; \mathcal{H}_{i,1}(\{z_{ik'}\}_{k'=1}^3), \mathcal{H}_{i,2}(\{z_{ik'}\}_{k'=1}^3)).
\end{aligned}
$$

This shows the existence of a function $\mathcal{M}_i(\cdot)$ by giving it an analytical expression.

C.2 Preliminary Results

This subsection derives several key implications of the identifying assumptions — Assumption 4.4 in particular. The derived results are used repeatedly in the subsequent proofs.

To begin with, I introduce logarithmic notation. Let $\mathscr{R}_i$, $\mathscr{L}_i$ and $\mathscr{M}_i$ be the observed supports of revenue r_{ik}, labor input ℓ_{ik} and material input m_{ik}, respectively. To facilitate exposition, I introduce a tilde notation to denote the logarithm of each variable. For instance, I write the logarithms of the firm's revenue, labor input, material input, and productivity as $\tilde{r}_{ik}$, $\tilde{\ell}_{ik}$, $\tilde{m}_{ik}$ and $\tilde{z}_{ik}$, respectively. Correspondingly, the observed supports for r_{ik}, ℓ_{ik} and m_{ik} are denoted by $\tilde{\mathscr{R}}_i$, $\tilde{\mathscr{L}}_i$ and $\tilde{\mathscr{M}}_i$, respectively. Also, the logarithms of the firm's output quantity and price are expressed, respectively, as

$$
\tilde{q}_{ik} \coloneqq \ln q_{ik} = \tilde{f}_i(\tilde{\ell}_{ik}, \tilde{m}_{ik}; \tilde{z}_{ik}), \tag{88}
$$

and

$$
\tilde{p}_{ik} \coloneqq \ln p_{ik} = \tilde{\wp}_i(\tilde{q}_{ik}, \tilde{A}_i(\tilde{\mathbf{q}}_i); \mathcal{I}_i), \tag{89}
$$

where $\tilde{f}_i(\cdot) \coloneqq (\ln \circ f_i \circ \exp)(\cdot)$, $\tilde{\wp}_i(\cdot) \coloneqq (\ln \circ \wp_i \circ \exp)(\cdot)$, and $\tilde{A}_i(\cdot) \coloneqq (\ln \circ A_i \circ \exp)(\cdot)$. In what follows, I let both the quantity index $\tilde{A}_i(\cdot)$ and the information set $\mathcal{I}_i$ be absorbed in the sector index i for the sake of brevity.

C.2.1 HSA Demand System

With the notation defined so far, the HSA demand system in Assumption 4.4 can be expressed as follows. First, it holds, by definition, that

$$\Phi_i \coloneqq \sum_{k=1}^{N_i} p_{ik}^* q_{ik}^*,$$

where p_{ik}^* and q_{ik}^* are the equilibrium (realized) values of the firm k's output price and quantity, respectively. Then, I can take

$$\Phi_i = \sum_{k=1}^{N_i} \varphi_i(q_{ik}^*), \tag{90}$$

where $r_{ik} = \varphi_i(q_{ik})$ with $\varphi_i(\cdot) \coloneqq (\exp \circ \tilde{\varphi}_i \circ \ln)(\cdot)$.

Next, let $\wp_i(q_{ik}, \mathbf{q}_{i,-k}) = \wp_{ik}(\mathbf{q}_i)$ be the residual inverse demand function faced by firm k in sector i. Under Assumption 4.4, it takes the form of

$$p_{ik} = \frac{\Phi_i}{q_{ik}} \Psi_i\left(\frac{q_{ik}}{A_i(\mathbf{q}_i)}\right) =: \wp_i(q_{ik}; \mathbf{q}_{i,-k}), \tag{91}$$

where

$$\Psi_i(q_{ik}) = \frac{\varphi_i(q_{ik})}{\Phi_i}, \tag{92}$$

with

$$\sum_{k=1}^{N_i} \Psi_i\left(\frac{q_{ik}}{A_i(\mathbf{q}_i)}\right) = 1. \tag{93}$$

Example C.1 (CES aggregator)**.** *For each sector $i \in \mathscr{N}$, consider the CES aggregator: $F_i(\{q_{ik}\}_{k\in\mathscr{N}_i}) \coloneqq \left(\sum_{k=1}^{N_i} \delta_i^{\sigma_i} q_{ik}^{\frac{\sigma_i-1}{\sigma_i}}\right)^{\frac{\sigma_i}{\sigma_i-1}}$, where σ_i represents the elasticity of substitution specific to the sector, and δ_i is a demand shifter specific to sector i.*[123] *Associated with this is the residual inverse demand curve faced by firm k: $p_{ik} = \frac{\Phi_i}{q_{ik}} \frac{\delta_i q_{ik}^{\frac{\sigma_i-1}{\sigma_i}}}{\sum_{k'=1}^{N_i} \delta_i q_{ik'}^{\frac{\sigma_i-1}{\sigma_i}}}$. Assumption 4.4 is then satisfied by setting $\Psi_i(x; \mathcal{I}_i) \coloneqq \delta_i B_0^{\frac{\sigma_i-1}{\sigma_i}} x^{\frac{\sigma_i-1}{\sigma_i}}$ with $A_i(\mathbf{q}_i) = \frac{1}{B_0} \sum_{k'=1}^{N_i} \delta_i q_{ik'}^{\frac{\sigma_i-1}{\sigma_i}}$, where B_0 is a normalization constant.*

[123] The CES aggregator is routinely assumed in the bulk of the macroeconomics literature on international pricing (Atkeson and Burstein, 2008; Amiti et al., 2014; Gaubert and Itskhoki, 2020).

Taking derivatives of the both hand sides of (93), one obtains

$$\frac{q_{ik}}{A_i(\mathbf{q}_i)}\frac{\partial A_i(\cdot)}{\partial q_{ik}} = \frac{\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\exp\{\tilde{\varphi}_i(\tilde{x}_{ik})\}}{\sum_{k'=1}^{N_i}\frac{d\tilde{r}_{ik'}}{d\tilde{x}_{ik'}}\exp\{\tilde{\varphi}_i(\tilde{x}_{ik'})\}}, \tag{94}$$

where $x_{ik} = \frac{q_{ik}}{A_i(\mathbf{q}_i)}$, and $\tilde{x}_{ik} = \ln x_{ik}$. Notice here that the right-hand side of (94) represents a weighted revenue-based market share with the weight attached to the derivatives of log revenue with respect to $\tilde{x}_{ik}$. Given this observation, denote $\tilde{u}_{ik} = \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\exp\{\tilde{\varphi}_i(\tilde{x}_{ik})\}$, and $\varpi_{ik} = \frac{\tilde{u}_{ik}}{\sum_{k'=1}^{N_i}\tilde{u}_{ik'}}$. Define moreover $\varrho_{ik} := \big(\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\big)^{-1}\frac{d^2\tilde{r}_{ik}}{d\tilde{x}_{ik}^2} + \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}$, and $t_{ik} := \frac{\varpi_{ik}}{q_{ik}}\big(\varrho_{ik} - \frac{\sum_{k'=1}^{N_i}\varrho_{ik'}\tilde{u}_{ik'}}{\sum_{k'=1}^{N_i}\tilde{u}_{ik'}}\big)$.

The identification of these variables proceeds in multiple steps. I first show that $\tilde{u}_{ik}$ and ϖ_{ik} are directly recovered from the observables (Fact C.2). This immediately restores the identification of $\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}$ (Lemma C.3), which is used in identifying firm-level output quantity and price (Proposition C.3). This recovers firm-level revenue in terms of the firm's output quantity (Corollary C.1). Then, the knowledge about $\tilde{u}_{ik}$ and ϖ_{ik} is again used to obtain the remaining two variables, namely, ϱ_{ik} and t_{ik} (Corollary C.2).

Fact C.2. *Suppose that Assumption 2.4 holds. Assume moreover that Assumption 4.4 holds with* (90)–(93). *Then,* $\{\tilde{u}_{ik'}\}_{k'=1}^{N_i}$ *and* $\{\varpi_{ik'}\}_{k'=1}^{N_i}$ *can be expressed as a function of firm-level labor and material input variables as well as aggregate variables.*

Proof. It holds from the first order condition for the profit maximization problem that

$$\begin{aligned}
&\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}p_{ik}^*(1-\varpi_{ik}) = mc_{ik} \\
&\therefore \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}p_{ik}^*q_{ik}^*(1-\varpi_{ik}) = mc_{ik}q_{ik}^* \\
&\quad\therefore \tilde{u}_{ik}\Big(1 - \frac{\tilde{u}_{ik}}{\sum_{k'=1}^{N_i}\tilde{u}_{ik'}}\Big) = mc_{ik}q_{ik}^*,
\end{aligned}$$

where the last implication follows from the definition of $\tilde{u}_{ik}$ and ϖ_{ik}. Here, notice that due to Assumption 2.4 (i), the right-hand side of the last equation equals the firm's total cost and can be expressed as $W^*\ell_{ik}^* + P_i^{M*}m_{ik}^*$, which is known to the econometrician. This means that this equation constitutes a system of N_i equations for N_i unknown variables $\{\tilde{u}_{ik'}\}_{k'=1}^{N_i}$. Solving this system of equations yields $\{\tilde{u}_{ik'}\}_{k'=1}^{N_i}$ as a function of firm-level labor and material inputs as well as aggregate variables.

Given the identification of $\{\tilde{u}_{ik'}\}_{k'=1}^{N_i}$, ϖ_{ik} is also identified by following its definition: $\varpi_{ik} = \frac{\tilde{u}_{ik}}{\sum_{k'=1}^{N_i}\tilde{u}_{ik'}}$.

This means that both $\tilde{u}_{ik'}$ and $\varpi_{ik'}$, for all $k' \in \mathscr{N}_i$, can be expressed in terms of firm-level labor and material input variables as well as aggregate variables, as claimed. □

C.2.2 Identification of the Values of Markup

Under the structure of the input markets imposed in the main text (these assumptions are presented in Section 2.3 and summarized below for ease of reference), firm-level markups are recovered from the observables.[124]

Assumption C.1 (Input Markets)**.** *(i) The input markets are perfectly competitive. (ii) All inputs are variable.*

Fact C.3. *Suppose that Assumptions 2.4 and C.1 hold. For each sector $i \in \mathscr{N}$ and each firm $k \in \mathscr{N}_i$, the equilibrium value of the firm's markup μ^*_{ik} can be recovered from the data.*

Proof. Under Assumption C.1, the equilibrium value of the firm's markup μ^*_{ik} can be expressed as:

$$\mu^*_{ik} := \frac{p^*_{ik}}{MC^*_{ik}} = \frac{Revenue^*_{ik}}{TC^*_{ik}} \frac{AC^*_{ik}}{MC^*_{ik}},$$

where MC^*_{ik}, AC^*_{ik}, and TC^*_{ik} represent the equilibrium values of the marginal, average, and total costs, respectively. Note here that $\frac{AC^*_{ik}}{MC^*_{ik}}$ is the elasticity of cost with respect to quantity (Syverson, 2019), which equals one due to Assumption 2.4 (i). Hence, I have

$$\mu^*_{ik} = \frac{Revenue^*_{ik}}{TC^*_{ik}},$$

i.e., the value of the firm's markup equals the ratio of revenue to total costs, both of which are observed in the data. Thus, the value of the firm-level markup μ^*_{ik} is identified from the observables, as desired. □

C.3 Recovering the Values of Firm-Level Quantity and Price

In this subsection, I first derive several preliminary results before moving to the identification of firm-level output quantity and price.

C.3.1 Preliminary Results

Let $\frac{\partial \tilde{f}_i(\cdot)^*}{\partial \tilde{\ell}_{ik}}$ and $\frac{\partial \tilde{f}_i(\cdot)^*}{\partial \tilde{m}_{ik}}$, respectively, denote the equilibrium values of the first-order derivatives of the log-production function with respect to log-labor and log-material, i.e.,

$$\frac{\partial \tilde{f}_i(\cdot)^*}{\partial \tilde{\ell}_{ik}} := \left. \frac{\partial \tilde{f}_i(\cdot)}{\partial \tilde{\ell}_{ik}} \right|_{(\tilde{\ell}_{ik}, \tilde{m}_{ik}) = (\tilde{\ell}^*_{ik}, \tilde{m}^*_{ik})},$$

[124] See Syverson (2019), De Loecker et al. (2020) and Kasahara and Sugita (2020) for a discussion.

and $\frac{\partial \tilde{f}_i(\cdot)^*}{\partial \tilde{m}_{ik}}$ is analogously defined.

It can easily be shown that $\frac{\partial \tilde{f}_i(\cdot)^*}{\partial \tilde{\ell}_{ik}}$ and $\frac{\partial \tilde{f}_i(\cdot)^*}{\partial \tilde{m}_{ik}}$ are identified from the data.

Proposition C.2. *Suppose that Assumptions 2.4 and C.1 hold. Then, the equilibrium values of the derivatives of the log-production function with respect to log-labor and log-material inputs can be recovered from the observables.*

Proof. Under Assumptions 2.4 and C.1, the firm's input cost-minimization problem is well-defined and has an interior solution. For a given level of output $\tilde{q}^*_{ik}$, the associated Lagrange function in terms of the logarithm variables reads[125]

$$\tilde{\mathcal{L}}(\tilde{\ell}_{ik}, \tilde{m}_{ik}, \xi_{ik}) := \exp\{\tilde{W} + \tilde{\ell}_{ik}\} + \exp\{\tilde{P}^M_i + \tilde{m}_{ik}\} - \xi_{ik}\Bigg(\exp\{\tilde{f}_i(\tilde{\ell}_{ik}, \tilde{m}_{ik}; \tilde{z}_{ik})\} - \exp\{\tilde{q}^*_{ik}\}\Bigg),$$

where ξ_{ik} represents the Lagrange multiplier indicating the marginal cost of producing an additional unit of output at the given level $\tilde{q}^*_{ik}$ (De Loecker and Warzynski, 2012; De Loecker et al., 2016, 2020). In equilibrium, the first order conditions at $\tilde{q}^*_{ik}$ look like

$$[\tilde{\ell}_{ik}] : \exp\{\tilde{W}^* + \tilde{\ell}^*_{ik}\} - \xi^*_{ik}\frac{\partial \tilde{f}_i(\cdot)^*}{\partial \tilde{\ell}_{ik}} \exp\{\tilde{f}_i(\tilde{\ell}^*_{ik}, \tilde{m}^*_{ik}; \tilde{z}_{ik})\} = 0 \tag{95}$$

$$[\tilde{m}_{ik}] : \exp\{\tilde{P^M_i}^* + \tilde{m}^*_{ik}\} - \xi^*_{ik}\frac{\partial \tilde{f}_i(\cdot)^*}{\partial \tilde{m}_{ik}} \exp\{\tilde{f}_i(\tilde{\ell}^*_{ik}, \tilde{m}^*_{ik}; \tilde{z}_{ik})\} = 0, \tag{96}$$

where $\tilde{\ell}^*_{ik}$ and $\tilde{m}^*_{ik}$, respectively, are the equilibrium quantities of labor and material inputs corresponding to the given output level q^*_{ik}. Taking the ratio between (95) and (96), I have

$$\frac{\frac{\partial \tilde{f}_i(\cdot)^*}{\partial \tilde{\ell}_{ik}}}{\frac{\partial \tilde{f}_i(\cdot)^*}{\partial \tilde{m}_{ik}}} = \frac{\exp\{\tilde{W}^* + \tilde{\ell}^*_{ik}\}}{\exp\{\tilde{P^M_i}^* + \tilde{m}^*_{ik}\}}. \tag{97}$$

Here, due to Assumption 2.4 (i),

$$\frac{\partial \tilde{f}_i(\cdot)^*}{\partial \tilde{\ell}_{ik}} + \frac{\partial \tilde{f}_i(\cdot)^*}{\partial \tilde{m}_{ik}} = 1,$$

[125] To simplify the exposition, I leverage the equivalence explained in Remark A.1, and consider the simultaneous decision of labor and material inputs, instead of the sequential formulation.

so that (97) gives

$$\frac{\partial \tilde{f}_i(\cdot)^*}{\partial \tilde{\ell}_{ik}} = \frac{\exp\{\tilde{W}^* + \tilde{\ell}^*_{ik}\}}{\exp\{\tilde{W}^* + \tilde{\ell}^*_{ik}\} + \exp\{\tilde{P_i^M}^* + \tilde{m}^*_{ik}\}}$$
$$\frac{\partial \tilde{f}_i(\cdot)^*}{\partial \tilde{m}_{ik}} = \frac{\exp\{\tilde{P_i^M}^* + \tilde{m}^*_{ik}\}}{\exp\{\tilde{W}^* + \tilde{\ell}^*_{ik}\} + \exp\{\tilde{P_i^M}^* + \tilde{m}^*_{ik}\}}.$$

Since both $\exp\{\tilde{W}^* + \tilde{\ell}^*_{ik}\}$ and $\exp\{\tilde{P_i^M}^* + \tilde{m}^*_{ik}\}$ are available in the data (Appendix B), I thus can identify $\frac{\partial \tilde{f}_i(\cdot)^*}{\partial \tilde{\ell}_{ik}}$ and $\frac{\partial \tilde{f}_i(\cdot)^*}{\partial \tilde{m}_{ik}}$ from the observables, as claimed. □

Next, I closely follow Kasahara and Sugita (2020) in identifying the equilibrium values of the firm's output quantity and price. Because of this, the notations are intentionally set closed to theirs.

To begin with, I admit a measurement error $\tilde{\eta}_{ik}$ in the observed log-revenue:[126]

$$\begin{aligned}
\tilde{r}_{ik} &= \tilde{\wp}_i(\tilde{q}_{ik}) + \tilde{q}_{ik} + \tilde{\eta}_{ik} \\
&= \tilde{\varphi}_i(\tilde{q}_{ik}) + \tilde{\eta}_{ik} \\
&= \tilde{\varphi}_i(\tilde{f}_i(\tilde{\ell}_{ik}, \tilde{m}_{ik}, \tilde{\mathcal{M}}_i(\tilde{\ell}_{ik}, \tilde{m}_{ik})) + \tilde{\eta}_{ik} \\
&= \tilde{\phi}_i(\tilde{\ell}_{ik}, \tilde{m}_{ik}) + \tilde{\eta}_{ik},
\end{aligned}$$

where $\tilde{\varphi}_i(\tilde{q}_{ik}) \coloneqq \tilde{\wp}_i(\tilde{q}_{ik}) + \tilde{q}_{ik}$, and $\tilde{\phi}_i(\cdot)$ is the nonparametric component of the revenue function in terms of labor and material inputs satisfying $\tilde{\phi}_i(\tilde{\ell}_{ik}, \tilde{m}_{ik}) = \tilde{\varphi}_i(\tilde{f}_i(\tilde{\ell}_{ik}, \tilde{m}_{ik}, \tilde{\mathcal{M}}_i(\tilde{\ell}_{ik}, \tilde{m}_{ik}))$. The additive separability of the log measurement error $\tilde{\eta}_{ik}$ is chosen to conform to the bulk of the literature on identification and estimation of production functions.[127]

Towards identification, it is posited that the econometrician has knowledge about the following regularity conditions.

Assumption C.2 (Regularity Conditions). *(i) (Strict exogeneity)* $E[\tilde{\eta}_{ik}|\tilde{\ell}_{ik}, \tilde{m}_{ik}] = 0$. *(ii) (Continuous differentiability)* $\phi_i(\cdot)$ *is at least first differentiable in each of its arguments. (iii) (Normalization) For each* $i \in \mathscr{N}$ *and each* $k \in \mathscr{N}_i$*, there exists a pair of labor and material inputs* $(\tilde{\ell}^\circ_{ik}, \tilde{m}^\circ_{ik}) \in \tilde{\mathscr{L}}_i \times \tilde{\mathscr{M}}_i$ *such that* $\tilde{f}_i(\tilde{\ell}^\circ_{ik}, \tilde{m}^\circ_{ik}; \tilde{z}_{ik}) = 0$. *(iv) (Monotonicity and*

[126] The measurement error is supposed to capture the variation in revenue that cannot be explained by firm-level input variables nor aggregate variables. This can be conceived as *i*) a shock to the firm's production that is unanticipated to the firm and hits after the firm's decision has been made, and/or *ii*) the coding error in the measurement used by the econometrician.

[127] The additive separability of the measurement errors in terms of logarithm variables is routinely employed in the literature (Olley and Pakes, 1996; Levinsohn and Petrin, 2003; Ackerberg et al., 2015; Gandhi et al., 2019).

differentiability) For each $i \in \mathscr{N}$, $\tilde{\varphi}_i(\cdot)$ is strictly increasing and invertible with its inverse $\tilde{\varphi}_i^{-1}(\tilde{r}_{ik})$, which is continuously diffrentiable with respect to $\tilde{r}_{ik} \in \tilde{\mathscr{R}}_i$.

Lemma C.2. *Suppose that Assumptions C.2 hold. Then, the logarithms of the firm-level revenue $\tilde{r}^*_{ik}$ and measurement error $\tilde{\eta}^*_{ik}$ can be identified.*

Proof. From Assumption C.2, I can identify $\tilde{\phi}_i(\cdot)$, $\bar{\tilde{r}}_{ik}$ and $\tilde{\eta}_{ik}$ according to $\tilde{\phi}_i(\tilde{\ell}_{ik}, \tilde{m}_{ik}) = E[\tilde{r}_{ik}|\tilde{\ell}_{ik}, \tilde{m}_{ik}]$; $\bar{\tilde{r}}_{ik} = \tilde{\phi}_i(\tilde{\ell}_{ik}, \tilde{m}_{ik})$; and $\tilde{\eta}_{ik} = \tilde{r}_{ik} - \bar{\tilde{r}}_{ik}$. □

Lemma C.3. *Suppose that Assumptions 2.4 and C.2 hold. Assume moreover that Assumption 4.4 holds with* (90)–(93)*. Then, the value of $\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}$ can be expressed in terms of firm-level labor and material input variables as well as aggregate variables.*

Proof. By Fact C.2, $\tilde{u}_{ik}$ can be expressed in terms of firm-level labor and material input variables as well as aggregate variables. It follows from Lemma C.2 that firm-level log revenue $\bar{\tilde{r}}_{ik}$ is identified. Tracing back the definition of $\tilde{u}_{ik}$ recovers the equilibrium value of $\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}$ as $\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}} = \frac{\tilde{u}_{ik}}{\exp\{\bar{\tilde{r}}_{ik}\}}$. □

C.3.2 Identification of the Values of Quantity and Price

The following lemma extends the result of Kasahara and Sugita (2020) by accounting for firms' strategic interactions.

Lemma C.4. *Suppose that Assumptions 2.4, C.1, and C.2 hold. Assume moreover that Assumption 4.4 holds with* (90)–(93)*. Then, the logarithms of the firm-level output quantity $\tilde{q}^*_{ik}$ and price $\tilde{p}^*_{ik}$ can be identified up to scale from the observables.*

Proof. The proof proceeds in three steps.
Step 1:

The first step identifies the firm's revenue free of the measurement errors $\bar{\tilde{r}}_{ik}$ in terms of $(\tilde{\ell}_{ik}, \tilde{m}_{ik})$, eliminating the measurement error $\tilde{\eta}_{ik}$. This is accomplished in Lemma C.2.
Step 2:

Next, I aim to identify the derivative of the inverse of the revenue function $\tilde{\varphi}_i(\cdot)$. By definition, it is true that

$$\tilde{f}_i(\tilde{\ell}_{ik}, \tilde{m}_{ik}, \tilde{\mathcal{M}}_i(\tilde{\ell}_{ik}, \tilde{m}_{ik})) = \tilde{\varphi}_i^{-1}(\bar{\tilde{r}}_{ik}). \tag{98}$$

Given that $\bar{\tilde{r}}_{ik} = \tilde{\phi}_i(\tilde{\ell}_{ik}, \tilde{m}_{ik})$ is identified above, one can take derivatives of (98) with respect to $\tilde{\ell}_{ik}$ and $\tilde{m}_{ik}$ to obtain

$$\frac{\partial \tilde{f}_i(\cdot)}{\partial \tilde{\ell}_{ik}} + \frac{\partial \tilde{f}_i(\cdot)}{\partial \tilde{z}_{ik}} \frac{\partial \tilde{\mathcal{M}}_i(\cdot)}{\partial \tilde{\ell}_{ik}} = \frac{\partial \tilde{\varphi}_i^{-1}(\cdot)}{\partial \bar{\tilde{r}}_{ik}} \frac{\partial \tilde{\phi}_i(\cdot)}{\partial \tilde{\ell}_{ik}} \tag{99}$$

$$\frac{\partial \tilde{f}_i(\cdot)}{\partial \tilde{m}_{ik}} + \frac{\partial \tilde{f}_i(\cdot)}{\partial \tilde{z}_{ik}} \frac{\partial \tilde{\mathcal{M}}_i(\cdot)}{\partial \tilde{m}_{ik}} = \frac{\partial \tilde{\varphi}_i^{-1}(\cdot)}{\partial \bar{\tilde{r}}_{ik}} \frac{\partial \tilde{\phi}_i(\cdot)}{\partial \tilde{m}_{ik}} \tag{100}$$

for all $(\tilde{\ell}_{ik}, \tilde{m}_{ik}) \in \tilde{\mathscr{L}}_i \times \tilde{\mathscr{M}}_i$. Notice here that $\frac{d\tilde{\varphi}_i^{-1}(\cdot)^*}{d\bar{\tilde{r}}_{ik}^*} = \left(\frac{d\tilde{r}_{ik}^*}{d\tilde{q}_{ik}^*}\right)^{-1}$, with the right-hand side being identified in Lemma C.3.[128] Thus, $\frac{d\tilde{\varphi}_i^{-1}(\cdot)^*}{d\bar{\tilde{r}}_{ik}}$ is identified.

Since the equilibrium values of $\frac{\partial \tilde{f}_i(\cdot)^*}{\partial \tilde{\ell}_{ik}}$ and $\frac{\partial \tilde{f}_i(\cdot)^*}{\partial \tilde{m}_{ik}}$ are identified in Proposition C.2, then (99) and (100) can be rearranged to identify, respectively, $\frac{\partial \tilde{f}_i(\cdot)^*}{\partial \tilde{z}_{ik}} \frac{\partial \tilde{\mathcal{M}}_i(\cdot)^*}{\partial \tilde{\ell}_{ik}}$ and $\frac{\partial \tilde{f}_i(\cdot)^*}{\partial \tilde{z}_{ik}} \frac{\partial \tilde{\mathcal{M}}_i(\cdot)^*}{\partial \tilde{m}_{ik}}$ as

$$\frac{\partial \tilde{f}_i(\cdot)^*}{\partial \tilde{z}_{ik}} \frac{\partial \tilde{\mathcal{M}}_i(\cdot)^*}{\partial \tilde{\ell}_{ik}} = \frac{\partial \tilde{\varphi}_i^{-1}(\cdot)^*}{\partial \bar{\tilde{r}}_{ik}} \frac{\partial \tilde{\phi}_i(\cdot)^*}{\partial \tilde{\ell}_{ik}} - \frac{\partial \tilde{f}_i(\cdot)^*}{\partial \tilde{\ell}_{ik}}, \tag{101}$$

and

$$\frac{\partial \tilde{f}_i(\cdot)^*}{\partial \tilde{z}_{ik}} \frac{\partial \tilde{\mathcal{M}}_i(\cdot)^*}{\partial \tilde{m}_{ik}} = \frac{\partial \tilde{\varphi}_i^{-1}(\cdot)^*}{\partial \bar{\tilde{r}}_{ik}} \frac{\partial \tilde{\phi}_i(\cdot)^*}{\partial \tilde{m}_{ik}} - \frac{\partial \tilde{f}_i(\cdot)^*}{\partial \tilde{m}_{ik}}. \tag{102}$$

Step 3:

The final step recovers the realized value of firm-level output quantity by means of integration:

$$\begin{aligned}
\tilde{q}_{ik}^* &= \tilde{f}_i(\tilde{\ell}_{ik}^*, \tilde{m}_{ik}^*, \tilde{z}_{ik}) \\
&= \int_{\tilde{\ell}_{ik}^\circ}^{\tilde{\ell}_{ik}^*} \left(\frac{\partial \tilde{f}_i}{\partial \tilde{\ell}_{ik}} + \frac{\partial \tilde{f}_i}{\partial \tilde{z}_{ik}} \frac{\partial \tilde{\mathcal{M}}_i}{\partial \tilde{\ell}_{ik}}\right)(s, \tilde{m}_{ik}^*) ds + \int_{\tilde{m}_{ik}^\circ}^{\tilde{m}_{ik}^*} \left(\frac{\partial \tilde{f}_i}{\partial \tilde{m}_{ik}} + \frac{\partial \tilde{f}_i}{\partial \tilde{z}_{ik}} \frac{\partial \tilde{\mathcal{M}}_i}{\partial \tilde{m}_{ik}}\right)(\tilde{\ell}_{ik}^\circ, s) ds,
\end{aligned}$$

where the value of $\tilde{f}_i(\tilde{\ell}_{ik}^\circ, \tilde{m}_{ik}^\circ, \tilde{z}_{ik})$ is assumed to be known to the econometrician (Assumption C.2 (iii)).

Lastly, I can also recover the realized value of the firm-level output price $\tilde{p}_{ik}^*$ through

$$\tilde{p}_{ik}^* = \bar{\tilde{r}}_{ik} - \tilde{q}_{ik}^*.$$

This completes the proof. □

Remark C.1. *(i) The proof of Lemma C.4 does not require the identification of firms' pro-*

[128]If competition in the output market is monopolistic, $\frac{d\tilde{r}_{ik}^*}{d\tilde{q}_{ik}^*}$ coincides with the inverse of the firm's markup $\tilde{\mu}_{ik}^*$, which is recovered in Fact C.3 (see Kasahara and Sugita, 2020).

ductivity. While Kasahara and Sugita (2020) exploit the panel structure of their dataset to identify firms' productivity, my framework is static by nature, prohibiting the use of time-series variation. Intuitively, I instead exploit cross-sectional variation to recover a function of firms' productivity, as embodied in Lemma lemma:IdentificationOfTotalDerivativeOfLogRevenueWrtLogX. (ii) Lemma C.4 does not invoke the feature of the Hicks-neutral productivity in the firm-level production function (Assumption 4.3). As a result, this lemma also applies to the case of non-Hicks-neutral productivity as studied in Demirer (2022) and Pan (2022). Under Hicks-neutrality, it holds $\frac{\partial \tilde{f}_i(\cdot)}{\partial \tilde{z}_{ik}} = 1$*. (iii) As discussed in Kasahara and Sugita (2020, 2023), Lemma C.4 identifies the firm-level quantity and price only up to a scale constant. Nevertheless, it is straightforward to verify that this is innocuous for the purposes of this paper, as the scale constants end up canceling out with each other. Hence, the presence of the scale constants is made implicit throughout the exposition.*

Having Lemma C.4 established, firm-level revenue can be traced in terms of firm-level output quantity. The following corollary is used to identify the second-order drivative of firm-level log revenue with respect to firm-level log quantity $\frac{d^2\tilde{r}_{ik}}{d\tilde{x}_{ik}^2}$.

Corollary C.1. *Suppose that the assumptions required in Lemma C.4 hold. Then, the firm's log revenue can be identified as a function of the firm's log quantity.*

Proof. The proof is omitted. □

Moreover, given Lemma C.4, firm-level quantity and price in level can immediately be recovered by reverting (88) and (89).

Proposition C.3. *Suppose that the assumptions required in Lemma C.4 hold. Then, the equilibrium values of the firm's output quantity* q_{ik}^* *and price* p_{ik}^* *are identified up to scale from the observables.*

Proof. The proof is omitted. □

Now, I am in a position to recover ϱ_{ik} and t_{ik}.

Corollary C.2. *Suppose that the assumptions required in Lemma C.4 hold. Then,* ϱ_{ik}*, and* t_{ik} *are identified.*

Proof. First, $\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}$ and $\frac{d^2\tilde{r}_{ik}}{d\tilde{x}_{ik}^2}$ are known from Lemma C.3 and Corollary C.1, respectively. Upon substituting these into the definition, ϱ_{ik} is recovered: $\varrho_{ik} = \left(\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\right)^{-1}\frac{d^2\tilde{r}_{ik}}{d\tilde{x}_{ik}^2} + \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}$. Next, $\{u_{ik'}, \varpi_{ik'}\}_{k'\in\mathcal{N}_i}$ are known from Fact C.2, and $\{q_{ik'}\}_{k'\in\mathcal{N}_i}$ are obtained in Proposition C.3. These can be combined with the identified $\{\varrho_{ik'}\}_{k'\in\mathcal{N}_i}$ to identify t_{ik} following its definition: $t_{ik} = \frac{\varpi_{ik}}{q_{ik}}\left(\varrho_{ik} - \frac{\sum_{k'=1}^{N_i}\varrho_{ik'}\tilde{u}_{ik'}}{\sum_{k'=1}^{N_i}\tilde{u}_{ik'}}\right)$. □

C.4 Recovering Demand-Side Elasticities

C.4.1 Quantity Index

I first identify the quantity index $A_i(\cdot)$ over the entire support $\mathscr{S}_i^{N_i}$. This is shown in Kasahara and Sugita (2020).

Lemma C.5 (Identification of A_i; Kasahara and Sugita (2020)). *Suppose that the same assumptions in Lemma C.4 are satisfied. Then, the quantity index $A_i(\mathbf{q}_i)$ is identified.*

Proof. See Kasahara and Sugita (2020). □

In Lemma C.5, the quantity index $A_i(\cdot)$ is nonparametrically identified as a function of $\mathbf{q}_i$, so that its derivatives can also be nonparametrically identified. The analytical expressions are summarized in the following corollary.

Corollary C.3 (Identification of $\frac{\partial A_i(\cdot)}{\partial q_{ik}}$ and $\frac{\partial^2 A_i(\cdot)}{\partial q_{ik} q_{ik'}}$). *Suppose that the same assumptions required in Lemma C.4 hold. Then, for each $i \in \mathscr{N}$, (i) $\frac{\partial A_i(\cdot)}{\partial q_{ik'}}$ and (ii) $\frac{\partial^2 A_i(\cdot)}{\partial q_{ik} \partial q_{ik'}}$ are identified for all $k, k' \in \mathscr{N}_i$.*

Proof. (i) Rearranging (94) yields $\frac{\partial A_i(\cdot)}{\partial q_{ik}} = \frac{A_i(\mathbf{q}_i)}{q_{ik}} \varpi_{ik}$, according to which the partial derivative of the quantity index with respect to the individual firm's output is identified.

(ii) One can apply another differentiation to the result obtained in part (i). The analytical expression for the second-order partial derivative of $A_i(\cdot)$ with respect to q_{ik} is given by

$$\frac{\partial^2 A_i(\cdot)}{\partial q_{ik}^2} = -(1 - \varrho_{ik}) \frac{A_i(\mathbf{q}_i)}{q_{ik}^2} (1 - \varpi_{ik}) \varpi_{ik} - \frac{A_i(\mathbf{q}_i)}{q_{ik}} \varpi_{ik} t_{ik}.$$

The mixed partial derivatives of $A_i(\cdot)$ with respect to q_{ik} and $q_{ik'}$, with $k' \neq k$, are given by

$$\frac{\partial^2 A_i(\cdot)}{\partial q_{ik} \partial q_{ik'}} = (1 - \varrho_{ik}) \frac{A_i(\mathbf{q}_i)}{q_{ik} q_{ik'}} \varpi_{ik} \varpi_{ik'} - \frac{A_i(\mathbf{q}_i)}{q_{ik}} \varpi_{ik} t_{ik'},$$

completing the proof. □

C.4.2 Residual Inverse Demand Functions

Using the results obtained so far, I can further recover the firm's residual inverse demand functions and their responsiveness with respect to the firm's quantity. I write $\frac{\partial \wp_{ik}(\cdot)}{\partial q_{ik'}} = \frac{\partial \wp_i(q_{ik}, \mathbf{q}_{i,-k})}{\partial q_{ik'}}$.

Lemma C.6. *Suppose that the same assumptions required in Lemma C.4 hold. Then, the first- and second-order derivatives of the residual inverse demand functions $\wp_i(\cdot)$ with respect to the firm's output quantity can be identified from the observables.*

Proof. For each $i \in \mathscr{N}$ and $k \in \mathscr{N}_i$, taking the partial derivatives of (91) with respect to q_{ik} and $q_{ik'}$ ($k' \neq k$), respectively, yield

$$\frac{\partial \wp_{ik}(\cdot)}{\partial q_{ik}} = -\frac{p_{ik}}{q_{ik}}\Big\{1 - \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}(1-\varpi_{ik})\Big\}, \tag{103}$$

and

$$\frac{\partial \wp_{ik}(\cdot)}{\partial q_{ik'}} = -\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}} p_{ik} \frac{\varpi_{ik'}}{q_{ik'}}.$$

Taking further the partial derivatives of (103), it is immediate to obtain

$$\begin{aligned}\frac{\partial^2 \wp_{ik}(\cdot)}{\partial q_{ik}^2} = \frac{p_{ik}}{q_{ik}}\Bigg[\Big\{1 - \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}(1-\varpi_{ik})\Big\}\Big\{2 - \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}(1-\varpi_{ik})\Big\} + (1-\varpi_{ik})\Big\{\frac{d^2\tilde{r}_{ik}}{d\tilde{x}_{ik}^2}(1-\varpi_{ik}) - \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\varrho_{ik}\varpi_{ik}\Big\}\Bigg] \\ + \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\frac{p_{ik}}{q_{ik}}\varpi_{ik} t_{ik},\end{aligned}$$

and

$$\frac{\partial^2 \wp_{ik}(\cdot)}{\partial q_{ik} q_{ik'}} = -\frac{p_{ik}}{q_{ik}q_{ik'}}\varpi_{ik'}\Bigg[\frac{d^2\tilde{r}_{ik}}{d\tilde{x}_{ik}^2}(1-\varpi_{ik}) - \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\Big\{\Big(1 - \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\Big) + \Big(\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}} + \varrho_{ik}\Big)\varpi_{ik}\Big\}\Bigg] + \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\frac{p_{ik}}{q_{ik}}\varpi_{ik} t_{ik'}.$$

for all $k' \neq k$, which completes the proof. □

Remark C.2. *Analogous results can be derived for monopolistic competition:* $\frac{\partial \wp_{ik}(\cdot)}{\partial q_{ik}} = -\frac{p_{ik}}{q_{ik}}\big(1 - \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\big)$, *and* $\frac{\partial \wp_{ik}(\cdot)}{\partial q_{ik'}} = 0$ *for all* $k' \neq k$*; and* $\frac{\partial^2 \wp_{ik}(\cdot)}{\partial q_{ik}^2} = \frac{p_{ik}}{q_{ik}}\Big\{\big(1 - \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\big)\big(2 - \frac{d\tilde{r}_{ik}}{\tilde{x}_{ik}}\big) + \frac{d^2\tilde{r}_{ik}}{d\tilde{x}_{ik}^2}\Big\}$, *and* $\frac{\partial^2 \wp_{ik}(\cdot)}{\partial q_{ik}\partial q_{ik'}} = 0$ *for all* $k' \neq k$.

C.4.3 Marginal Revenue Functions

For each sector $i \in \mathscr{N}$ and for each firm $k \in \mathscr{N}_i$, let $mr_{ik} : \mathscr{S}_i \times \mathscr{S}_i^{N_i - 1} \to \mathbb{R}$ be the firm's marginal revenue function; that is, $mr_{ik}(q_{ik}, \mathbf{q}_{i,-k}; \mathcal{I}_i) := \frac{\partial \wp_{ik}(\cdot)}{\partial q_{ik}} q_{ik} + p_{ik}$.

Lemma C.7 (Identification of Marginal Revenue Function)**.** *Suppose that the assumptions required in Lemma C.4 are satisfied. Then, the equilibrium values of (i) the firm-level marginal revenue function* $mr_{ik}(\cdot)$ *and (ii) its partial derivatives* $\frac{\partial mr_{ik}(\cdot)}{\partial q_{ik'}}$, *for all* $k' \in \mathscr{N}_i$, *are identified.*

Proof. (i) By the setup, $r_{ik} = \exp\{\tilde{\varphi}_i(\tilde{x}_{ik})\}$, wehre $\tilde{x}_{ik} = \ln x_{ik}$ with $x_{ik} = x_{ik}(q_{ik}, \mathbf{q}_{i,-k})$. By the definition of the marginal revenue, it follows

$$mr_{ik} = \frac{dr_{ik}}{dq_{ik}} = \exp\{\tilde{\varphi}_i(\tilde{x}_{ik})\}\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\frac{d\tilde{x}_{ik}}{dx_{ik}}\frac{\partial x_{ik}(\cdot)}{\partial q_{ik}} = \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}} p_{ik}(1-\varpi_{ik}) =: mr_i(q_{ik}, \mathbf{q}_{i,-k}).$$

This restores the identification of the equilibrium value of the firm's marginal revenue function.

(ii) Taking the derivative of part (i) with respect to q_{ik} yields

$$\frac{\partial mr_{ik}(\cdot)}{\partial q_{ik}} = \frac{p_{ik}}{q_{ik}}(1-\varpi_{ik})\left[\frac{d^2\tilde{r}_{ik}}{d\tilde{x}_{ik}^2}(1-\varpi_{ik}) - \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\left\{\left(1-\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\right)+\left(\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}+\varrho_{ik}\right)\varpi_{ik}\right\}\right] + \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}p_{ik}\varpi_{ik}t_{ik}.$$

Analogously, the derivative with respect to $q_{ik'}$ $(k' \neq k)$ leads to

$$\frac{\partial mr_{ik}(\cdot)}{\partial q_{ik'}} = -p_{ik}\frac{\varpi_{ik'}}{q_{ik'}}\left[\frac{d^2\tilde{r}_{ik}}{d\tilde{x}_{ik}^2}(1-\varpi_{ik}) + \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\left\{\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}-\left(\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}+\varrho_{ik}\right)\varpi_{ik}\right\}\right] + \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}p_{ik}\varpi_{ik}t_{ik'}$$

This restores the identification of the equilibrium value of derivatives of the firm's marginal revenue function.

□

Remark C.3. *Notice that there are general relationships between the derivatives of a demand function and those of a marginal revenue function, namely,* $\frac{\partial \wp_{ik}(\cdot)}{\partial q_{ik}}q_{ik} + p_{ik} = mr_i(q_{ik}, \mathbf{q}_{i,-k})$, $\frac{\partial^2 \wp_{ik}(\cdot)}{\partial q_{ik}^2}q_{ik} + 2\frac{\partial \wp_{ik}(\cdot)}{\partial q_{ik}} = \frac{\partial mr_{ik}(\cdot)}{\partial q_{ik}}$, *and* $\frac{\partial \wp_{ik}(\cdot)}{\partial q_{ik}\partial q_{ik'}}q_{ik} + \frac{\partial \wp_{ik}(\cdot)}{\partial q_{ik'}} = \frac{\partial mr_{ik}(\cdot)}{\partial q_{ik'}}$ *for all* $k' \neq k$*. These equalities offer an alternative route from Lemma C.6 to Lemma C.7, or the other way around.*

Remark C.4. *Analogous results are true for the case of monopolistic competition:* $mr_{ik}(\cdot) = \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}p_{ik}$, $\frac{\partial mr_{ik}(\cdot)}{\partial q_{ik}} = \frac{p_{ik}}{q_{ik}}\left\{\frac{d^2\tilde{r}_{ik}}{d\tilde{x}_{ik}^2} - \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\left(1-\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\right)\right\}$ *and* $\frac{\partial mr_{ik}(\cdot)}{\partial q_{ik'}} = 0$ *for all* $k' \neq k$*.*

Example C.2 (CES Sectoral Aggregator)**.** *Consider that the sectoral aggregator in sector* i *takes the form of a CES function with the elasticity of substitution being* σ_i*. In this case, it is straightforward to see that* $\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}} = \frac{\sigma_i - 1}{\sigma_i}$, $\frac{d^2\tilde{r}_{ik}}{d\tilde{x}_{ik}^2} = 0$, *and thus* $\varpi_{ik} = \frac{\sigma_i - 1}{\sigma_i}$ *and* $t_{ik} = 0$ *for all* $k \in \mathscr{N}_i$*.*

C.4.4 Aggregate Quantity and Price

I can further recover the sectoral aggregator $F_i(\cdot)$, and its partial derivatives with respect to q_{ik} (denoted by $\frac{\partial F_i(\cdot)}{\partial q_{ik}}$) as well as the partial derivatives of $\mathcal{P}_i(\cdot)$ with respect to q_{ik} (denoted by $\frac{\partial \mathcal{P}_i(\cdot)}{\partial q_{ik}}$) for all $k \in \mathscr{N}_i$, provided the following normalization condition.

Assumption C.3 (Normalization of HSA Demand System)**.** *There exists a collection of constants* $\{c_{ik}\}_{k=1}^{N_i}$ *such that* $F_i(\{c_{ik}\}_{k=1}^{N_i}) = 1$*.*

Lemma C.8 (Identification of Sectoral Aggregators)**.** *Suppose that the assumptions required in Lemma C.4 are satisfied. Assume moreover that Assumption C.3 holds. Then, (i) the sectoral aggregator* $F_i(\cdot)$*, and (ii) the derivatives* $\frac{\partial F_i(\cdot)}{\partial q_{ik}}$ *and* $\frac{\partial \mathcal{P}_i(\cdot)}{\partial q_{ik}}$*, for each* $k \in \mathscr{N}_i$*, are* *identified as a function of* $\mathbf{q}_i$*. (iii) In particular, evaluated at the realized values, it holds that* $\frac{\partial F_i(\cdot)^*}{\partial q_{ik}} = \frac{p^*_{ik}}{P^*_i}$ *and* $\frac{\partial \mathcal{P}_i(\cdot)^*}{\partial q_{ik}} = -\frac{p^*_{ik}}{Q^*_i}$.

Proof. (i) By Proposition 1 (i) and Remark 3 (self-duality) of Matsuyama and Ushchev (2017), there exists a unique monotone, convex, continuous, and homothetic rational preference over the support of $\mathbf{q}_i$ associated to the HSA inverse demand system (91)–(93). Clearly, this preference corresponds to the sectoral aggregator $F_i(\cdot)$. Moreover, a variant of Proposition 1 (ii) of Matsuyama and Ushchev (2017) implies that $F_i(\cdot)$ can be expressed as[129]

$$\ln F_i(\mathbf{q}_i) = \ln A_i(\mathbf{q}_i) + \sum_{k=1}^{N_i} \int_{c_{ik}}^{q_{ik}/A_i(\mathbf{q}_i)} \frac{\Psi_i(\zeta)}{\zeta} d\zeta, \tag{104}$$

where $\{c_{ik}\}_{k=1}^{N_i}$ are constants satisfying Assumption C.3.

Since, by Lemma C.5, $A_i(\cdot)$ is identified, it remains to prove that for each $k \in \mathscr{N}$, the integrand $\frac{\Psi_i(\zeta)}{\zeta}$ is identified for all $\zeta \in [c_{ik}, \frac{q_{ik}}{A_i(\mathbf{q}_i)}]$. Observe that $\varphi_i(\cdot)$ in (92) is known (Corollary C.1). Notice moreover that for the realized values $\{q^*_{ik}\}_{k=1}^{N_i}$, I can recover Φ_i using (90):

$$\Phi_i = \sum_{k=1}^{N_i} \varphi_i(q^*_{ik}),$$

where Φ_i is a constant that the firms take as given. Then, the identification of $\frac{\Psi_i(\zeta)}{\zeta}$, for $\zeta \in [c_{ik}, \frac{q_{ik}}{A_i(\mathbf{q}_i)}]$, comes directly from its construction (92). Tracing (104) therefore restores the identification of $F_i(\cdot)$ as a function of $\mathbf{q}_i$.

(ii) Taking partial derivatives of (104) with respect to q_{ik}: for all $\mathbf{q}_i \in \mathscr{S}_i^{N_i}$,

$$\frac{\frac{\partial F_i(\cdot)}{\partial q_{ik}}}{F_i(\mathbf{q}_i)} = \frac{\frac{\partial A_i(\cdot)}{\partial q_{ik}}}{A_i(\mathbf{q}_i)} + \frac{1}{q_{ik}} \Psi_i\Big(\frac{q_{ik}}{A_i}\Big) - \Big(\sum_{k'=1}^{N_i} \Psi_i\Big(\frac{q_{ik'}}{A_i}\Big)\Big) \frac{1}{A_i(\mathbf{q}_i)} \frac{\partial A_i(\cdot)}{\partial q_{ik}},$$

so that, by construction,

$$\frac{\partial F_i(\cdot)}{\partial q_{ik}} = \frac{F_i(\mathbf{q}_i)}{\Phi_i} \frac{1}{q_{ik}} \varphi\Big(\frac{q_{ik}}{A_i(\mathbf{q}_i)}\Big).$$

This expression recovers $\frac{\partial F_i(\cdot)}{\partial q_{ik}}$ as a function of $\mathbf{q}_i$.

Moreover, it hods by (90) that $\mathcal{P}_i(\mathbf{q}_i) F_i(\mathbf{q}_i) = \Phi_i$. Then, taking the partial derivatives of

[129] See also Kasahara and Sugita (2020).

the both hand sides with respect to q_{ik}, I obtain

$$\frac{\partial \mathcal{P}_i(\cdot)}{\partial q_{ik}} F_i(\mathbf{q}_i) + \mathcal{P}_i(\mathbf{q}_i) \frac{\partial F_i(\cdot)}{\partial q_{ik}} = 0.$$

Rearranging this identifies $\frac{\partial \mathcal{P}_i(\cdot)}{\partial q_{ik}}$ as a function of $\mathbf{q}_i$.

(iii) For the realized values $\mathbf{q}_i^*$, part (ii) of this lemma further simplifies to

$$\frac{\partial F_i(\cdot)^*}{\partial q_{ik}} = \frac{F_i(\mathbf{q}_i^*)}{\Phi_i} \frac{1}{q_{ik}^*} \varphi\Big(\frac{q_{ik}^*}{A_i(\mathbf{q}_i^*)}\Big) = \frac{p_{ik}^*}{\mathcal{P}_i(\mathbf{q}_i^*)} = \frac{p_{ik}^*}{P_i^*},$$

and

$$\frac{\partial \mathcal{P}_i(\cdot)^*}{\partial q_{ik}} = -\frac{\mathcal{P}_i(\mathbf{q}_i^*)}{F_i(\mathbf{q}_i^*)} \frac{p_{ik}^*}{P_i^*} = -\frac{p_{ik}^*}{F_i(\mathbf{q}_i^*)} = -\frac{p_{ik}^*}{Q_i^*}.$$

This completes the proof. □

Remark C.5. *As discussed in Kasahara and Sugita (2020, 2023), the HSA demand is identified only up to a scale constant. Nevertheless, in my application, the sectoral price indices are observed in the data, and sectoral revenues can be recovered through the preceding identification argument. I thus anchor the scale around the sectoral price indices.*

C.5 Recovering Λ and Γ

Once the partial derivatives of the sector- and firm-level production functions as well as the firms' prices and quantities are identified, I can also recover the matrices $\Lambda_{i,1}$ and $\Lambda_{i,2}$ in (32), and the matrices Γ_1 and Γ_2 in (43), all of which jointly serve as a "bridge" between the partial derivatives (i.e., production- and demand-side elasticities) and the total derivatives (i.e., comparative statics). The identification is constructive in the sense that these are recovered simply following the construction derived in Appendix A.

C.5.1 Identification of Λ

Fact C.4 (Identification of $\Lambda_{i,1}$ and $\Lambda_{i,2}$)**.** *Suppose that Proposition C.3 and Lemma C.7 hold. Then, for each sector $i \in \mathscr{N}$, both matrices $\Lambda_{i,1}$ and $\Lambda_{i,2}$ in* (32) *are identified.*

Proof. First, it immediately follows from Lemma C.7 that $\Lambda_{i,1} := \big[\frac{\partial mr_{ik}(\cdot)^*}{\partial q_{ik'}}\big]_{k,k' \in \mathscr{N}_i}$ are identified. Next, $\{q_{ik}^*\}_{k=1}^{N_i}$ are identified by Proposition C.3. Since moreover labor and material inputs are available in the data (Fact B.4), the matrix $\Lambda_{i,2}$ in (32) is identified, as desired. □

Remark C.6. *In view of Fact C.4, each entry of the matrix $\Lambda_{i,1}^{-1}\Lambda_{i,2}$ (i.e., $\lambda_{ik,k'}^{-1}$) is also identified.*

Fact C.5 (Identification of $\bar{\lambda}^L_{ik}$ and $\bar{\lambda}^M_{ik}$). *Suppose that the assumptions required in Fact C.4 are satisfied. Then, for each sector $i \in \mathscr{N}$ and each firm $k \in \mathscr{N}_i$, both $\bar{\lambda}^L_{ik}$ and $\bar{\lambda}^M_{ik}$ are identified from the observables.*

Proof. For each sector $i \in \mathscr{N}$, q^*_{ik} is identified for all $k \in \mathscr{N}_i$ (Proposition C.3). Since $\lambda^{-1}_{ik,k'}$ is identified for all $k, k' \in \mathscr{N}_i$ (Fact C.4), both $\bar{\lambda}^L_{ik}$ and $\bar{\lambda}^M_{ik}$ are identified by tracing their construction: $\bar{\lambda}^L_{ik} = \sum_{k'=1}^{N_i} \lambda^{-1}_{ik,k'} \frac{\ell^*_{ik'}}{q^*_{ik'}}$ and $\bar{\lambda}^M_{ik} = \sum_{k'=1}^{N_i} \lambda^{-1}_{ik,k'} \frac{m^*_{ik'}}{q^*_{ik'}}$, where ℓ^*_{ik} and m^*_{ik} are observed in the data (Fact B.4). □

Fact C.6 (Identification of $\bar{\lambda}^L_{i\cdot}$ and $\bar{\lambda}^M_{i\cdot}$). *Suppose that the assumptions required in Fact C.4 are satisfied. Assume moreover that Lemma C.8 holds. Then, for each sector $i \in \mathscr{N}$, both $\bar{\lambda}^L_{i\cdot}$ and $\bar{\lambda}^M_{i\cdot}$ are identified.*

Proof. First, for each sector $i \in \mathscr{N}$, p^*_{ik} is identified for all $k \in \mathscr{N}_i$ (Proposition C.3). Second, $\bar{\lambda}^L_{ik}$ and $\bar{\lambda}^M_{ik}$ are identified by Fact C.5. Moreover, in view of Lemma C.8, $\frac{\partial \mathcal{P}_i(\cdot)^*}{\partial q_{ik}}$ can be expressed in terms of $\{p^*_{ik'}\}_{k' \in \mathscr{N}_i}$ and Q^*_i. Hence, both $\bar{\lambda}^L_{i\cdot}$ and $\bar{\lambda}^M_{i\cdot}$ are identified according to (39). □

C.5.2 Identification of Γ

Given that material input is composed according to a Cobb-Douglas aggregator (19), the equilibrium material cost index corresponding to (40) is given by

$$P_i^{M*} = \prod_{j=1}^{N} \frac{1}{\gamma_{i,j}^{\gamma_{i,j}}} \Big\{ (1-\tau_i) P^*_j \Big\}^{\gamma_{i,j}}.$$

Fact C.7. *Under the specification (19), $\frac{\partial \mathcal{P}^M_i(\cdot)^*}{\partial P_j}$ and $\frac{\partial \mathcal{P}^M_i(\cdot)^*}{\partial \tau_n}$ in (41) are identified from the observables.*

Proof. Under the specification (19), it holds that $\frac{\partial \mathcal{P}^M_i(\cdot)^*}{\partial P_j} = \gamma_{i,j} \frac{P_i^{M*}}{P^*_j}$ and $\frac{\partial \mathcal{P}^M_i(\cdot)}{\partial \tau_n} = -\frac{P_i^{M*}}{1-\tau_i} \mathbb{1}_{\{n=i\}}$. The values of these terms are directly obtained from the data (Appendix B). Hence, $\frac{\partial \mathcal{P}^M_i(\cdot)^*}{\partial P_j}$ and $\frac{\partial \mathcal{P}^M_i(\cdot)^*}{\partial \tau_n}$ are identified. □

Fact C.8. *Suppose that the assumptions required in Fact C.6 are satisfied. Then, the matrices Γ_1 and Γ_2 in (43) are identified.*

Proof. In view of Fact C.7, $\{\frac{\partial \mathcal{P}^M_i(\cdot)^*}{\partial P_j}\}_{i,j \in \mathscr{N}}$ are identified. Moreover, $\{\bar{\lambda}^L_{j\cdot}\}_{j=1}^N$ and $\{\bar{\lambda}^M_{j\cdot}\}_{j=1}^N$ are identified due to Fact C.6. Thus, both Γ_1 and Γ_2 in (43) can be recovered by following their definitions. □

C.6 Recovering Comparative Statics

With the results obtained above (Appendices C.3, C.4 and C.5), I now turn to the identification of the comparative statics of sector- and firm-level variables. As a preliminary, this requires the identification of the first- and second-order derivatives of firm-level production functions. This is accomplished by following the share regression approach of Gandhi et al. (2019), and is deferred to Appendix C.7. Hence, this section takes these as identified. The identification of the comparative statics is constructive, so that I can follow the theoretical results established in Appendix A.

To begin with, I study the identification of the responsiveness of wage.

Fact C.9 (Identification of D_{ik})**.** *Suppose that the assumptions required in Fact C.6 are satisfied. Then, for each sector $i \in \mathscr{N}$ and each firm $k \in \mathscr{N}_i$, the matrix D_{ik} in* (57) *is identified.*

Proof. First, it holds by Assumption 2.4 (i) that the firm's marginal cost equals its average cost, so that $\xi^*_{ik} = \frac{TC^*_{ik}}{q^*_{ik}}$. This expression recovers ξ^*_{ik} because the total cost is directly observed in the data (Appendix B) and the firm-level output quantity is recovered by Proposition C.3. Next, both $\bar{\lambda}^L_{ik}$ and $\bar{\lambda}^M_{ik}$ are identified by Fact C.5, and moreover the first- and second-order derivatives of the firm-level production function are identified (Appendix C.7). Then, I can recover the matrix D_{ik} by tracing its definition (57). □

Proposition C.4 (Identification of $\frac{dW^*}{d\tau_n}$)**.** *Suppose that the assumptions required in Fact C.6 are satisfied. Then, $\frac{dW^*}{d\tau_n}$ is identified.*

Proof. From Fact C.7, it is known that $\frac{\partial \mathcal{P}^M_i(\cdot)^*}{\partial \tau_n} = -\frac{P^{M*}_i}{1-\tau_i}\mathbb{1}_{\{n=i\}}$. In addition, it holds from Fact C.8 that both Γ_1 and Γ_2 are identified. Thus, $\vartheta_{1,i}$ and $\vartheta_{2,i}$ in (60) can also be identified by following their construction. Since moreover each entry of the matrix D_{ik} is identified (Fact C.9), the identification of $\frac{dW^*}{d\tau_n}$ obtains through (64). □

Next, I turn to the identification of the responsiveness of the sectoral variables (i.e., the sectoral price indices and material cost indices).

Proposition C.5 (Identification of $\frac{dP^{M*}_i}{d\tau_n}$)**.** *Suppose that the assumptions required in Fact C.6 are satisfied. Then, for each sector $i \in \mathscr{N}$, $\frac{dP^{M*}_i}{d\tau_n}$ is identified.*

Proof. In light of Fact C.7, $\frac{\partial \mathcal{P}^M_i(\cdot)^*}{\partial \tau_n}$ is identified. Both Γ_1 and Γ_2 are recovered in Fact C.8. Given the identification of $\frac{dW^*}{d\tau_n}$ (Proposition C.4), I can identify $\frac{dP^{M*}_i}{d\tau_n}$ according to (44). □

Proposition C.6 (Identification of $\frac{dP^*_i}{d\tau_n}$)**.** *Suppose that the assumptions required in Fact C.6 are satisfied. Then, for each sector $i \in \mathscr{N}$, $\frac{dP^*_i}{d\tau_n}$ is identified.*

Proof. Due to Fact C.6, both $\bar{\lambda}^L_{i\cdot}$ and $\bar{\lambda}^M_{i\cdot}$ are identified. Given the identifications of $\frac{dW^*}{d\tau_n}$ (Proposition C.4) and $\frac{dP_i^{M*}}{d\tau_n}$ (Proposition C.5), I can identify $\frac{dP_i^*}{d\tau_n}$ according to (39). □

Lastly, I move to the identification of the responsiveness of firm-level output and input variables.

Proposition C.7 (Identification of $\frac{dq^*_{ik}}{d\tau_n}$ and $\frac{dp^*_{ik}}{d\tau_n}$). *Suppose that the assumptions required in Fact C.6 are satisfied. Then, for each sector $i \in \mathscr{N}$ and each firm $k \in \mathscr{N}_i$, $\frac{dq^*_{ik}}{d\tau_n}$ and $\frac{dp^*_{ik}}{d\tau_n}$ are identified.*

Proof. First, observe that both $\bar{\lambda}^L_{ik}$ and $\bar{\lambda}^M_{ik}$ are identified for each $i \in \mathscr{N}$ and each $k \in \mathscr{N}_i$ (Fact C.5). Given the identification of $\frac{dW^*}{d\tau_n}$ (Proposition C.4) and $\frac{dP_i^{M*}}{d\tau_n}$ (Proposition C.5), I can identify $\frac{dq^*_{ik}}{d\tau_n}$ according to (32).

Next, $\frac{dp^*_{ik}}{d\tau_n}$ is in turn recovered through $\frac{dp^*_{ik}}{d\tau_n} = \sum_{k'=1}^{N_i} \frac{\partial \wp_{ik}(\cdot)^*}{\partial q_{ik'}} \frac{dq^*_{ik'}}{d\tau_n}$, where the identification of $\frac{\partial \wp_{ik}(\cdot)^*}{\partial q_{ik'}}$ (for all $k' \in \mathscr{N}_i$) is given in Lemma C.6. □

Proposition C.8 (Identification of $\frac{d\ell^*_{ik}}{d\tau_n}$ and $\frac{dm^*_{ik}}{d\tau_n}$). *Suppose that the assumptions required in Fact C.6 are satisfied. Then, for each sector $i \in \mathscr{N}$ and each firm $k \in \mathscr{N}_i$, $\frac{d\ell^*_{ik}}{d\tau_n}$ and $\frac{dm^*_{ik}}{d\tau_n}$ are identified.*

Proof. It follows from Fact C.9 that the matrix D_{ik} is identified for each $i \in \mathscr{N}$ and each $k \in \mathscr{N}_i$. Given the identifications of $\frac{dW^*}{d\tau_n}$ (Proposition C.4) and $\frac{dP_i^{M*}}{d\tau_n}$ (Proposition C.5), I can identify $\frac{d\ell^*_{ik}}{d\tau_n}$ and $\frac{dm^*_{ik}}{d\tau_n}$ according to (56). □

Notice that if material input is composed according to a Cobb-Douglas aggregator (19), the equilibrium derived demand for sectoral intermediate good corresponding to (65) is given by (20):

$$m^*_{ik,j} = \gamma_{i,j} \frac{P_i^{M*}}{(1-\tau_i)P_j^*} m^*_{ik}.$$

Proposition C.9 (Identification of $\frac{dm^*_{ik,j}}{d\tau_n}$). *Suppose that the assumptions required in Fact C.6 are satisfied. Then, for each $i, j \in \mathscr{N}$ and each $k \in \mathscr{N}_i$, $\frac{dm^*_{ik,j}}{d\tau_n}$ is identified.*

Proof. Under the specification (19), it holds that $\frac{\partial m_{ik,j}(\cdot)^*}{\partial P_{j'}} = -\frac{1}{P_{j'}} m_{ik,j} \mathbb{1}_{\{j'=j\}} + \frac{\gamma_{i,j'}}{P^*_{j'}} m^*_{ik,j}$ for all $j' \in \mathscr{N}$, $\frac{\partial m_{ik,j}(\cdot)^*}{\partial \tau_n} = -\frac{m_{ik,j}}{1-\tau_i} \mathbb{1}_{\{n=i\}}$, and $\frac{\partial m_{ik,j}(\cdot)^*}{\partial m_{ik}} = \frac{m^*_{ik,j}}{m^*_{ik}}$. Note that these three terms can be directly recovered from the data (Appendix B).

Hence, given the identification of $\left\{\frac{dP^*_{j'}}{d\tau_n}\right\}_{j'=1}^N$ (Proposition C.6) and $\frac{dm^*_{ik}}{d\tau_n}$ (Proposition C.8), I can identify $\frac{dm^*_{ik,j}}{d\tau_n}$ according to (66), which proves the claim. □

Remark C.7. *Alternatively, one may directly work on the total differentiation of* (20)*, which is given by*

$$\frac{dm^*_{ik,j}}{d\tau_n} = \left\{ \frac{1}{1-\tau_i} \mathbb{1}_{\{n=i\}} + \frac{1}{P_i^{M*}} \frac{dP_i^{M*}}{d\tau_n} - \frac{1}{P_j^*} \frac{dP_j^*}{d\tau_n} + \frac{1}{m^*_{ik}} \frac{dm^*_{ik}}{d\tau_n} \right\} m^*_{ik,j}.$$

In this case, the identification of $\frac{dm^*_{ik,j}}{d\tau_n}$ *follows from Propositions C.5, C.6 and C.8 as well as Appendix B.*

C.7 Recovering the First- and Second-Order Partial Derivatives of the Firm-Level Production Functions

The goal of this subsection is to identify the equilibrium values of the second-order derivatives of $f_i(\cdot)$ with respect to ℓ_{ik} and m_{ik}.[130] To begin with, observe that under Assumption 4.3, there exits a function $g_i : \mathscr{L}_i \times \mathscr{M}_i \to \mathbb{R}$ such that

$$f_i(\ell_{ik}, m_{ik}; z_{ik}) = z_{ik} g_i(\ell_{ik}, m_{ik}), \tag{105}$$

for all $(\ell_{ik}, m_{ik}, z_{ik}) \in \mathscr{L}_i \times \mathscr{M}_i \times \mathscr{Z}_i$. I define $\tilde{g}_i : \tilde{\mathscr{L}}_i \times \tilde{\mathscr{M}}_i \to \mathbb{R}$ such that

$$\tilde{f}_i(\tilde{\ell}_{ik}, \tilde{m}_{ik}; \tilde{z}_{ik}) = \tilde{z}_{ik} + \tilde{g}_i(\tilde{\ell}_{ik}, \tilde{m}_{ik}). \tag{106}$$

My identification strategy is based on the following relationships between the partial derivatives of $\tilde{g}_i(\cdot)$ and those of $f_i(\cdot)$.

Fact C.10. *Under Assumption 4.3, it holds that for all* $(\ell_{ik}, m_{ik}, z_{ik}) \in \mathscr{L}_i \times \mathscr{M}_i \times \mathscr{Z}_i$*,*

(i) $\frac{\partial \tilde{f}_i(\cdot)}{\partial \tilde{\ell}_{ik}} = \frac{\partial \tilde{g}_i(\cdot)}{\partial \tilde{\ell}_{ik}}$ *and* $\frac{\partial \tilde{f}_i(\cdot)}{\partial \tilde{m}_{ik}} = \frac{\partial \tilde{g}_i(\cdot)}{\partial \tilde{m}_{ik}}$*;*

(ii) $\frac{\partial f_i(\cdot)}{\partial \ell_{ik}} = \frac{\partial \tilde{g}_i(\cdot)}{\partial \tilde{\ell}_{ik}} \frac{f_i(\cdot)}{\ell_{ik}}$ *and* $\frac{\partial f_i(\cdot)}{\partial m_{ik}} = \frac{\partial \tilde{g}_i(\cdot)}{\partial \tilde{m}_{ik}} \frac{f_i(\cdot)}{m_{ik}}$*;*

(iii) $\frac{\partial^2 f_i(\cdot)}{\partial \ell_{ik}^2} = \frac{f_i(\cdot)}{\ell_{ik}^2} \left\{ \frac{\partial^2 \tilde{g}_i(\cdot)}{\partial \tilde{\ell}_{ik}^2} + \left(\frac{\partial \tilde{g}_i(\cdot)}{\partial \tilde{\ell}_{ik}} \right)^2 - \frac{\partial \tilde{g}_i(\cdot)}{\partial \tilde{\ell}_{ik}} \right\}$, $\frac{\partial^2 f_i(\cdot)}{\partial m_{ik}^2} = \frac{f_i(\cdot)}{m_{ik}^2} \left\{ \frac{\partial^2 \tilde{g}_i(\cdot)}{\partial \tilde{m}_{ik}^2} + \left(\frac{\partial \tilde{g}_i(\cdot)}{\partial \tilde{m}_{ik}} \right)^2 - \frac{\partial \tilde{g}_i(\cdot)}{\partial \tilde{m}_{ik}} \right\}$ *and* $\frac{\partial^2 f_i(\cdot)}{\partial \ell_{ik} \partial m_{ik}} = \frac{f_i(\cdot)}{\ell_{ik} m_{ik}} \left(\frac{\partial^2 \tilde{g}_i(\cdot)}{\partial \tilde{\ell}_{ik} \partial \tilde{m}_{ik}} + \frac{\partial \tilde{g}_i(\cdot)}{\partial \tilde{\ell}_{ik}} \frac{\partial \tilde{g}_i(\cdot)}{\partial \tilde{m}_{ik}} \right)$,

where $f_i(\cdot) \coloneqq f_i(\ell_{ik}, m_{ik}; z_{ik})$ *and* $\tilde{g}_i(\cdot) \coloneqq \tilde{g}_i(\tilde{\ell}_{ik}, \tilde{m}_{ik})$*.*

Proof. The proof is omitted. □

[130] Note that the equilibrium values of the first-order derivatives are already identified in Proposition C.2.

The identification results of Gandhi et al. (2019) rest on Fact C.10 (i) and the timing assumption encoded in (6). I further leverage the insights from Facts C.10 (ii) and (iii). In particular, invoking (iii) in equilibrium, I have

$$\frac{\partial^2 f_i(\cdot)^*}{\partial m_{ik}^2} = \frac{q_{ik}^*}{(m_{ik}^*)^2}\left\{\frac{\partial^2 \tilde{g}_i(\cdot)^*}{\partial \tilde{m}_{ik}^2} + \left(\frac{\partial \tilde{g}_i(\cdot)^*}{\partial \tilde{m}_{ik}}\right)^2 - \frac{\partial \tilde{g}_i(\cdot)^*}{\partial \tilde{m}_{ik}}\right\} \tag{107}$$

and also in light of Young's theorem,

$$\frac{\partial^2 f_i(\cdot)^*}{\partial m_{ik}\partial \ell_{ik}} = \frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik}\partial m_{ik}} = \frac{q_{ik}^*}{\ell_{ik}^* m_{ik}^*}\left\{\frac{\partial^2 \tilde{g}_i(\cdot)^*}{\partial \tilde{\ell}_{ik}\partial \tilde{m}_{ik}} + \left(\frac{\partial \tilde{g}_i(\cdot)^*}{\partial \tilde{\ell}_{ik}}\right)\left(\frac{\partial \tilde{g}_i(\cdot)^*}{\partial \tilde{m}_{ik}}\right)\right\}. \tag{108}$$

Once these are obtained, I can further invoke Euler's theorem for homogeneous functions to derive

$$\frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik}^2} = -\frac{m_{ik}^*}{\ell_{ik}^*}\frac{\partial^2 f_i(\cdot)^*}{\partial m_{ik}\partial \ell_{ik}} = \left(\frac{m_{ik}^*}{\ell_{ik}^*}\right)^2 \frac{\partial^2 f_i(\cdot)^*}{\partial m_{ik}^2}. \tag{109}$$

Since q_{ik}^* can be identified from Proposition C.3, it remains to identify (the equilibrium values of) the first- and second-order derivatives of $\tilde{g}_i(\cdot)$ with respect to $\tilde{\ell}_{ik}$ and $\tilde{m}_{ik}$. To this end, I follow Gandhi et al. (2019) in nonparametrically identifying the first-oder partial derivatives of $\tilde{g}(\cdot)$ as a function of $\tilde{\ell}_{ik}$ and $\tilde{m}_{ik}$.

The identification equation builds on the one-step profit maximization set out in Appendix A.1. Under Assumption 4.3, multiplying (28) by m_{ik} and dividing by $p_{ik}q_{ik}$ leads to

$$\frac{1}{\mu_{ik}}\frac{\partial \tilde{g}_i(\cdot)}{\partial \tilde{m}_{ik}} = s_{ik}^m,$$

where $s_{ik}^m := \frac{P_i^M m_{ik}}{p_{ik}q_{ik}}$ is the material input cost relative to the revenue. Taking the logarithm of this expression, I obtain

$$\ln s_{ik}^m = \ln \frac{\partial \tilde{g}_i(\cdot)}{\partial \tilde{m}_{ik}} - \ln \mu_{ik}. \tag{110}$$

However, in general this relationship cannot be directly fed into the data when the output market is imperfectly competitive, because the firm-level markup μ_{ik} needs to be identified (Kasahara and Sugita, 2020). Yet, in my setup, owing to Assumption 2.4 (i), μ_{ik} is recovered in advance of solving (110) for the first-order derivative of $\tilde{g}_i$ with respect to $\tilde{m}_{ik}$ (Fact C.3).

Taking stock of this, I adopt the same empirical specification as Gandhi et al. (2019):

$$\tilde{s}_{ik}^{m,\tilde{\mu}} = \ln \mathcal{E}_i^m + \ln \frac{\partial \tilde{g}_i}{\partial \tilde{m}_{ik}}(\tilde{\ell}_{ik}, \tilde{m}_{ik}) - \tilde{\varepsilon}_{ik}^m, \tag{111}$$

where $\tilde{s}_{ik}^{m,\tilde{\mu}} := \ln s_{ik}^m + \ln \mu_{ik}$ can readily be calculated from the data, and $\tilde{\varepsilon}_{ik}^m$ is a measurement error with $E[\tilde{\varepsilon}_{ik}^m \mid \tilde{\ell}_{ik}, \tilde{m}_{ik}] = 0$. The measurement error $\tilde{\varepsilon}_{ik}^m$ captures any unmodeled, non-systematic noise, and is associated with the constant $\mathcal{E}_i^m$ through $\mathcal{E}_i^m = E[\exp\{\tilde{\varepsilon}_{ik}^m\}]$. Inclusion of the mean $\mathcal{E}_i^m$ is based on the suggestion made in Gandhi et al. (2019).

My identification result heavily draws from Gandhi et al. (2019), and is summarized in the following lemma.

Lemma C.9 (Theorem 2 of Gandhi et al. (2019))**.** *Suppose that Assumptions 2.4 and 4.3 hold. Then, the share regression* (111) *identifies the first-order derivatives of* $\tilde{g}_i(\cdot)$ *with respect to log-labor and log-material inputs for all* $(\tilde{\ell}_{ik}, \tilde{m}_{ik}) \in \tilde{\mathscr{L}}_i \times \tilde{\mathscr{M}}_i$.

Proof. First, I start by writing (111) as

$$\tilde{s}_{ik}^{m,\tilde{\mu}} = \ln D_{ik}^m(\tilde{\ell}_{ik}, \tilde{m}_{ik}) - \tilde{\varepsilon}_{ik}^m, \tag{112}$$

where $\ln D_{ik}^m(\tilde{\ell}_{ik}, \tilde{m}_{ik}) := \ln \mathcal{E}_i^m + \ln \frac{\partial \tilde{g}_i}{\partial \tilde{m}_{ik}}(\tilde{\ell}_{ik}, \tilde{m}_{ik})$. I can nonparametrically identify $\ln D_{ik}^m(\tilde{\ell}_{ik}, \tilde{m}_{ik})$ according to

$$\ln D_{ik}^m(\tilde{\ell}_{ik}, \tilde{m}_{ik}) = E\big[\tilde{s}_{ik}^{m,\tilde{\mu}} | \tilde{\ell}_{ik}, \tilde{m}_{ik}\big]$$

for all $(\tilde{\ell}_{ik}, \tilde{m}_{ik}) \in \tilde{\mathscr{L}}_i \times \tilde{\mathscr{M}}_i$. The error term $\tilde{\varepsilon}_{ik}^m$ is identified through the specification (112):

$$\tilde{\varepsilon}_{ik}^m = \ln D_{ik}^m(\tilde{\ell}_{ik}, \tilde{m}_{ik}) - \tilde{s}_{ik}^{m,\tilde{\mu}} \tag{113}$$

which in turn identifies the mean $\mathcal{E}_i^m$:

$$\mathcal{E}_i^m = E\big[\exp\{\tilde{\varepsilon}_{ik}^m\}\big] \tag{114}$$

Next, plugging these back into the the definition of $\ln D_{ik}^m$, I identify the log-labor input elasticity of the log-production function:

$$\ln \frac{\partial \tilde{g}_i}{\partial \tilde{m}_{ik}}(\tilde{\ell}_{ik}, \tilde{m}_{ik}) = \ln D_{ik}^m(\tilde{\ell}_{ik}, \tilde{m}_{ik}) - \ln \mathcal{E}_i^m = \ln \frac{D_{ik}^m(\tilde{\ell}_{ik}, \tilde{m}_{ik})}{\mathcal{E}_i^m},$$

yielding

$$\frac{\partial \tilde{g}_i(\tilde{\ell}_{ik}, \tilde{m}_{ik})}{\partial \tilde{m}_{ik}} = \frac{D^m_{ik}(\tilde{\ell}_{ik}, \tilde{m}_{ik})}{\mathcal{E}^m_i} \tag{115}$$

for all $(\tilde{\ell}_{ik}, \tilde{m}_{ik}) \in \tilde{\mathscr{L}}_i \times \tilde{\mathscr{M}}_i$.

Lastly, given the identification of $\frac{\partial \tilde{g}_i(\cdot)}{\partial \tilde{m}_{ik}}$, one can invoke Euler's theorem for homogeneous functions under Assumption 2.4 (i) and Fact C.10 (i) to recover $\frac{\partial \tilde{g}_i(\cdot)}{\partial \tilde{\ell}_{ik}}$ for all $(\tilde{\ell}_{ik}, \tilde{m}_{ik}) \in \tilde{\mathscr{L}}_i \times \tilde{\mathscr{M}}_i$, completing the proof. □

As soon as $\frac{\partial \tilde{g}_i(\cdot)}{\partial \tilde{\ell}_{ik}}$ and $\frac{\partial \tilde{g}_i(\cdot)}{\partial \tilde{m}_{ik}}$ are identified as functions of $\tilde{\ell}_{ik}$ and $\tilde{m}_{ik}$, I can also recover the second-order derivatives of $\tilde{g}_i(\cdot)$.

Corollary C.4. *The second-order derivatives of $\tilde{g}_{ik}(\cdot)$ with respect to log-labor and log-material inputs, i.e., $\frac{\partial^2 \tilde{g}_i(\cdot)}{\partial \tilde{\ell}^2_{ik}}$, $\frac{\partial^2 \tilde{g}_i(\cdot)}{\partial \tilde{m}^2_{ik}}$, and $\frac{\partial^2 \tilde{g}_i(\cdot)}{\partial \tilde{\ell}_{ik} \tilde{m}_{ik}}$, are nonparametrically identified for all $(\tilde{\ell}_{ik}, \tilde{m}_{ik}) \in \tilde{\mathscr{L}}_i \times \tilde{\mathscr{M}}_i$.*

Now, I prove that it is possible to identify the values of the second-order derivative of the production function corresponding to the equilibrium labor and material inputs.

Lemma C.10. *Suppose that the assumptions required in Proposition C.3 and Lemma C.9 are satisfied. The equilibrium values of the second-order derivatives of the production function are identified from the observables.*

Proof. By Proposition C.3, q^*_{ik} is recovered. Moreover, Lemma C.9 identifies the value of $\frac{\partial \tilde{g}_i(\cdot)}{\partial \tilde{\ell}_{ik}}$ and $\frac{\partial \tilde{g}_i(\cdot)}{\partial \tilde{m}_{ik}}$ at the equilibrium values of inputs $(\tilde{\ell}^*_{ik}, \tilde{m}^*_{ik})$, while Corollary C.4 recovers the equilibrium values of $\frac{\partial^2 \tilde{g}_i(\cdot)}{\partial \tilde{\ell}^2_{ik}}$, $\frac{\partial^2 \tilde{g}_i(\cdot)}{\partial \tilde{m}^2_{ik}}$ and $\frac{\partial^2 \tilde{g}_i(\cdot)}{\partial \tilde{\ell}_{ik} \partial \tilde{m}_{ik}}$. Hence, by tracing (107), (108) and (109), I can recover the equilibrium values of the second-order derivatives of the production function, as claimed. □

C.8 Identification of the Object of Interest

Theorem C.1 (Identification of $\frac{dY_i(s)}{ds}$)**.** *Suppose that Assumptions 4.1–4.5 hold. Assume moreover that the regularity conditions (Assumptions C.2 and C.3) are satisfied. Then, the value of $\frac{dY_i(s)}{ds}$ evaluated at any point on $[\boldsymbol{\tau}^0, \boldsymbol{\tau}^1]$ is identified from the observables.*

Proof. Observe that $\frac{dY_i(s)}{ds}$ evaluated at an arbitrary point $s \in [\boldsymbol{\tau}^0, \boldsymbol{\tau}^1]$ can be decomposed as

$$\left.\frac{dY_i(s)}{ds}\right|_{s=\tau_n} = \sum_{k=1}^{N_i} \frac{dp^*_{ik}}{d\tau_n} q^*_{ik} + \sum_{k=1}^{N_i} p^*_{ik} \frac{dq^*_{ik}}{d\tau_n} - \left(\sum_{k=1}^{N_i} \sum_{j=1}^{N} \frac{dP^*_j}{d\tau_n} m^*_{ik,j} + \sum_{k=1}^{N_i} \sum_{j=1}^{N} P^*_j \frac{dm^*_{ik,j}}{d\tau_n} \right),$$

For all $i, j \in \mathscr{N}$ and $k \in \mathscr{N}_i$, I can recover p_{ik}^* and q_{ik}^* (Proposition C.3), $\frac{dp_{ik}^*}{d\tau_n}$ and $\frac{dq_{ik}^*}{d\tau_n}$ (Proposition C.7), $\frac{dP_j^*}{d\tau_n}$ (Proposition C.6), and $\frac{dm_{ik,j}^*}{d\tau_n}$ (Proposition C.9) over the empirical support. Hence, I can recover the value of $\frac{dY_i(s)}{ds}$ at any point on $\mathscr{T}$. □

Proof of Theorem 4.1. By applying Theorem C.1 repeatedly, the object of interest $\Delta Y(\tau_n^0, \tau_n^1)$ can be recovered according to (15):

$$\Delta Y(\tau_n^0, \tau_n^1) = \sum_{i=1}^{N} \int_{\tau_n^0}^{\tau_n^1} \frac{dY_i(s)}{ds} ds,$$

which proves the theorem. □

A version of Theorem 4.1 remains valid for the case of monopolistic competition with the solution concept appropriately modified.

Corollary C.5. *Suppose that the same assumptions as Theorem 4.1 are satisfied. Assume that firms operate within a structure of monopolistic competition in the output market. Then, the object of interest* (14) *is identified from the observables.*

Proof. The proof is analogous to that of Theorem 4.1 and only requires modifying the responsiveness of the firm's inverse demand and marginal revenue functions, accordingly (as explained in Remarks C.2 and C.4). □

Remark C.8. *(i) It is also possible to consider a hybrid environment in which some sectors are oligopolistic, while others are monopolistic. (ii) Corollary C.5 does not mean that my framework can be agnostic about the nature of the market competition. My framework requires specifying the market competition before analysis.*

C.9 Systematic Patterns Induced by Identification Assumptions

The identification assumptions induce several important patterns in the recovered firm-level responses, which in turn affects the policy parameter $\Delta Y(\tau_n^0, \tau_n^1)$. This subsection explores such patterns by classifying them into three categories, namely, *i*) patterns induced by the production-side assumptions, *ii*) those induced by the demand-side assumptions, and *iii*) those induced by the both types of assumptions.

C.9.1 Systematic Patterns Induced by Production-Side Assumptions

First, I look at the consequences of the assumptions imposed on the firm-level production function. The following lemma states that the firm's input choices are proportional to the firm's own output quantity and the inverse of the firm's own productivity.

Lemma C.11. *Suppose that Assumptions 2.4 and 4.3 hold. Then, for each $i \in \mathscr{N}$, there exist $\beta_i^\ell, \beta_i^m \in \mathbb{R}_+$ such that $\ell_{ik}^* = \beta_i^\ell z_{ik}^{-1} q_{ik}^*$ and $m_{ik}^* = \beta_i^m z_{ik}^{-1} q_{ik}^*$.*

Proof. Under Assumption 2.4, the firm's cost-minimization problem implies

$$TC_{ik}(W, P_i^M; q_{ik}^*) = MC_{ik}(W, P_i^M) q_{ik}^*,$$

where $TC_{ik}(\cdot; q_{ik}^*)$ and $MC_{ik}(\cdot)$, respectively, are the firm k's total cost function conditional on output quantity q_{ik}^*, and marginal cost function. Taking derivatives of this equation with respect to W and P_i^M yields

$$\frac{\partial TC_{ik}(\cdot)}{\partial W} = \frac{\partial MC_{ik}(\cdot)}{\partial W} q_{ik}^* \qquad \text{and} \qquad \frac{\partial TC_{ik}(\cdot)}{\partial P_i^M} = \frac{\partial MC_{ik}(\cdot)}{\partial P_i^M} q_{ik}^*.$$

In view of Shephard's lemma, these can equivalently be written as

$$\ell_{ik}^* = \frac{\partial MC_{ik}(\cdot)}{\partial W} q_{ik}^* \qquad \text{and} \qquad m_{ik}^* = \frac{\partial MC_{ik}(\cdot)}{\partial P_i^M} q_{ik}^*.$$

Since $\frac{\partial MC_{ik}(\cdot)}{\partial W}$ and $\frac{\partial MC_{ik}(\cdot)}{\partial P_i^M}$ do not involve the firm's choice variables (i.e., ℓ_{ik} and m_{ik}), these can be treated as constants. I thus define $\beta_{ik}^\ell := \frac{\partial MC_{ik}(\cdot)}{\partial W}$ and $\beta_{ik}^m := \frac{\partial MC_{ik}(\cdot)}{\partial P_i^m}$, so that

$$\ell_{ik}^* = \beta_{ik}^\ell q_{ik}^* \qquad \text{and} \qquad m_{ik}^* = \beta_{ik}^m q_{ik}^*. \tag{116}$$

Combined with Hicks-neutrality (Assumption 4.3), (116) suggests

$$z_{ik} g_i(\beta_{ik}^\ell, \beta_{ik}^m) = 1.$$

Under Assumption 2.4, this is true if and only if there exist $\beta_i^\ell, \beta_i^m \in \mathbb{R}_+$ such that $\beta_{ik}^\ell = \beta_i^\ell z_{ik}^{-1}$ and $\beta_{ik}^m = \beta_i^m z_{ik}^{-1}$ with $g_i(\beta_i^\ell, \beta_i^m) = 1$. Substituting this back into (116) leads to

$$\ell_{ik}^* = \beta_i^\ell z_{ik}^{-1} q_{ik}^* \qquad \text{and} \qquad m_{ik}^* = \beta_i^m z_{ik}^{-1} q_{ik}^*,$$

as desired. □

By construction, β_i^ℓ and β_i^m convey partial information about the marginal cost common to all firms. With this insight in mind, the following corollary is straightforward.

Corollary C.6. *Suppose that the assumptions required in Lemma C.11 are satisfied. Then, for each $i \in \mathscr{N}$ and each $k \in \mathscr{N}_i$, $mc_{ik} = mc_i z_{ik}^{-1}$ with $mc_i = \beta_i^\ell W + \beta_i^m P_i^M$, where β_i^ℓ and β_i^m are the constants appearing in Lemma C.11.*

Proof. Assumption 2.4 implies

$$W\ell_{ik}^{*} + P_i^M m_{ik}^{*} = mc_{ik} q_{ik}^{*}.$$

From Lemma C.11, this further implies

$$(\beta_i^\ell W + \beta_i^m P_i^M) z_{ik}^{-1} q_{ik}^{*} = mc_{ik} q_{ik}^{*},$$

so that

$$mc_{ik} = (\beta_i^\ell W + \beta_i^m P_i^M) z_{ik}^{-1}.$$

Upon defining $mc_i = \beta_i^\ell W + \beta_i^m P_i^M$, the claim is proved. □

C.9.2 Systematic Patterns Induced by Demand-Side Assumptions

Next, I derive several theoretical results that follow from the assumptions imposed on the demand side (i.e., the sectoral aggregator). Here, it is postulated that firms engage in oligopolistic competition in the output market as in the main text, while the case of monopolistic competition is postponed until Appendix C.9.4.

The following lemma pushes Lemma C.7 forward to derive the system of firms' pricing equations in equilibrium.

Lemma C.12 (Firms' Pricing Equations in Oligopolistic Competition)**.** *Suppose that Assumption 4.4 holds. Then, for each* $i \in \mathcal{N}$, $p_{ik}^{*} = (\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}})^{-1} \frac{1}{1-\varpi_{ik}} mc_{ik}$ *for all* $k \in \mathcal{N}_i$.

Proof. For each firm $k \in \mathcal{N}_i$, the profit-maximization problem with respect to quantity implies

$$mr_{ik} = mc_{ik}. \tag{117}$$

Under Assumption 4.4, the left-hand side of (117) reads[131]

$$mr_{ik} = \frac{dr_{ik}}{dq_{ik}} = \exp\{\tilde{\varphi}_i(\tilde{x}_{ik})\} \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}} \frac{d\tilde{x}_{ik}}{dx_{ik}} \frac{\partial x_{ik}(\cdot)}{\partial q_{ik}} = p_{ik} \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}} (1 - \varpi_{ik}).$$

Thus, (117) implies

$$p_{ik}^{*} \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}} (1 - \varpi_{ik}) = mc_{ik},$$

[131]See also the proof of Lemma C.7.

so that

$$p_{ik}^* = \left(\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\right)^{-1}(1-\varpi_{ik})^{-1}mc_{ik},$$

which proves the statement of this lemma. □

The following two facts pertain to the analytical expressions of the derivatives of the firm's marginal revenue function (derived in the proof of Lemma C.7).

Fact C.11. *Suppose that Assumption 4.4 holds. Then,* $\sum_{k'=1}^{N_i} q_{ik'}t_{ik'} = 0$.

Proof. It immediately follows from the definitions of t_{ik} and ϖ_{ik} that

$$\begin{aligned}
\sum_{k'=1}^{N_i} q_{ik'}t_{ik'} &= \sum_{k'=1}^{N_i} q_{ik'}\frac{1}{q_{ik'}}\varpi_{ik'}\left(\varrho_{ik'} - \frac{\sum_{k''=1}^{N_i}\varrho_{ik''}\tilde{u}_{ik''}}{\sum_{k''=1}^{N_i}\tilde{u}_{ik''}}\right)\\
&= \sum_{k'=1}^{N_i}\varpi_{ik'}\varrho_{ik'} - \frac{\sum_{k''=1}^{N_i}\varrho_{ik''}\tilde{u}_{ik''}}{\sum_{k''=1}^{N_i}\tilde{u}_{ik''}}\sum_{k'=1}^{N_i}\varpi_{ik'}\\
&= \frac{\sum_{k'=1}^{N_i}\varrho_{ik'}\tilde{u}_{ik'}}{\sum_{k''=1}^{N_i}\tilde{u}_{ik''}} - \frac{\sum_{k''=1}^{N_i}\varrho_{ik''}\tilde{u}_{ik''}}{\sum_{k''=1}^{N_i}\tilde{u}_{ik''}}\\
&= 0,
\end{aligned}$$

as desired. □

Now, to simplify the exposition, I introduce two additional notations. Denote

$$\begin{aligned}
\hat{B}_{ik} &:= \frac{d^2\tilde{r}_{ik}}{d\tilde{x}_{ik}^2}(1-\varpi_{ik}) - \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\left\{\left(1 - \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\right) + \left(\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}} + \varrho_{ik}\right)\varpi_{ik}\right\}\\
\check{B}_{ik} &:= \frac{d^2\tilde{r}_{ik}}{d\tilde{x}_{ik}^2}(1-\varpi_{ik}) - \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\left\{\left(-\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\right) + \left(\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}} + \varrho_{ik}\right)\varpi_{ik}\right\},
\end{aligned}$$

so that

$$\begin{aligned}
\frac{\partial mr_{ik}(\cdot)^*}{\partial q_{ik}} &= \frac{p_{ik}^*}{q_{ik}^*}(1-\varpi_{ik})\hat{B}_{ik} + \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}p_{ik}^*\varpi_{ik}t_{ik}\\
\frac{\partial mr_{ik}(\cdot)^*}{\partial q_{ik'}} &= -p_{ik}^*\frac{\varpi_{ik'}}{q_{ik'}^*}\check{B}_{ik} + \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}p_{ik}^*\varpi_{ik}t_{ik'}.
\end{aligned}$$

The following fact is immediate.

Fact C.12. *For all* $i \in \mathscr{N}$ *and* $k \in \mathscr{N}_i$, $\hat{B}_{ik} - \check{B}_{ik} = -\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}$.

Proof. By definition, it is straightforward to verify that

$$\begin{aligned}\hat{B}_{ik} - \check{B}_{ik} &= \frac{d^2\tilde{r}_{ik}}{d\tilde{x}_{ik}^2}(1-\varpi_{ik}) - \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\left\{\left(1-\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\right)+\left(\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}+\varrho_{ik}\right)\varpi_{ik}\right\}\\ &\qquad - \frac{d^2\tilde{r}_{ik}}{d\tilde{x}_{ik}^2}(1-\varpi_{ik}) + \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\left\{\left(-\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\right)+\left(\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}+\varrho_{ik}\right)\varpi_{ik}\right\}\\ &= -\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}},\end{aligned}$$

as claimed. □

C.9.3 Systematic Patterns Induced by Production- and Demand-Side Assumptions

Finally, I am in a position to derive key results for the systematic patterns of the recovered responses. With the production- and demand-side assumptions combined, the following proposition states that the elasticity of the firm's output quantity is constant for all firms in the same sector.

Proposition C.10 (Elasticity of Firm-Level Quantity)**.** *Suppose that Assumptions 2.4, 4.3, 4.4 and A.1 hold. Then, for each* $i \in \mathscr{N}$, $\frac{dq_{ik}^*/d\tau_n}{q_{ik}^*} = \vec{c}_i^q$ *for all* $k \in \mathscr{N}_i$ *with* $\vec{c}_i^q := -\frac{\beta_i^\ell \frac{dW^*}{d\tau_n} + \beta_i^m \frac{dP_i^{M*}}{d\tau_n}}{\beta_i^\ell W^* + \beta_i^m P_i^{M*}}$*, where* β_i^ℓ *and* β_i^ℓ *are the constants appearing in Lemma C.11.*

Proof. Observe that (31) (for the realized ℓ_{ik}^* and m_{ik}^*) can equivalently be rewritten as

$$\begin{bmatrix} \frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i1}} & \frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i2}} & \cdots & \frac{\partial mr_{i1}(\cdot)^*}{\partial q_{iN_i}} \\ \frac{\partial mr_{i2}(\cdot)^*}{\partial q_{i1}} & \frac{\partial mr_{i2}(\cdot)^*}{\partial q_{i2}} & \cdots & \frac{\partial mr_{i2}(\cdot)^*}{\partial q_{iN_i}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial mr_{iN_i}(\cdot)^*}{\partial q_{i1}} & \frac{\partial mr_{iN_i}(\cdot)^*}{\partial q_{i2}} & \cdots & \frac{\partial mr_{iN_i}(\cdot)^*}{\partial q_{iN_i}} \end{bmatrix} \begin{bmatrix} q_{i1} & 0 & \cdots & 0 \\ 0 & q_{i2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & q_{iN_i} \end{bmatrix} \begin{bmatrix} \frac{1}{q_{i1}} & 0 & \cdots & 0 \\ 0 & \frac{1}{q_{i2}} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{1}{q_{iN_i}} \end{bmatrix} \begin{bmatrix} \frac{dq_{i1}^*}{d\tau_n} \\ \frac{dq_{i2}^*}{d\tau_n} \\ \vdots \\ \frac{dq_{iN_i}^*}{d\tau_n} \end{bmatrix}$$
$$= \begin{bmatrix} \frac{1}{q_{i1}} & 0 & \cdots & 0 \\ 0 & \frac{1}{q_{i2}} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{1}{q_{iN_i}} \end{bmatrix} \begin{bmatrix} \ell_{i1}^* & m_{i1}^* \\ \ell_{i2}^* & m_{i2}^* \\ \vdots & \vdots \\ \ell_{iN_i}^* & m_{iN_i}^* \end{bmatrix} \begin{bmatrix} \frac{dW^*}{d\tau_n} \\ \frac{dP_i^{M*}}{d\tau_n} \end{bmatrix},$$

which can further be rearranged as

$$\begin{bmatrix} q_{i1} & 0 & \dots & 0 \\ 0 & q_{i2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & q_{iN_i} \end{bmatrix} \begin{bmatrix} \frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i1}} & \frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i2}} & \dots & \frac{\partial mr_{i1}(\cdot)^*}{\partial q_{iN_i}} \\ \frac{\partial mr_{i2}(\cdot)^*}{\partial q_{i1}} & \frac{\partial mr_{i2}(\cdot)^*}{\partial q_{i2}} & \dots & \frac{\partial mr_{i2}(\cdot)^*}{\partial q_{iN_i}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial mr_{iN_i}(\cdot)^*}{\partial q_{i1}} & \frac{\partial mr_{iN_i}(\cdot)^*}{\partial q_{i2}} & \dots & \frac{\partial mr_{iN_i}(\cdot)^*}{\partial q_{iN_i}} \end{bmatrix} \begin{bmatrix} q_{i1} & 0 & \dots & 0 \\ 0 & q_{i2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & q_{iN_i} \end{bmatrix} \begin{bmatrix} \frac{dq^*_{i1}/d\tau_n}{q^*_{i1}} \\ \frac{dq^*_{i2}/d\tau_n}{q^*_{i2}} \\ \vdots \\ \frac{dq^*_{iN_i}/d\tau_n}{q^*_{iN_i}} \end{bmatrix} = \begin{bmatrix} \ell^*_{i1} & m^*_{i1} \\ \ell^*_{i2} & m^*_{i2} \\ \vdots & \vdots \\ \ell^*_{iN_i} & m^*_{iN_i} \end{bmatrix} \begin{bmatrix} \frac{dW^*}{d\tau_n} \\ \frac{dP_i^{M*}}{d\tau_n} \end{bmatrix}. \tag{118}$$

Due to the invertibility (Assumption A.1), (118) can uniquely be solved for $[\frac{dq^*_{i1}/d\tau_n}{q^*_{i1}} \frac{dq^*_{i2}/d\tau_n}{q^*_{i2}} \dots \frac{dq^*_{iN_i}/d\tau_n}{q^*_{iN_i}}]^T$. Thus, it suffices to very that

$$\begin{bmatrix} \frac{dq^*_{i1}/d\tau_n}{q^*_{i1}} \\ \frac{dq^*_{i2}/d\tau_n}{q^*_{i2}} \\ \vdots \\ \frac{dq^*_{iN_i}/d\tau_n}{q^*_{iN_i}} \end{bmatrix} = -\frac{\beta_i^\ell \frac{dW^*}{d\tau_n} + \beta_i^m \frac{dP_i^{M*}}{d\tau_n}}{\beta_i^\ell W^* + \beta_i^m P_i^{M*}} \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} \tag{119}$$

satisfies (118).

Now, provided (119), the left-hand side of (118) boils down to

$$\begin{aligned} &\bar{c}_i^q \begin{bmatrix} q_{i1} & 0 & \dots & 0 \\ 0 & q_{i2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & q_{iN_i} \end{bmatrix} \begin{bmatrix} \frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i1}} & \frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i2}} & \dots & \frac{\partial mr_{i1}(\cdot)^*}{\partial q_{iN_i}} \\ \frac{\partial mr_{i2}(\cdot)^*}{\partial q_{i1}} & \frac{\partial mr_{i2}(\cdot)^*}{\partial q_{i2}} & \dots & \frac{\partial mr_{i2}(\cdot)^*}{\partial q_{iN_i}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial mr_{iN_i}(\cdot)^*}{\partial q_{i1}} & \frac{\partial mr_{iN_i}(\cdot)^*}{\partial q_{i2}} & \dots & \frac{\partial mr_{iN_i}(\cdot)^*}{\partial q_{iN_i}} \end{bmatrix} \begin{bmatrix} q_{i1} & 0 & \dots & 0 \\ 0 & q_{i2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & q_{iN_i} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} \\ &= \bar{c}_i^q \begin{bmatrix} \frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i1}} q^*_{i1} q^*_{i1} & \frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i2}} q^*_{i1} q^*_{i2} & \dots & \frac{\partial mr_{i1}(\cdot)^*}{\partial q_{iN_i}} q^*_{i1} q^*_{iN_i} \\ \frac{\partial mr_{i2}(\cdot)^*}{\partial q_{i1}} q^*_{i2} q^*_{i1} & \frac{\partial mr_{i2}(\cdot)^*}{\partial q_{i2}} q^*_{i2} q^*_{i2} & \dots & \frac{\partial mr_{i2}(\cdot)^*}{\partial q_{iN_i}} q^*_{i2} q^*_{iN_i} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial mr_{iN_i}(\cdot)^*}{\partial q_{i1}} q^*_{iN_i} q^*_{i1} & \frac{\partial mr_{iN_i}(\cdot)^*}{\partial q_{i2}} q^*_{iN_i} q^*_{i2} & \dots & \frac{\partial mr_{iN_i}(\cdot)^*}{\partial q_{iN_i}} q^*_{iN_i} q^*_{iN_i} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}, \end{aligned} \tag{120}$$

where $\bar{c}_i^q := -\frac{\beta_i^\ell \frac{dW^*}{d\tau_n} + \beta_i^m \frac{dP_i^{M*}}{d\tau_n}}{\beta_i^\ell W^* + \beta_i^m P_i^{M*}}$. Notice here that

$$\frac{\partial mr_{ik}(\cdot)^*}{\partial q_{ik}} q_{ik}^* q_{ik}^* = p_{ik}^* q_{ik}^* (1 - \varpi_{ik}) \hat{B}_{ik} + \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}} p_{ik}^* q_{ik}^* \varpi_{ik} q_{ik}^* t_{ik},$$

and

$$\frac{\partial mr_{ik}(\cdot)^*}{\partial q_{ik'}} q_{ik}^* q_{ik'}^* = -p_{ik}^* q_{ik}^* \varpi_{ik'} \check{B}_{ik} + \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}} p_{ik}^* q_{ik}^* \varpi_{ik} q_{ik'}^* t_{ik'},$$

for all $k' \neq k$.

(i) the 1st row. The first row of (120), denoted as LHS_1, reads

$$\begin{aligned}
LHS_1 = \bar{c}_i^q \Big\{ & p_{i1}^* q_{i1}^* (1 - \varpi_{i1}) \hat{B}_{i1} + \frac{d\tilde{r}_{i1}}{d\tilde{x}_{i1}} p_{i1}^* q_{i1}^* \varpi_{i1} q_{i1}^* t_{i1} \\
& - p_{i1}^* q_{i1}^* \varpi_{i2} \check{B}_{i1} + \frac{d\tilde{r}_{i1}}{d\tilde{x}_{i1}} p_{i1}^* q_{i1}^* \varpi_{i1} q_{i2}^* t_{i2} \\
& - \dots \\
& - p_{i1}^* q_{i1}^* \varpi_{iN_i} \check{B}_{i1} + \frac{d\tilde{r}_{i1}}{d\tilde{x}_{i1}} p_{i1}^* q_{i1}^* \varpi_{i1} q_{iN_i}^* t_{iN_i} \Big\} \\
= \ & \bar{c}_i^q p_{i1}^* q_{i1}^* (1 - \varpi_{i1}) (\hat{B}_{i1} - \check{B}_{i1}) \\
= \ & -\bar{c}_i^q mc_i z_{i1}^{-1} q_{i1}^* \\
= \ & \Big(\beta_i^\ell \frac{dW^*}{d\tau_n} + \beta_i^m \frac{dP_i^{M*}}{d\tau_n} \Big) z_{i1}^{-1} q_{i1}^*,
\end{aligned}$$

where the second equality is a consequence of Fact C.11, the third equality is due to Lemma C.12 and Fact C.12, and the fourth equality follows from Corollary C.6.

The first row of the right-hand side of (118), denoted as RHS_1, is

$$RHS_1 = \ell_{i1}^* \frac{dW^*}{d\tau_n} + m_{i1}^* \frac{dP_i^{M*}}{d\tau_n} = \Big(\beta_i^\ell \frac{dW^*}{d\tau_n} + \beta_i^m \frac{dP_i^{M*}}{d\tau_n} \Big) z_{i1}^{-1} q_{i1}^*,$$

where the second equality comes from Lemma C.11.

Clearly, $LHS_1 = RHS_1$, meaning that (119) is true for the first row of (118).

(ii) the N_ith row. The last row of (120), denoted as LHS_{N_i}, reads

$$\begin{aligned} LHS_{N_i} = \bar{c}_i^q \Big\{ & - p_{iN_i}^* q_{iN_i}^* \varpi_{i1} \check{B}_{iN_i} + \frac{d\tilde{r}_{iN_i}}{d\tilde{x}_{i1}} p_{iN_i}^* q_{iN_i}^* \varpi_{iN_i} q_{i1}^* t_{i1} \\ & - \dots \\ & - p_{iN_i}^* q_{iN_i}^* \varpi_{i,N_i-1} \check{B}_{iN_i} + \frac{d\tilde{r}_{iN_i}}{d\tilde{x}_{iN_i}} p_{iN_i}^* q_{iN_i}^* \varpi_{iN_i} q_{i,N_i-1}^* t_{i,N_i-1} \\ & + p_{iN_1}^* q_{iN_1}^* (1 - \varpi_{iN_1}) \hat{B}_{iN_1} + \frac{d\tilde{r}_{iN_1}}{d\tilde{x}_{iN_1}} p_{iN_1}^* q_{iN_1}^* \varpi_{iN_1} q_{iN_1}^* t_{iN_1} \Big\} \\ = {} & \bar{c}_i^q p_{iN_i}^* q_{iN_i}^* (1 - \varpi_{iN_i}) (\hat{B}_{iN_i} - \check{B}_{iN_i}) \\ = {} & -\bar{c}_i^q mc_i z_{iN_i}^{-1} q_{iN_i}^* \\ = {} & \Big(\beta_i^\ell \frac{dW^*}{d\tau_n} + \beta_i^m \frac{dP_i^{M*}}{d\tau_n} \Big) z_{iN_i}^{-1} q_{iN_i}^*, \end{aligned}$$

while the right-hand side of (118), denoted as RHS_{N_i}, is

$$RHS_{N_i} = \ell_{iN_i}^* \frac{dW^*}{d\tau_n} + m_{iN_i}^* \frac{dP_i^{M*}}{d\tau_n} = \Big(\beta_i^\ell \frac{dW^*}{d\tau_n} + \beta_i^m \frac{dP_i^{M*}}{d\tau_n} \Big) z_{iN_i}^{-1} q_{iN_i}^*.$$

Clearly, $LHS_{N_i} = RHS_{N_i}$, meaning that (119) is true for the last row of (118).

(iii) the kth row ($k = 2, 3, \dots, N_i - 1$). The kth row of (120), denoted as LHS_k, reads

$$\begin{aligned} LHS_k = \bar{c}_i^q \Big\{ & - p_{ik}^* q_{ik}^* \varpi_{i1} \check{B}_{ik} + \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}} p_{ik}^* q_{ik}^* \varpi_{ik} q_{i1}^* t_{i1} \\ & - \dots \\ & - p_{ik}^* q_{ik}^* \varpi_{i,k-1} \check{B}_{ik} + \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}} p_{ik}^* q_{ik}^* \varpi_{ik} q_{i,k-1}^* t_{i,k-1} \\ & + p_{ik}^* q_{ik}^* (1 - \varpi_{i,k}) \hat{B}_{ik} + \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}} p_{ik}^* q_{ik}^* \varpi_{ik} q_{i,k}^* t_{i,k} \\ & - p_{ik}^* q_{ik}^* \varpi_{i,k+1} \check{B}_{ik} + \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}} p_{ik}^* q_{ik}^* \varpi_{ik} q_{i,k+1}^* t_{i,k+1} \\ & - \dots \\ & - p_{ik}^* q_{ik}^* \varpi_{iN_i} \check{B}_{ik} + \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}} p_{ik}^* q_{ik}^* \varpi_{ik} q_{iN_i}^* t_{iN_i} \Big\} \\ = {} & \bar{c}_i^q p_{ik}^* q_{ik}^* (1 - \varpi_{ik}) (\hat{B}_{ik} - \check{B}_{ik}) \\ = {} & -\bar{c}_i^q mc_i z_{ik}^{-1} q_{ik}^* \\ = {} & \Big(\beta_i^\ell \frac{dW^*}{d\tau_n} + \beta_i^m \frac{dP_i^{M*}}{d\tau_n} \Big) z_{ik}^{-1} q_{ik}^*, \end{aligned}$$

where the second equality is a consequence of Fact C.11, the third equality is due to Lemma C.12 and Fact C.12, and the fourth equality follows from Corollary C.6.

The kth row of the right-hand side of (118), denoted as RHS_k, is

$$RHS_k = \ell^*_{ik} \frac{dW^*}{d\tau_n} + m^*_{ik} \frac{dP_i^{M*}}{d\tau_n} = \Big(\beta_i^\ell \frac{dW^*}{d\tau_n} + \beta_i^m \frac{dP_i^{M*}}{d\tau_n}\Big) z_{ik}^{-1} q^*_{ik},$$

where the second equality comes from Lemma C.11.

Clearly, $LHS_k = RHS_k$, meaning that (119) is true for the kth row of (118) for $k = 2, \ldots, N_i - 1$.

Hence, I have shown that (119) is certainly a unique solution for (118), completing the proof. □

Corollary C.7 (Elasticities of Firm-Level Quantity and Price)**.** *Suppose that the assumptions required in Proposition C.10 are satisfied. Then, for each* $i \in \mathscr{N}$, *(i)* $\frac{dp^*_{ik}/d\tau_n}{p^*_{ik}} = \bar{c}_i^p$, *where* $\bar{c}_i^p = -\bar{c}_i^q$; *and (ii)* $\frac{dp^*_{ik}}{d\tau_n} q^*_{ik} + p^*_{ik} \frac{dq^*_{ik}}{d\tau_n} = 0$ *for all* $k \in \mathscr{N}_i$.

Proof. (i) By construction,

$$\begin{aligned}
\frac{dp^*_{ik}}{d\tau_n} &= \sum_{k'=1}^{N_i} \frac{\partial \wp_{ik}(\cdot)^*}{\partial q_{ik'}} \frac{dq^*_{ik'}}{d\tau_n} \\
&= \frac{\partial \wp_{ik}(\cdot)^*}{\partial q_{ik}} \frac{dq^*_{ik}}{d\tau_n} + \sum_{k' \neq k} \frac{\partial \wp_{ik}(\cdot)^*}{\partial q_{ik'}} \frac{dq^*_{ik'}}{d\tau_n} \\
&= -p^*_{ik} \Big\{ 1 - \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}} (1 - \varpi_{ik}) \Big\} \frac{dq^*_{ik}/d\tau_n}{q^*_{ik}} - \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}} p^*_{ik} \sum_{k' \neq k} \varpi_{ik'} \frac{dq^*_{ik'}/d\tau_n}{q^*_{ik'}} \\
&= \bar{c}_i^q \Big[-p^*_{ik} \Big\{ 1 - \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}} (1 - \varpi_{ik}) \Big\} - \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}} p^*_{ik} (1 - \varpi_{ik}) \Big] \\
&= -\bar{c}_i^q p^*_{ik},
\end{aligned}$$

where the third equality utilizes the analytical expressions for the price elasticities derived in Lemma C.6, and the fourth equality is a consequence of Proposition C.10. Rearranging this leads to

$$\frac{dp^*_{ik}/d\tau_n}{p^*_{ik}} = -\bar{c}_i^q.$$

By setting $\bar{c}_i^p = -\bar{c}_i^q$, the claim is proved.

(ii) It is straightforward to show that

$$\frac{dp_{ik}^*}{d\tau_n} q_{ik}^* + p_{ik}^* \frac{dq_{ik}^*}{d\tau_n} = p_{ik}^* q_{ik}^* \left(\frac{dp_{ik}^*/d\tau_n}{p_{ik}^*} + \frac{dq_{ik}^*/d\tau_n}{q_{ik}^*} \right) = p_{ik}^* q_{ik}^* (-\bar{c}_i^q + \bar{c}_i^q) = 0,$$

where the second equality is due to part (i) of this corollary and Proposition C.10. This completes the proof of this corollary. □

Proposition C.10 and Corollary C.7 (i), respectively, state that the elasticities of a firm's output quantity and price with respect to a subsidy change do not vary across firms. Corollary C.7 (ii) refers to the responsiveness of the firm-level revenue: The price effect exactly cancels out the quantity effect. This also means that the first two terms inside the curly bracket in (16) (i.e., the revenue effects) vanish, leaving the cost effects as the sole component (see Section 5.2).[132]

To facilitate interpretation, it is useful to look at Corollary C.7 (ii) in terms of the elasticity of price with respect to quantity: for each $i \in \mathscr{N}$,

$$\frac{dp_{ik}^*/p_{ik}^*}{dq_{ik}^*/q_{ik}^*} = -1, \tag{121}$$

for all $k \in \mathscr{N}_i$.[133] This expression entails two observations. First, the elasticity being the same across firms in the same sector is a natural consequence of Assumption 4.4. Second, the unitary elasticity suggests that the sectoral demand, coupled with the strategic interactions, is "strong enough" to affect the price level in a way that exactly offsets the effect of a change in quantity demanded, keeping the sectoral aggregator's total expenditure unchanged. In contrast, the demand in monopolistic competition (i.e., in the absence of strategic forces) is inelastic due to a firm's market power (as elaborated on in Appendix C.9.4).

C.9.4 Systematic Patterns Induced by Production- and Demand-Side Assumptions (Monopolistic Competition)

The following lemma is a monopolistic competition counterpart of Lemma C.12.

Lemma C.13 (Firm's Pricing Equations in Monopolistic Competition)**.** *Assume that firms in each sector engage in monopolistic competition in the output market. Suppose that Assumption 4.4 holds. Then, for each* $i \in \mathscr{N}$, $p_{ik}^* = (\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}})^{-1} mc_{ik}$ *for all* $k \in \mathscr{N}_i$.

[132]While this is an artifact of the functional form assumptions, it is worth emphasizing that these assumptions include the specifications commonly employed in the existing literature, as seen in Example C.1.

[133]The fact that the constant $\bar{c}_i$ depends on the macro and micro complementarities offers an alternative view, namely, the complementarities are determined in a way that the quantity elasticities become common across firms in the same sector.

Proof. For each firm $k \in \mathcal{N}_i$, fhe profit-maximization problem with respect to quantity implies

$$mr_{ik} = mc_{ik}. \tag{122}$$

Under Assumption 4.4, the left-hand side of (122) reads (with a slight abuse of notation)

$$mr_{ik} = \frac{dr_{ik}}{dq_{ik}} = \exp\{\tilde{\varphi}_i(\tilde{x}_{ik})\}\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\frac{d\tilde{x}_{ik}}{dx_{ik}}\frac{\partial x_{ik}(\cdot)}{\partial q_{ik}} = \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}p_{ik}.$$

Thus, (122) implies

$$p^*_{ik}\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}} = mc_{ik},$$

so that

$$p^*_{ik} = \left(\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\right)^{-1} mc_{ik},$$

proving the statement of this lemma. □

Proposition C.11 (Elasticity of Firm-Level Quantity in Monopolistic Competition)**.** *Assume that firms in each sector engage in monopolistic competition in the output market. Suppose that Assumptions 2.4, 4.3 and 4.4 hold. Then, for each $i \in \mathcal{N}$, there exists a sector-specific constant $\bar{c}^q_i \in \mathbb{R}\backslash\{0\}$ such that $\frac{dq^*_{ik}/d\tau_n}{q^*_{ik}} = \bar{c}^q_i$ for all $k \in \mathcal{N}_i$, if and only if there exists a sector-specific constant $\bar{d}^q_i \in \mathbb{R}\backslash\{0\}$ such that $\left(\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\right)^{-1}\left\{\frac{d^2\tilde{r}_{ik}}{d\tilde{x}^2_{ik}} - \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\left(1 - \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\right)\right\} = \bar{d}^q_i$ for all $k \in \mathcal{N}_i$.*

Proof. First of all, in monopolistic competition, the equation corresponding to (118) can be written as

$$\begin{bmatrix} q_{i1} & 0 & \dots & 0 \\ 0 & q_{i2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & q_{iN_i} \end{bmatrix} \begin{bmatrix} \frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i1}} & 0 & \dots & 0 \\ 0 & \frac{\partial mr_{i2}(\cdot)^*}{\partial q_{i2}} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \frac{\partial mr_{iN_i}(\cdot)^*}{\partial q_{iN_i}} \end{bmatrix} \begin{bmatrix} q_{i1} & 0 & \dots & 0 \\ 0 & q_{i2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & q_{iN_i} \end{bmatrix} \begin{bmatrix} \frac{dq_{i1}^*/d\tau_n}{q_{i1}^*} \\ \frac{dq_{i2}^*/d\tau_n}{q_{i2}^*} \\ \vdots \\ \frac{dq_{iN_i}^*/d\tau_n}{q_{iN_i}^*} \end{bmatrix}$$

$$= \begin{bmatrix} \ell_{i1}^* & m_{i1}^* \\ \ell_{i2}^* & m_{i2}^* \\ \vdots & \vdots \\ \ell_{iN_i}^* & m_{iN_i}^* \end{bmatrix} \begin{bmatrix} \frac{dW^*}{d\tau_n} \\ \frac{dP_i^{M*}}{d\tau_n} \end{bmatrix}, \tag{123}$$

where $\frac{\partial mr_{ik}(\cdot)^*}{\partial q_{ik}} = \frac{p_{ik}^*}{q_{ik}^*}\Big\{\frac{d^2\tilde{r}_{ik}}{d\tilde{x}_{ik}^2} - \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\big(1 - \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\big)\Big\}$.[134]

($\Longrightarrow$). Suppose that for each sector $i \in \mathscr{N}$, there exists a sector-specific constant $\bar{c}_i^q \in \mathbb{R}\backslash\{0\}$ such that $\frac{dq_{ik}^*/d\tau_n}{q_{ik}^*} = \bar{c}_i^q$ for all $k \in \mathscr{N}_i$. Then, it follows from (123) that for each $i \in \mathscr{N}$,

$$\bar{c}_i^q \frac{\partial mr_{ik}(\cdot)^*}{\partial q_{ik}} q_{ik}^* q_{ik}^* = \ell_{ik}^* \frac{dW^*}{d\tau_n} + m_{ik}^* \frac{dP_i^{M*}}{d\tau_n},$$

for all $k \in \mathscr{N}_i$. In view of Lemma C.11, Corollary C.6 and Lemma C.13, this yields

$$\Big(\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\Big)^{-1}\Big\{\frac{d^2\tilde{r}_{ik}}{d\tilde{x}_{ik}^2} - \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\big(1 - \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\big)\Big\} = (\bar{c}_i^q)^{-1}\frac{\beta_i^\ell \frac{dW^*}{d\tau_n} + \beta_i^m \frac{dP_i^{M*}}{d\tau_n}}{\beta_i^\ell W^* + \beta_i^m P_i^{M*}},$$

where β_i^ℓ and β_i^m are the constants appearing in Lemma C.11. Since the right-hand side of this expression is free from the firm-specific index k, the implication is true by setting $\bar{d}_i^q := (\bar{c}_i^q)^{-1}\frac{\beta_i^\ell \frac{dW^*}{d\tau_n} + \beta_i^m \frac{dP_i^{M*}}{d\tau_n}}{\beta_i^\ell W^* + \beta_i^m P_i^{M*}}$.

($\Longleftarrow$). Suppose that for each sector $i \in \mathscr{N}$, there exists a sector-specific constant $\bar{d}_i^q \in \mathbb{R}\backslash\{0\}$ such that $\Big(\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\Big)^{-1}\Big\{\frac{d^2\tilde{r}_{ik}}{d\tilde{x}_{ik}^2} - \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\big(1 - \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\big)\Big\} = \bar{d}_i^q$ for all $k \in \mathscr{N}_i$. Then, it follows from (123) that for each $i \in \mathscr{N}$,

$$\frac{\partial mr_{ik}(\cdot)^*}{\partial q_{ik}} q_{ik}^* q_{ik}^* \frac{dq_{ik}^*/d\tau_n}{q_{ik}^*} = \ell_{ik}^* \frac{dW^*}{d\tau_n} + m_{ik}^* \frac{dP_i^{M*}}{d\tau_n},$$

[134]See Remark C.4.

for all $k \in \mathscr{N}_i$. In view of Lemma C.11, Corollary C.6 and Lemma C.13, this yields

$$\frac{dq_{ik}^*/d\tau_n}{q_{ik}^*} = (\bar{d}_i^q)^{-1} \frac{\beta_i^\ell \frac{dW^*}{d\tau_n} + \beta_i^m \frac{dP_i^{M*}}{d\tau_n}}{\beta_i^\ell W^* + \beta_i^m P_i^{M*}},$$

where β_i^ℓ and β_i^m are the constants appearing in Lemma C.11. Since the right-hand side of this expression is free from the firm-specific index k, the implication is true by setting $\bar{c}_i^q := (\bar{d}_i^q)^{-1} \frac{\beta_i^\ell \frac{dW^*}{d\tau_n} + \beta_i^m \frac{dP_i^{M*}}{d\tau_n}}{\beta_i^\ell W^* + \beta_i^m P_i^{M*}}$.

This completes the proof of this proposition. □

The following corollary corresponds to, but is not quite the same as Corollary C.7 (i).

Corollary C.8. *Assume that firms in each sector engage in monopolistic competition in the output market. Suppose that the assumptions required in Proposition C.11 are satisfied. In addition, assume that for each $i \in \mathscr{N}$, (i) there exists a sector-specific constant $\bar{d}_i^q \in \mathbb{R}\backslash\{0\}$ such that $\left(\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\right)^{-1} \left\{\frac{d^2\tilde{r}_{ik}}{d\tilde{x}_{ik}^2} - \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\left(1 - \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\right)\right\} = \bar{d}_i^q$ for all $k \in \mathscr{N}_i$; and (ii) there exists a sector-specific constant $\bar{e}_i^q \in \mathbb{R}$ such that $1 - \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}} = \bar{e}_i^q$ for all $k \in \mathscr{N}_i$. Then, there exists a sector-specific constant $\bar{c}_i^p \in \mathbb{R}$ such that $\frac{dp_{ik}^*/d\tau_n}{p_{ik}^*} = \bar{c}_i^p$ for all $k \in \mathscr{N}_i$.*

Proof. By construction,

$$\frac{dp_{ik}^*}{d\tau_n} = \frac{\partial \wp_{ik}(\cdot)^*}{\partial q_{ik}} \frac{dq_{ik}^*}{d\tau_n} = -\bar{e}_i^q \frac{p_{ik}^*}{q_{ik}^*} \frac{dq_{ik}^*}{d\tau_n},$$

where the second equality is the result of Remark C.2 and the hypothesis (ii) of this corollary. This can be rearranged to

$$\frac{dp_{ik}^*/d\tau_n}{p_{ik}^*} = -\bar{e}_i^q \frac{dq_{ik}^*/d\tau_n}{q_{ik}^*}.$$

In view of Proposition C.11, the hypothesis (i) of this corollary implies that there exists a sector-specific constant $\bar{c}_i^q \in \mathbb{R}\backslash\{0\}$ such that $\frac{dq_{ik}^*/d\tau_n}{q_{ik}^*} = \bar{c}_i^q$ for all $k \in \mathscr{N}_i$. Hence,

$$\frac{dp_{ik}^*/d\tau_n}{p_{ik}^*} = -\bar{e}_i^q \bar{c}_i^q.$$

Since the right-hand side of this expression is free from the firm-specific index k, the claim of this corollary is true by choosing $\bar{c}_i^p := -\bar{c}_i^q \bar{e}_i^q$. □

This corollary means that the elasticity of the firm's price might be constant for all firms, but it is not the same in magnitude as the elasticity of the firm's quantity. In regard to the added term $\bar{e}_i^q$, it is worth noting that in equilibrium under monopolistic competition, $\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}$ dictates the inverse of the firm's markup (see the proof of Lemma C.4). Hence, the hypothesis (ii) of this corollary essentially requires that the equilibrium markup is the same for all firms.[135] An intuition is that each monopolist can exercise market power against the demand side.

The next corollary appears similar to Corollary C.7 (ii), but its implication is quite the opposite in effect: Corollary C.7 (ii) does not hold in monopolistic competition.

Corollary C.9. *Assume that firms in each sector engage in monopolistic competition in the output market. Suppose that the assumptions required in Proposition C.11 are satisfied. In addition, assume that for each* $i \in \mathscr{N}$, *(i) there exists a sector-specific constant* $\bar{d}_i^q \in \mathbb{R}\backslash\{0\}$ *such that* $\left(\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\right)^{-1}\left\{\frac{d^2\tilde{r}_{ik}}{d\tilde{x}_{ik}^2} - \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\left(1 - \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\right)\right\} = \bar{d}_i^q$ *for all* $k \in \mathscr{N}_i$*; and (ii) there exists a sector-specific constant* $\bar{e}_i^q \in \mathbb{R}$ *such that* $1 - \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}} = \bar{e}_i^q$ *for all* $k \in \mathscr{N}_i$*. Then, for each sector* $i \in \mathscr{N}$, $\frac{dp_{ik}^*}{d\tau_n}q_{ik}^* + p_{ik}^*\frac{dq_{ik}^*}{d\tau_n} = 0$ *for all* $k \in \mathscr{N}_i$ *if and only if* $\bar{e}_i^q = 1$.

Proof. In view of Proposition C.11, it follows from the hypothesis (i) that for each $i \in \mathscr{N}$, there exists a sector-specific constant $\bar{c}_i^q \in \mathbb{R}\backslash\{0\}$ such that $\frac{dq_{ik}^*/d\tau_n}{q_{ik}^*} = \bar{c}_i^q$ for all $k \in \mathscr{N}_i$. Moreover, it holds by Corollary C.8 that there exists a sector-specific constant $\bar{c}_i^p \in \mathbb{R}$ such that $\frac{dp_{ik}^*}{d\tau_n}p_{ik}^* = \bar{c}_i^p$ for all $k \in \mathscr{N}_i$. In particular, $\bar{c}_i^p = -\bar{c}_i^q\bar{e}_i^q$.

Now, pick an arbitrary k. It is then straightforward to show that

$$\frac{dp_{ik}^*}{d\tau_n}q_{ik}^* + p_{ik}^*\frac{dq_{ik}^*}{d\tau_n} = 0 \Longleftrightarrow \bar{e}_i^q = 1.$$

The proof is completed as soon as noticing that this equivalence result does not depend on the particular choice of k. □

Notice that $\bar{e}_i^q = 1$ means that the firm's markup is infinity and so is the firm's output price, a case that is unlikely be interesting on either theoretical or empirical grounds. Because of this, Corollary C.9 effectively states that the firm-level price effect will never exactly offsets the quantity effect, leaving a non-zero revenue effect. This observation is summarized in the following corollary.

Corollary C.10. *Assume that firms in each sector engage in monopolistic competition in the output market. Suppose that the assumptions required in Corollary C.8 are satisfied. Then,* $\frac{dp_{ik}^*/p_{ik}^*}{dq_{ik}^*/q_{ik}^*} \in (-1, 0)$.

Proof. It follows from Corollary C.8 that $\frac{dp_{ik}^*/d\tau_n}{p_{ik}^*} = -\bar{e}_i^q\bar{c}_i^q$. Since under monopolistic competition, $\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}$ is equal to the inverse of the firm's markup, it holds that $0 < \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}} < 1$, so that

[135] This can be true in the case of a CES sectoral aggregator as shown in Example C.2.

$0 < \bar{e}_i^q < 1$. Combining these leads to

$$\frac{dp_{ik}^*/d\tau_n}{p_{ik}^*}(\bar{c}_i^q)^{-1} \in (-1, 0).$$

Noticing that $\bar{c}_i^q = \frac{dq_{ik}^*/d\tau_n}{q_{ik}^*}$ completes the proof. $\square$

This corollary implies that the price elasticity in monopolistic competition is inelastic due to the firm's market power, marking a sharp contrast with the unitary elasticity in the oligopolistic environment (121). Specifically, Corollary C.10 means that monopolistic firms can receive positive revenue effects from increasing their output quantities.

C.10 Further Systematic Patterns Induced by Cobb-Douglas Firm-Level Production Function

This subsection further elaborates on the results derived in Appendix C.9.3 in the context of a Cobb-Douglas firm-level production function. The key takeaways from this subsection are twofold: (i) under oligopolistic competition with Cobb-Douglas firm-level production functions and HSA sectoral aggregators (embedding CES sectoral aggregators), the general equilibrium feedback through the change in wage is muted; and (ii) the causal policy estimand $\Delta Y(\tau_n^0, \tau_n^1)$ (defined in (14)) can be written in terms firm-level input variables and aggregate variables, free from measurement errors.

General equilibrium feedback. First, I derive analytical expressions for $\bar{\lambda}_{ik}^L$ and $\bar{\lambda}_{ik}^M$. Plugging in (32) back to (31), the equilibrium variables satisfy

$$\begin{bmatrix} \frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i1}} & \frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i2}} & \cdots & \frac{\partial mr_{i1}(\cdot)^*}{\partial q_{iN_i}} \\ \frac{\partial mr_{i2}(\cdot)^*}{\partial q_{i1}} & \frac{\partial mr_{i2}(\cdot)^*}{\partial q_{i2}} & \cdots & \frac{\partial mr_{i2}(\cdot)^*}{\partial q_{iN_i}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial mr_{iN_i}(\cdot)^*}{\partial q_{i1}} & \frac{\partial mr_{iN_i}(\cdot)^*}{\partial q_{i2}} & \cdots & \frac{\partial mr_{iN_i}(\cdot)^*}{\partial q_{iN_i}} \end{bmatrix} \begin{bmatrix} \bar{\lambda}_{i1}^L & \bar{\lambda}_{i1}^M \\ \vdots & \vdots \\ \bar{\lambda}_{iN_i}^L & \bar{\lambda}_{iN_i}^M \end{bmatrix} = \begin{bmatrix} \frac{\ell_{i1}^*}{q_{i1}^*} & \frac{m_{i1}^*}{q_{i1}^*} \\ \vdots & \vdots \\ \frac{\ell_{iN_i}^*}{q_{iN_i}^*} & \frac{m_{iN_i}^*}{q_{iN_i}^*} \end{bmatrix}. \tag{124}$$

Proposition C.12. *Assume that firms in each sector engage in oligopolistic competition in the output market. Suppose that Assumption 4.4 holds. Suppose further that the firm-level production function $f_i(\cdot)$ takes the form of a Cobb-Douglas function: for each $k \in \mathcal{N}_i$,*

$$f_i(\ell_{ik}, m_{ik}; z_{ik}) := z_{ik}\ell_{ik}^{\alpha} m_{ik}^{1-\alpha}, \tag{125}$$

where α stands for the output share of labor input. Then, (124) *is true if and only if* $\bar{\lambda}^L_{ik} = -\frac{\ell^*_{ik}}{\xi^*_{ik}}$ *and* $\bar{\lambda}^M_{ik} = -\frac{m^*_{ik}}{\xi^*_{ik}}$ *for all* $k \in \mathscr{N}_i$*.*

Proof. Due to Assumption A.1, the premultiplying term of the left-hand side of (124) is invertible, yielding a unique solution. Hence, it is enough to show that (124) holds with $\bar{\lambda}^L_{ik} = -\frac{\ell^*_{ik}}{\xi^*_{ik}}$ and $\bar{\lambda}^M_{ik} = -\frac{m^*_{ik}}{\xi^*_{ik}}$ for all $k \in \mathscr{N}_i$. I prove this claim column by column. First, the firm's marginal cost ξ^*_{ik} is given by $\xi^*_{ik} = mc_i z^{-1}_{ik}$, where mc_i represents a part of the marginal cost common across all firms. Next, define $\bar{t}_{ik}$ by $t_{ik} = \frac{\bar{t}_{ik}}{q^*_{ik}}$. Third, let

$$E_{ik} \coloneqq \frac{d^2\tilde{r}_{ik}}{d\tilde{x}^2_{ik}}(1-\varpi_{ik}) + \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\left\{\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}} - \left(\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}} + \rho_{ik}\right)\varpi_{ik}\right\}.$$

Then, the derivatives of the marginal revenue function can be expressed as

$$\frac{\partial mr_{ik}(\cdot)^*}{\partial q_{ik}} = \frac{p^*_{ik}}{q^*_{ik}}(1-\varpi_{ik})\left(E_{ik} - \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\right) + \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}} p^*_{ik}\varpi_{ik} t_{ik},$$

and

$$\frac{\partial mr_{ik}(\cdot)^*}{\partial q_{ik'}} = -p^*_{ik}\frac{\varpi_{ik'}}{q^*_{ik'}}E_{ik} + \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}} p^*_{ik}\varpi_{ik} t_{ik'},$$

for all $k' \neq k$. Then, I have

$$\frac{\partial mr_{ik}(\cdot)^*}{\partial q_{ik}}\frac{\ell^*_{ik}}{\xi^*_{ik}} = \frac{\ell^*_{ik}}{q^*_{ik}} p^*_{ik}\frac{1}{mc_i z^{-1}_{ik}}(1-\varpi_{ik})\left(E_{ik} - \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\right) + \frac{\ell^*_{ik}}{q^*_{ik}}\frac{1}{q^*_{ik}}\frac{1}{mc_i z^{-1}_{ik}}\tilde{u}_{ik}\varpi_{ik}\bar{t}_{ik},$$

and

$$\frac{\partial mr_{ik}(\cdot)^*}{\partial q_{ik'}}\frac{\ell^*_{ik'}}{\xi^*_{ik'}} = -\frac{\ell^*_{ik}}{q^*_{ik}} p^*_{ik}\frac{1}{mc_i z^{-1}_{ik}}\varpi_{ik'}E_{ik} + \frac{\ell^*_{ik}}{q^*_{ik}}\frac{1}{q^*_{ik}}\frac{1}{mc_i z^{-1}_{ik}}\tilde{u}_{ik}\varpi_{ik}\bar{t}_{ik'},$$

for all $k' \neq k$. Hence, for each row k, the first column of the left-hand side of (124) reads

$$\begin{aligned}\sum_{k'=1}^{N_i} \frac{\partial mr_{ik}(\cdot)^*}{\partial q_{ik'}} \frac{\ell^*_{ik'}}{\xi^*_{ik'}} &= \frac{\partial mr_{ik}(\cdot)^*}{\partial q_{ik}} \frac{\ell^*_{ik}}{\xi^*_{ik}} + \sum_{k' \neq k} \frac{\partial mr_{ik}(\cdot)^*}{\partial q_{ik'}} \frac{\ell^*_{ik'}}{\xi^*_{ik'}} \\ &= \frac{\ell^*_{ik}}{q^*_{ik}} p^*_{ik} \frac{1}{mc_i z_{ik}^{-1}} \Big(1 - \sum_{k'=1}^{N_i} \varpi_{ik'}\Big) E_{ik} - \frac{\ell^*_{ik}}{q^*_{ik}} \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}} p^*_{ik}(1 - \varpi_{ik}) \frac{1}{mc_i z_{ik}^{-1}} \\ &\quad + \frac{\ell^*_{ik}}{q^*_{ik}} \frac{1}{q^*_{ik}} \frac{1}{mc_i z_{ik}^{-1}} \tilde{u}_{ik} \varpi_{ik} \sum_{k'=1}^{N_i} \bar{t}_{ik'} \\ &= -\frac{\ell^*_{ik}}{q^*_{ik}},\end{aligned}$$

where the last equality is a consequence of $\sum_{k'=1}^{N_i} \varpi_{ik'} = 1$, and $\sum_{k'=1}^{N_i} \bar{t}_{ik'} = 0$, as well as $\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}} p^*_{ik}(1 - \varpi_{ik}) \frac{1}{mc_i z_{ik}^{-1}} = 1$ by the first order condition. That is, the left-hand side of (124) certainly coincides with the right-hand side of (124). An analogous argument holds for the case of $\bar{\lambda}^M_{ik} = -\frac{m^*_{ik}}{\xi^*_{ik}}$. This proves the statement.

□

Next, I derive analytical expressions for the matrix D_{ik} defined in (57).

Lemma C.14. *Suppose that the assumptions required in Proposition C.12 hold. Then,* $d_{ik,11} = -\frac{1}{\alpha} \frac{(\ell^*_{ik})^2 m^*_{ik}}{\xi^*_{ik} q^*_{ik}} \big\{(1-\alpha)\frac{1}{m^*_{ik}} + \alpha^2 \xi^*_{ik} \frac{1}{(\ell^*_{ik})^2}\big\}$, $d_{ik,12} = d_{ik,21} = 0$, *and* $d_{ik,22} = -\frac{m^*_{ik}}{P_i^{M*}}$

Proof. For the sake of exposition, I introduce the following notation: in (57),

$$d_{ik,0} := -\left(\xi^*_{ik} \frac{q^*_{ik}}{\ell^*_{ik}} \frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik} \partial m_{ik}}\right)^{-1}$$

and

$$\begin{bmatrix} \bar{d}_{ik,11} & \bar{d}_{ik,12} \\ \bar{d}_{ik,21} & \bar{d}_{ik,22} \end{bmatrix} := \begin{bmatrix} \frac{\partial f_i(\cdot)^*}{\partial m_{ik}} & -\xi^*_{ik} \frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik} \partial m_{ik}} \\ -\frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}} & \xi^*_{ik} \frac{\partial^2 f_i(\cdot)^*}{\partial \ell^2_{ik}} \end{bmatrix} \begin{bmatrix} 1 - \frac{\ell^*_{ik}}{q^*_{ik}} \frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}} & -\frac{m^*_{ik}}{q^*_{ik}} \frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}} \\ \bar{\lambda}^L_{ik} & \bar{\lambda}^M_{ik} \end{bmatrix}.$$

(i) $d_{ik,0}$
Given the expression of the firm's production function (125), it holds

$$d_{ik,0} = -\frac{1}{\alpha(1-\alpha)} \frac{(\ell^*_{ik})^2 m^*_{ik}}{\xi^*_{ik} (q^*_{ik})^2}.$$

(ii) $d_{ik,11}$

Once again, since the firm's production function is given by (125),

$$\bar{d}_{ik,11} = \frac{\partial f_i(\cdot)^*}{\partial m_{ik}}\Big(1 - \frac{\ell^*_{ik}}{q^*_{ik}}\frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}}\Big) + \Big(-\xi^*_{ik}\frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik}\partial m_{ik}}\Big)\Big(-\frac{m^*_{ik}}{q^*_{ik}}\frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}}\Big)$$
$$= (1-\alpha)q^*_{ik}\Big\{(1-\alpha)\frac{1}{m^*_{ik}} + \alpha^2\xi^*_{ik}\frac{1}{(\ell^*_{ik})^2}\Big\},$$

and thus

$$d_{ik,11} = d_{ik,0}\bar{d}_{ik,11} = -\frac{1}{\alpha}\frac{(\ell^*_{ik})^2 m^*_{ik}}{\xi^*_{ik}q^*_{ik}}\Big\{(1-\alpha)\frac{1}{m^*_{ik}} + \alpha^2\xi^*_{ik}\frac{1}{(\ell^*_{ik})^2}\Big\}.$$

(iii) $d_{ik,12}$

In view of Proposition C.12, $\bar{\lambda}^M_{ik} = -\frac{m^*_{ik}}{\xi^*_{ik}}$. Thus, it follows

$$\bar{d}_{ik,12} = \frac{\partial f_i(\cdot)^*}{\partial m_{ik}}\Big(-\frac{m^*_{ik}}{q^*_{ik}}\frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}}\Big) + \Big(-\xi^*_{ik}\frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik}\partial m_{ik}}\Big)\bar{\lambda}^M_{ik}$$
$$= (1-\alpha)\frac{q^*_{ik}}{m^*_{ik}}\Big(-\frac{m^*_{ik}}{q^*_{ik}}\alpha\frac{q^*_{ik}}{\ell^*_{ik}}\Big) + \Big\{-\xi^*_{ik}\alpha(1-\alpha)\frac{q^*_{ik}}{\ell^*_{ik}m^*_{ik}}\Big\}\Big(-\frac{m^*_{ik}}{\xi^*_{ik}}\Big)$$
$$= -\alpha(1-\alpha)\frac{q^*_{ik}}{\ell^*_{ik}} + \alpha(1-\alpha)\frac{q^*_{ik}}{\ell^*_{ik}}$$
$$= 0,$$

and then

$$d_{ik,12} = d_{ik,0}\bar{d}_{ik,12} = 0.$$

(iv) $d_{ik,21}$

In view of Proposition C.12, $\bar{\lambda}^L_{ik} = -\frac{\ell^*_{ik}}{\xi^*_{ik}}$. Thus, it follows

$$\bar{d}_{ik,21} = -\frac{\partial f_i(\cdot)}{\partial \ell_{ik}}\Big(1 - \frac{\ell_{ik}}{q_{ik}}\frac{\partial f_i(\cdot)}{\partial \ell_{ik}}\Big) + \xi^*_{ik}\frac{\partial^2 f_i(\cdot)}{\partial \ell^2_{ik}}\bar{\lambda}^L_{ik}$$
$$= -\alpha\frac{q^*_{ik}}{\ell^*_{ik}}\Big(1 - \frac{\ell^*_{ik}}{q^*_{ik}}\alpha\frac{q^*_{ik}}{\ell^*_{ik}}\Big) + \xi^*_{ik}\Big\{-\alpha(1-\alpha)\frac{q^*_{ik}}{(\ell^*_{ik})^2}\Big\}\Big(-\frac{\ell^*_{ik}}{\xi^*_{ik}}\Big)$$
$$= -\alpha(1-\alpha)\frac{q^*_{ik}}{\ell^*_{ik}} + \alpha(1-\alpha)\frac{q^*_{ik}}{\ell^*_{ik}}$$
$$= 0,$$

and then

$$d_{ik,21} = d_{ik,0}\bar{d}_{ik,21} = 0.$$

(v) $d_{ik,22}$

In view of Proposition C.12, $\bar{\lambda}^M_{ik} = -\frac{m^*_{ik}}{\xi^*_{ik}}$. It holds that

$$\begin{aligned}
\bar{d}_{ik,22} &= \Big(-\frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}}\Big)\Big(-\frac{m^*_{ik}}{q^*_{ik}}\frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}}\Big) + \xi^*_{ik}\frac{\partial^2 f_i(\cdot)^*}{\partial \ell^2_{ik}}\bar{\lambda}^M_{ik} \\
&= \frac{m^*_{ik}}{q^*_{ik}}\Big(\frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}}\Big)^2 + \xi^*_{ik}\frac{\partial^2 f_i(\cdot)^*}{\partial \ell^2_{ik}}(-\frac{m^*_{ik}}{\xi^*_{ik}}) \\
&= \alpha\frac{m^*_{ik}q^*_{ik}}{(\ell^*_{ik})^2},
\end{aligned}$$

and then

$$\begin{aligned}
d_{ik,22} &= d_{ik,0}\bar{d}_{ik,22} \\
&= -\frac{1}{\alpha(1-\alpha)}\frac{(\ell^*_{ik})^2 m^*_{ik}}{\xi^*_{ik}(q^*_{ik})^2}\alpha\frac{m^*_{ik}q^*_{ik}}{(\ell^*_{ik})^2} \\
&= -\frac{1}{1-\alpha}\frac{(m^*_{ik})^2}{\xi^*_{ik}q^*_{ik}} \\
&= -\frac{W^*\ell^*_{ik} + P^M_i m^*_{ik}}{P^M_i m^*_{ik}}\frac{(m^*_{ik})^2}{\xi^*_{ik}q^*_{ik}} \\
&= -\frac{m^*_{ik}}{P^M_i},
\end{aligned}$$

where the fourth equality follows from the fact that the firm's production function is given by (125), and the last equality is due to the equivalence about the total cost: $\xi^*_{ik}q^*_{ik} = W^*\ell^*_{ik} + P^M_i m^*_{ik}$. This completes the proof. □

Now, I can directly calculate the general equilibrium feedback effect through the change in wage, which turns out to be zero.

Corollary C.11. *Suppose the assumptions required in Lemma C.14 hold. Then, (i) Assumption A.4 is satisfied; and (ii)* $\frac{dW^*}{d\tau_n} = 0$.

Proof. (i) By Lemma C.14, $\sum_{i=1}^{N}\sum_{k=1}^{N_i} d_{ik,11} < 0$ and $\sum_{k=1}^{N_i} d_{ik,12} = 0$ for all $i \in \mathscr{N}$. Hence, it holds that $\sum_{i=1}^{N}\sum_{k=1}^{N_i}(d_{ik,11} + \vartheta_{1,i}d_{ik,12}) < 0$, which means that Assumption A.4 is satisfied.

(ii) Given the first part of this corollary, $\frac{dW^*}{d\tau_n}$ is well defined according to (64). It follows moreover from Lemma C.14 that $\sum_{k=1}^{N_i} d_{ik,12} = 0$ for all $i \in \mathscr{N}$. Thus, it holds by (64) that $\frac{dW^*}{d\tau_n} = 0$, completing the proof. □

Corollary C.11 has an important implication for policy analysis: When the firm's production technology takes the form of a Cobb-Douglas aggregator, and the sectoral aggregator is given by an HSA demand system, the general equilibrium feedback through the change in wage is muted.

Causal policy estimand. Proposition C.12 has also an implication for the causal policy effect $\Delta Y(\tau_n^0, \tau_n^1)$. First, I derive analytical expressions for the micro complementarities $\bar{\lambda}_{i\cdot}^L$ and $\bar{\lambda}_{i\cdot}^M$.

Lemma C.15. *Suppose that the assumptions required in Proposition C.12 are satisfied. Then,* $\bar{\lambda}_{i\cdot}^L = P_i^* \sum_{k'=1}^{N_i} \varpi_{ik'} \frac{\ell_{ik'}^*}{TC_{ik'}^*}$*, and* $\bar{\lambda}_{i\cdot}^M = P_i^* \sum_{k'=1}^{N_i} \varpi_{ik'} \frac{m_{ik'}^*}{TC_{ik'}^*}$.

Proof. It follows from Lemma C.8 that $\frac{\partial \mathcal{P}_i(\cdot)^*}{\partial q_{ik}} = -\frac{p_{ik}^*}{Q_i^*}$. From Proposition C.12, $\bar{\lambda}_{ik}^L = -\frac{\ell_{ik}^*}{\xi_{ik}^*}$. Substituting these into the definition of $\bar{\lambda}_{i\cdot}^L$ yields

$$\bar{\lambda}_{i\cdot}^L := \sum_{k'=1}^{N_i} \frac{\partial \mathcal{P}_i(\cdot)^*}{\partial q_{ik'}} \bar{\lambda}_{ik'}^L = \sum_{k'=1}^{N_i} \Big(-\frac{p_{ik'}^*}{Q_i^*} \Big) \Big(-\frac{\ell_{ik'}^*}{\xi_{ik'}^*} \Big) = P_i^* \sum_{k'=1}^{N_i} \frac{p_{ik'}^* q_{ik'}^*}{P_i^* Q_i^*} \frac{\ell_{ik'}^*}{\xi_{ik'}^* q_{ik'}^*} = P_i^* \sum_{k'=1}^{N_i} \varpi_{ik} \frac{\ell_{ik'}^*}{TC_{ik'}^*}.$$

Analogously, $\bar{\lambda}_{i\cdot}^M = P_i^* \sum_{k'=1}^{N_i} \varpi_{ik} \frac{m_{ik'}^*}{TC_{ik'}^*}$. This completes the proof. □

This lemma, in turn, leads to analytical expressions for the macro complementarities.

Corollary C.12. *Suppose that the assumptions required in Lemma C.15 hold. Suppose moreover that Assumption B.3 holds. Then,* $\Gamma_1 = [\gamma_{i,j} P_i^{M*} \sum_{k'=1}^{N_j} \varpi_{jk'} \frac{\ell_{jk'}^*}{TC_{jk'}^*}]_{i,j \in \mathscr{N}}$ *and* $\Gamma_2 = [\gamma_{i,j} P_i^{M*} \sum_{k'=1}^{N_j} \varpi_{jk'} \frac{m_{jk'}^*}{TC_{jk'}^*}]_{i,j \in \mathscr{N}}$.

Proof. Under Assumption B.3, $\frac{\partial \mathcal{P}_i^M(\cdot)^*}{\partial P_j} = \gamma_{i,j} \frac{P_i^{M*}}{P_j^*}$, as seen in Fact C.7. Moreover, it follows from Lemma C.15 that $\bar{\lambda}_{i\cdot}^L = P_i^* \sum_{k'=1}^{N_i} \varpi_{ik'} \frac{\ell_{ik'}^*}{TC_{ik'}^*}$. Putting these together, the definition of Γ_1 yields

$$\Gamma_1 := \Big[\frac{\partial \mathcal{P}_i^M(\cdot)^*}{\partial P_j} \bar{\lambda}_{j\cdot}^L \Big]_{i,j \in \mathscr{N}} = \Big[\gamma_{i,j} P_i^{M*} \sum_{k'=1}^{N_j} \varpi_{jk'} \frac{\ell_{jk'}^*}{TC_{jk'}^*} \Big]_{i,j \in \mathscr{N}}.$$

Analogously, it holds that $\Gamma_2 = [\gamma_{i,j} P_i^{M*} \sum_{k'=1}^{N_j} \varpi_{jk'} \frac{m_{jk'}^*}{TC_{jk'}^*}]_{i,j \in \mathscr{N}}$, as desired. □

Now, I show that the causal policy estimand $\Delta Y(\tau_n^0, \tau_n^1)$ depends only on firm-level input variables and aggregate variables.

Proposition C.13. *Suppose that the assumptions required in Corollary C.12 hold. Then, the object of interest* $\Delta Y(\tau_n^0, \tau_n^1)$ *given in* (15) *can be expressed in terms of firm-level input variables and aggregate variables.*

Proof. From Fact C.2, $\tilde{u}_{ik}$ and ϖ_{ik} can be expressed as a function of firm-level input variables and aggregate variables. In view of Corollary C.12, both Γ_1 and Γ_2 are then written in terms of firm-level input variables and aggregate variables. This in turn means that $\frac{dP_i^*}{d\tau_n}$ and $\frac{dP_i^{M*}}{d\tau_n}$ are also a function of firm-level input variables and aggregate variables according to (39) and (44), respectively. This observation can further be coupled with Lemma C.14 to express $\frac{dm_{ik}^*}{d\tau_n}$ in terms of firm-level input variables and aggregate variables. In view of Proposition C.9, $\frac{dm_{ik,j}^*}{d\tau_n}$ can be written as a function of firm-level input variables and aggregate variables. Combining Lemma C.14 and Corollary C.11, I also have $\frac{d\ell_{ik}^*}{d\tau_n} = 0$.

Here, in light of Corollary C.7, the marginal responsiveness of sectoral GDP (16) only depends on (in addition to P_j^* and $m_{ik,j}^*$) $\frac{dP_i}{d\tau_n}$ and $\frac{dm_{ik,j}^*}{d\tau_n}$, each of which can be written as a function of firm-level input variables and aggregate variables. Hence, the target parameter $\Delta Y(\tau_n^0, \tau_n^1)$ in (15) is also expressed only in terms of firm-level input variables and aggregate variables, as claimed. □

Provided that firm-level input variables and aggregate variables are observable, this proposition has an important implication for policy evaluation. Proposition C.13 states that if the sectoral aggregator takes a form of an HSA demand system and the firm-level production function is given by a nested Cobb-Douglas aggregator, then the policy effect $\Delta Y(\tau_n^0, \tau_n^1)$ does not depend on the firm-level output variables, such as firm-level price, quantity and revenue. While my framework posits that firms' revenues are prone to measurement error, this proposition guarantees that the policy effect can be computed free from such errors. In this case, "estimating" the policy effect essentially boils down to an accounting exercise.

C.11 Comparison to the Literature

This subsection compares the identification analysis of my paper to those of the existing literature.

My framework relates to the literature on production function identification and estimation in two ways. First, my approach extends the existing methodology to the case of oligopolistic competition. The existing work has customarily assumed perfect competition (e.g., Ackerberg et al., 2015; Gandhi et al., 2019) or monopolistic competition (e.g., Kasahara and Sugita, 2020). Doraszelski and Jaumandreu (2019), Brand (2020), and Bond et al. (2021) draw attention to the risk of simply applying the standard control function approach to the case of oligopolistic competition, but they do not provide a methodology to account for firms' strategic interactions in recovering firms' production functions. Second, in my framework, the econometrician (or policymaker) only has access to data on firms' revenues, being unable to observe firms' prices and quantities. It has long been recognized that the use of the quantity measure of revenue data — revenue data deflated by price index — as a proxy for quantity data induces an omitted price bias (Klette and Griliches, 1996) and masks the demand-side heterogeneity encoded in firm-specific price variables.[136] Kasahara and Sugita (2020, 2023) develop a methodology that recovers firm-level price and quantity from firm-level revenue, but do not consider firms' strategic competition. Blum et al. (2023), Ackerberg and De Loecker (2024), and Doraszelski and Jaumandreu (2024) study the production function estimation under oligopolistic competition but require data on firm-level prices and quantities. My approach blends these two approaches and proposes a methodology that can be used to recover firms' prices and quantities from revenues in an oligopolistic environment.

The idea behind my identification strategy resembles exact hat algebra (Dekle et al., 2007, 2008), a method routinely used to generate a counterfactual prediction in the literature (e.g., Caliendo and Parro, 2015; Adão et al., 2017, 2020).[137] My approach is distinct in two ways, however. First, exact hat algebra is not principally concerned with the identification and estimation of the comparative statics; it only calculates the comparative statics, taking model parameters as known (Dingel and Tintelnot, 2023). My paper provides a unified framework for the identification, estimation, and prediction of both "model parameters" and counterfactual outcomes. Second, the presumption of exact hat algebra is that all endogenous equilibrium variables are observable. This requirement, however, is not fulfilled in my case as firm-level quantity and price are not available in the data (see Section 3). In Section 4.2, I provide a path forward to move on in the presence of these unobservable endogenous variables.

The left-hand side of (16) alone may be of limited practical relevance because it only measures the impact of an infinitesimally small policy change around the status quo policy regime $\boldsymbol{\tau}^0$. My target parameter (14), in contrast, can be used to analyze a large policy reform from $\boldsymbol{\tau}^0$ to $\boldsymbol{\tau}^1$.[138] While useful as an approximation around the equilibrium in response to a

[136] See, for example, Klette and Griliches (1996), Doraszelski and Jaumandreu (2019), Flynn et al. (2019), Bond et al. (2021) and Kirov et al. (2022) for the details.

[137] See Costinot and Rodríguez-Clare (2014) for an outline of the method.

[138] In a related vein, Baqaee and Farhi (2022) investigate the consequences of discrete changes in distortions. Assuming away from any distortions in the initial state of the economy, they provide a second-order approximation for the responses of real GDP and welfare. Accordingly, the discrete changes in their characterization need to be small enough to make the second-order approximation sufficiently good. In contrast, my paper derives an exact formula that is valid for discrete changes of arbitrary size (as long as the support condition is satisfied) from the (possibly inefficient) current policy regime.

small shock, the common practice of setting $\boldsymbol{\tau}^0 = \mathbf{0}$ (e.g., Liu, 2019; Baqaee and Farhi, 2022) is less likely to yield an economically interesting policy objective, and is not sufficient to recover the effect of a large policy reform, such as the one considered in (14). Moreover, such an estimate may not be accurate because the responsiveness of GDP generally depends on the level of the underlying policy regime. This observation is examined through a numerical simulation (Appendix F) and also found in an empirical application (Section 5).

There are two remarks on (23). First, to recover the firm's production function over the entire empirical support, the literature typically further assumes that the firm's productivity follows a Markov process (e.g., Ackerberg et al., 2015; Gandhi et al., 2019). In contrast, my analysis is only concerned with identifying the equilibrium values of the relevant functions and variables (see Section 4.1), thereby abstracting from the stochastic process of the firm's productivity. This is plausible in view of the fact that the economic model of my framework is static in nature, and thus my empirical analysis does not exploit the time-series feature of the data (see Section 3). Second, plugging (23) into (4), the firm's production function can be written in a way that does not depend on competitors' variables.[139] This observation is combined with the repeated sample paradigm (see Section 3) to restore the identification of firm-level variables under the "large n" asymptotics (see Ackerberg and De Loecker (2024)).

[139]The competitor's productivity matters only through aggregate productivities, which are effectively absorbed in the sectoral index.

D Extensions

D.1 Dynamic Environment

The CHIPS and Science Act consists of two parts: *i*) Investment in construction, expansion, or modernization of facilities producing semiconductors, and *ii*) tax credit for capital investments in semiconductors. In the main text, I focus on the second part only; as far as the tax credits and the static analysis are concerned, the empirical analysis of this paper is consistent with the model. To explicitly account for the investment part, the model of this paper needs to be extended to include the firm's dynamic capital accumulation. But the extension is not trivial because it requires an explicit consideration of not only the firm's own future choices but also competitors' future choices. This convoluted, forward-looking nature opens up another source of multiplicity of equilibria and is left to be explored.

D.2 Long-Run Perspective

This paper focuses on the short-run policy effects, excluding the firms' endogenous entry and exit decisions in reaction to a change in policy. At first glance, this might appear to be restrictive because the present paper studies merely a "special case" of the "full-fledged model." In practice, however, the short-run analysis deserves separate attention in its own right mainly for two reasons. First, the short-run analysis *per se* is useful as a tool for "validation" of the policy under consideration.[140] In the short run, the model prediction can be compared to what has actually happened in the data since implementation. If the data turn out to be substantially different from the model prediction, the policymaker can/should revise the model and update the policy prescription. In contrast, when the observed outcomes are largely in line with the model prediction, it is a strong indication that the model is plausible, granting the policymaker confidence about the policy in place. Second, the short-run analysis is a necessary step to separately identify the intensive and extensive margin causal effects.[141] While the short-run analysis identifies the intensive margin causal effect as explored in the main text, the long-run analysis directly identifies the total causal effect. Thus, the extensive margin causal effect is only identified as a residual between these two effects. That is, the short-run perspective is necessary to separately identify the extensive

[140] This insight is employed in the empirical microeconomic literature. See, e.g., Low and Meghir (2017) and references therein.

[141] For example, the international trade literature studies the "trade elasticities" for the both intensive and extensive margins (e.g., Chaney, 2008; Adão et al., 2020; Boehm et al., 2023). Other works decompose the total growth/difference in the value of trade into the intensive and extensive margins (e.g., Feenstra, 1994; Hummels and Klenow, 2005; Kehoe and Ruhl, 2013). My framework pertains to the intensive and extensive margin causal policy effects.

margin causal effect.

To illustrate the idea, I briefly sketch the definition and identification of the extensive margin causal effects.

D.2.1 Illustrative Example

Definition. Consider the same setup as in the main text but deviate by allowing for the firm's endogenous entry and exit. Consider a policy reform from $\boldsymbol{\tau}^0$ to $\boldsymbol{\tau}^1$. Let $\mathcal{N}_i^0$ and $\mathcal{N}_i^1$ be the index sets for firms in sector i under $\boldsymbol{\tau}^0$ and $\boldsymbol{\tau}^1$, respectively. Let u signify the competitiveness of the market under $\mathcal{N}_i^u$, and $y_{ik}^u(\boldsymbol{\tau})$ be the firm-level value-added of firm k in sector i under u and $\boldsymbol{\tau}$. The competitiveness is determined by the membership of firms in the same sector. The total causal effect of the policy reform is defined as

$$\Delta Y(\boldsymbol{\tau}^0, \boldsymbol{\tau}^1) := \sum_{i=1}^{N} \sum_{k \in \mathcal{N}_i^1} y_{ik}^1(\boldsymbol{\tau}^1) - \sum_{i=1}^{N} \sum_{k \in \mathcal{N}_i^0} y_{ik}^0(\boldsymbol{\tau}^0).$$

By the technique of add and subtract, it can be decomposed into the intensive and extensive margin causal effects:

$$\underbrace{\Delta Y(\boldsymbol{\tau}^0, \boldsymbol{\tau}^1)}_{\text{the total causal effect}} = \underbrace{\sum_{i=1}^{N} \sum_{k \in \mathcal{N}_i^1} y_{ik}^1(\boldsymbol{\tau}^1) - \sum_{i=1}^{N} \sum_{k \in \mathcal{N}_i^0} y_{ik}^0(\boldsymbol{\tau}^1)}_{\text{the extensive margin causal effect}} + \underbrace{\sum_{i=1}^{N} \sum_{k \in \mathcal{N}_i^0} y_{ik}^0(\boldsymbol{\tau}^1) - \sum_{i=1}^{N} \sum_{k \in \mathcal{N}_i^0} y_{ik}^0(\boldsymbol{\tau}^0)}_{\text{the intensive margin causal effect}}.$$

The first term of the right-hand side of this expression is a *ceteris paribus* difference in GDP due to a change in the number of firms, thus representing the *extensive margin causal effect*. The second term fixes the number of firms at the status quo level while only changing the level of subsidy; thus, this term is the *intensive margin causal effect*, as discussed in the main text.

Identification. Notice here that the second half (i.e., the intensive margin causal effect) is identified by the short-run analysis of this paper. As shown below, the long-run analysis directly identifies the total causal effect. Hence, the extensive margin causal effect is identified as a residual.

To simplify the exposition, suppose that the market competitiveness is summarized in a single variable: Let $\mathbf{a}^u \in \mathbb{R}$ be the index of the market competitiveness corresponding to u.

Under the assumption of the HSA demand system, I can write as

$$y_{ik}(\boldsymbol{\tau}, \mathbf{a}^u) = y^u_{ik}(\boldsymbol{\tau}),$$

for any $\boldsymbol{\tau} \in \mathscr{T}$. Assume that a version of the "within the support" condition (a version of Assumption 4.2) holds for $[\mathbf{a}^0, \mathbf{a}^1]$ as well. The total causal effect can be expressed as

$$\Delta Y(\boldsymbol{\tau}^0, \boldsymbol{\tau}^1) = \sum_{i=1}^{N} \sum_{k \in \mathcal{N}_i^1} y_{ik}(\boldsymbol{\tau}^1, \mathbf{a}^1) - \sum_{i=1}^{N} \sum_{k \in \mathcal{N}_i^0} y_{ik}(\boldsymbol{\tau}^0, \mathbf{a}^0).$$

From this expression, the identification analysis can further be broken down into the following four components:

$$\begin{aligned}
\Delta Y(\boldsymbol{\tau}^0, \boldsymbol{\tau}^1) = \sum_{i=1}^{N} \Bigg\{ & \underbrace{\sum_{k \in \mathcal{N}_i^0 \cap \mathcal{N}_i^1} \big(y_{ik}(\boldsymbol{\tau}^1, \mathbf{a}^1) - y_{ik}(\boldsymbol{\tau}^0, \mathbf{a}^0) \big)}_{\text{continuing firms}} \\
& + \underbrace{\sum_{k \in \mathcal{N}_i^1 \backslash \mathcal{N}_i^0} \big(y_{ik}(\boldsymbol{\tau}^1, \mathbf{a}^1) - y_{ik}(\boldsymbol{\tau}^0, \mathbf{a}^0) \big)}_{\text{new entrants}} + \underbrace{\sum_{k \in \mathcal{N}_i^0 \backslash \mathcal{N}_i^1} \big(y_{ik}(\boldsymbol{\tau}^1, \mathbf{a}^1) - y_{ik}(\boldsymbol{\tau}^0, \mathbf{a}^0) \big)}_{\text{exiting firms}} \\
& + \underbrace{\sum_{k \in \mathcal{N}_i^1 \backslash \mathcal{N}_i^0} y_{ik}(\boldsymbol{\tau}^0, \mathbf{a}^0) - \sum_{k \in \mathcal{N}_i^0 \backslash \mathcal{N}_i^1} y_{ik}(\boldsymbol{\tau}^1, \mathbf{a}^1)}_{\text{a normalization constant}} \Bigg\}
\end{aligned}$$

The first term is the causal effect that stems from the continuing firms' (i.e., firms that operate both before and after the policy reform) moving from the current state of the economy $(\boldsymbol{\tau}^0, \mathbf{a}^0)$ to an alternative state of the economy $(\boldsymbol{\tau}^1, \mathbf{a}^1)$. The second and third terms represent the causal effect arising from new entrants (i.e., firms that do not operate before the policy reform but become active after the policy reform) and from exiting firms (i.e., firms that are active before the policy reform but cease to operate after the policy reform), respectively. Note that these terms involve counterfactual outcomes because $\{y_{ik}(\boldsymbol{\tau}^0, \mathbf{a}^0) : k \in \mathcal{N}_i^1 \backslash \mathcal{N}_i^0\}$ and $\{y_{ik}(\boldsymbol{\tau}^1, \mathbf{a}^1) : k \in \mathcal{N}_i^0 \backslash \mathcal{N}_i^1\}$ are not observed in the data. This fact points to the importance of a structural model in defining and identifying the causal policy effects. The last term is the difference between the sum of firm-level value-added that would have been created by the entering firms if they were to be operative before the policy reform, and the sum of firm-level value-added that would have been yielded by the exiting firms if they were to continue to operate under the post-policy environment. This term acts as a normalization constant, reflecting the free entry condition as well as other model specifications.

For the first three terms (i.e., for continuing firms, new entrants and exiting firms), the summand can be rearranged as

$$\begin{aligned} y_{ik}(\boldsymbol{\tau}^1, \mathbf{a}^1) - y_{ik}(\boldsymbol{\tau}^0, \mathbf{a}^0) &= y_{ik}(\boldsymbol{\tau}^1, \mathbf{a}^1) - y_{ik}(\boldsymbol{\tau}^0, \mathbf{a}^1) + y_{ik}(\boldsymbol{\tau}^0, \mathbf{a}^1) - y_{ik}(\boldsymbol{\tau}^0, \mathbf{a}^0) \\ &= \int_{\boldsymbol{\tau}^0}^{\boldsymbol{\tau}^1} \frac{\partial y_{ik}(s, \mathbf{a}^1)}{\partial s} ds + \int_{\mathbf{a}^0}^{\mathbf{a}^1} \frac{\partial y_{ik}(\boldsymbol{\tau}^0, s)}{\partial s} ds. \end{aligned}$$

The left-hand side of this equation is identified as soon as both $\frac{\partial y_{ik}(s,\mathbf{a}^1)}{\partial s}$ and $\frac{\partial y_{ik}(\boldsymbol{\tau}^0,s)}{\partial s}$ are identified. The details of the identification depend on the specification of market competitiveness $\mathbf{a}$ and are beyond the scope of this paper. The identification of the fourth term (i.e., the normalization constant) hinges on the formulation of the free-entry condition, which determines the number of firms $\mathcal{N}_i^1$. Further investigation is left for future work.

D.3 Other Causal Parameters of Interest

The discussion of the main text of this paper concentrated around the policy parameter (14) (i.e., the *ceteris paribus* difference in GDP as a result of a policy reform) for the sake of exposition. However, the approach of this paper applies more broadly. This subsection explores the versatility of my framework by showing how it can be used to define other economically interesting causal policy parameters studied in the literature. All the parameters in this subsection are identified under the same set of assumptions as in Theorem 4.1.

D.3.1 Various Formulations

First, the researcher may want to restrict attention to a subset $\mathscr{N}^{sub} \subset \mathscr{N}$ of sectors (e.g., broadly defined sectors). In such a case, the object of interest takes the form of

$$\sum_{i \in \mathscr{N}^{sub}} Y_i(\boldsymbol{\tau}^1) - \sum_{i \in \mathscr{N}^{sub}} Y_i(\boldsymbol{\tau}^0).$$

Second, under Assumption 2.1, the policy parameter (14) is essentially equivalent to writing as

$$\frac{1}{N} \sum_{i=1}^{N} Y_i(\boldsymbol{\tau}^1) - \frac{1}{N} \sum_{i=1}^{N} Y_i(\boldsymbol{\tau}^0).$$

This expression allows for the interpretation as the average treatment effect (ATE) of the policy change on sectoral GDP.

Another economically interesting policy parameter would be the growth rate $\%\Delta Y(\tau_n^0, \tau_n^1)$ of the kind studied in Arkolakis et al. (2012) and Adão et al. (2017). This is just a version of (14) and can be defined as

$$\%\Delta Y(\tau_n^0, \tau_n^1) \coloneqq \frac{1}{Y^{\boldsymbol{\tau}^0}} \Delta Y(\tau_n^0, \tau_n^1).$$

Furthermore, the elasticity-type policy parameter $\frac{d \ln Y}{d\tau_n}$ around $\boldsymbol{\tau}^0$ (e.g., Caliendo and Parro (2015), Liu (2019), Baqaee and Farhi (2022)) can also be viewed as a version of (14) at the limit of $\boldsymbol{\tau}^1 \to \boldsymbol{\tau}^0$, i.e.,

$$\left. \frac{d \ln Y^{\boldsymbol{\tau}}}{d\tau_n} \right|_{\boldsymbol{\tau}=\boldsymbol{\tau}^0} = \lim_{\boldsymbol{\tau}^1 \to \boldsymbol{\tau}^0} \%\Delta Y(\tau_n^0, \tau_n^1).$$

D.3.2 Aggregate Variables

Consumption. The causal policy effect on final consumption is given by

$$\Delta C(\tau_n^0, \tau_n^1) \coloneqq C(\boldsymbol{\tau}^1) - C(\boldsymbol{\tau}^0) = \int_{\tau_n^0}^{\tau_n^1} \frac{dC}{d\tau_n} d\tau_n,$$

where $C(\boldsymbol{\tau})$ represents the equilibrium consumption under policy regime $\boldsymbol{\tau}$. Assuming that government spending G is fixed, it can be rewritten as

$$\frac{dC}{d\tau_n} = \frac{dY}{d\tau_n} = \sum_{i=1}^{N} \frac{dY_i}{d\tau_n},$$

where the identification of $\frac{dY_i}{d\tau_n}$ is studied in the main text.

Labor, material, and output quantities. In equilibrium, labor employed in sector i is defined as

$$L_i^* \coloneqq \sum_{k=1}^{N_i} \ell_{ik}^*.$$

The policy effect on labor employed in sector i, $\Delta L_i(\tau_n^0, \tau_n^1)$, is given by

$$\Delta L_i(\tau_n^0, \tau_n^1) \coloneqq L_i(\boldsymbol{\tau}^1) - L_i(\boldsymbol{\tau}^0) = \sum_{k=1}^{N_i} \int_{\tau_n^0}^{\tau_n^1} \frac{d\ell_{ik}^*}{d\tau_n} d\tau_n,$$

where $L(\boldsymbol{\tau})$ denotes the total labor employed in sector i under policy $\boldsymbol{\tau}$. From this equality, $\Delta L_i(\tau_n^0, \tau_n^1)$ is identified as soon as $\frac{d\ell_{ik}^*}{d\tau_n}$ is identified for all $k \in \mathscr{N}_i$ and $\tau_n \in [\tau_n^0, \tau_n^1]$.[142]

Analogous arguments hold for the output price and quantity as well as for the quantity of material input.

Unilateral and bilateral trade flows. The equilibrium volume of unilateral trade flow from sector j to i is defined as

$$U_{i,j}^* \coloneqq \sum_{k=1}^{N_i} m_{ik,j}^*.$$

The policy effect on the unilateral trade flow is given by

$$\Delta U_{i,j}(\tau_n^0, \tau_n^1) \coloneqq U_{i,j}(\boldsymbol{\tau}^0) - U_{i,j}(\boldsymbol{\tau}^1) = \sum_{k=1}^{N_i} \int_{\tau_n^0}^{\tau_n^1} \frac{dm_{ik,j}^*}{d\tau_n} d\tau_n,$$

where $U_{i,j}(\boldsymbol{\tau})$ represents the unilateral trade flow from sector j to i under policy $\boldsymbol{\tau}$. It follows from this expression that the causal effect $\Delta U_{i,j}(\tau_n^0, \tau_n^1)$ is recovered through the identification of $\frac{dm_{ik,j}^*}{d\tau_n}$.[143]

The policy effect on the bilateral trade flow between sector i and j, denoted by $B_{i,j}$, can similarly be analyzed by noticing $B_{i,j} = U_{i,j} + U_{j,i}$.

D.3.3 Various Treatment Effects

As stated in the main text, the construction of the policy parameter (14) shares the common vein with the treatment effects. In fact, multitudes of "treatment effects" can be analyzed within my framework. As an example, consider the firm k's net profit, defined by

$$\pi_{ik}^* \coloneqq p_{ik}^* q_{ik}^* - (W^* \ell_{ik}^* + P_i^{M*} m_{ik}^*).$$

This represents the firm's profit after all taxes and subsidies are applied.

Individual-level treatment effects. The individual-level treatment effect on a firm's profit is given by

$$\Delta \pi_{ik}(\tau_n^0, \tau_n^1) \coloneqq \pi_{ik}(\boldsymbol{\tau}^1) - \pi_{ik}(\boldsymbol{\tau}^0) = \int_{\tau_n^0}^{\tau_n^1} \frac{d\pi_{ik}^*}{d\tau_n} d\tau_n,$$

[142] This is established in Proposition C.8.
[143] This is established in Proposition C.9.

where $\pi_{ik}(\boldsymbol{\tau})$ denotes the firm k's equilibrium profit π^*_{ik} under policy regime $\boldsymbol{\tau}$. Here, it is straightforward to verify that $\frac{d\pi^*_{ik}}{d\tau_n}$ is identified under the same set of assumptions as Theorem 4.1, and thus so is the individual treatment effect $\Delta\pi_{ik}(\tau^0_n, \tau^1_n)$.

Average treatment effects. For each sector $i \in \mathscr{N}$, the sector-level average treatment effect is given by

$$\Delta\Pi_i(\tau^0_n, \tau^1_n) \coloneqq \frac{1}{N_i}\sum_{k=1}^{N_i} \pi_{ik}(\boldsymbol{\tau}^1) - \frac{1}{N_i}\sum_{k=1}^{N_i} \pi_{ik}(\boldsymbol{\tau}^0) = \frac{1}{N_i}\sum_{k=1}^{N_i} \Delta\pi_{ik}(\tau^0_n, \tau^1_n).$$

Moreover, the economy-wide average treatment effect (i.e., producer surplus) is given by

$$\Delta\Pi(\tau^0_n, \tau^1_n) \coloneqq \frac{1}{N}\sum_{i=1}^{N}\frac{1}{N_i}\sum_{k=1}^{N_i} \pi_{ik}(\boldsymbol{\tau}^1) - \frac{1}{N}\sum_{i=1}^{N}\frac{1}{N_i}\sum_{k=1}^{N_i} \pi_{ik}(\boldsymbol{\tau}^0) = \frac{1}{N}\sum_{i=1}^{N} \Delta\Pi_i(\tau^0_n, \tau^1_n).$$

Given the identification of the individual-level treatment effect $\Delta\pi_{ik}(\tau^0_n, \tau^1_n)$, the sector-level average treatment effect $\Delta\Pi_i(\tau^0_n, \tau^1_n)$ is also identified, which in turn recovers the economy-wide average treatment effect $\Delta\Pi(\tau^0_n, \tau^1_n)$.

Remark D.1. *The recent international trade literature has applied the statistical treatment effect approach to study the average treatment effects of a trade policy change on bilateral international trade flows (e.g., Baier and Bergstrand, 2007, 2009; Egger et al., 2008, 2011). Such an estimand can be mirrored in my framework, as sketched in Appendices D.3.1 and D.3.2.*

Distributional treatment effects. Given that individual-level treatment effects $\Delta\pi_{ik}(\tau^0_n, \tau^1_n)$ are identified and the firm-level profits under the current policy regime $\pi_{ik}(\boldsymbol{\tau}^0)$ are directly observed in the data, it is possible to recover the firm's profit under an alternative policy $\boldsymbol{\tau}^1$:

$$\pi_{ik}(\boldsymbol{\tau}^1) = \pi_{ik}(\boldsymbol{\tau}^0) + \Delta\pi_{ik}(\tau^0_n, \tau^1_n).$$

This means that one can recover the joint distribution of $\pi_{ik}(\boldsymbol{\tau}^0)$ and $\pi_{ik}(\boldsymbol{\tau}^1)$, a basis on which a variety of distributional criteria for policy evaluation are defined and identified. For example, the policymaker may be interested in the proportion of firms that benefit from the policy reform from $\boldsymbol{\tau}^1$ to $\boldsymbol{\tau}^0$.[144] In such a case, the object of interest is given by

$$Prop_i(\boldsymbol{\tau}^0, \boldsymbol{\tau}^1) \coloneqq Pr(\pi_{ik}(\boldsymbol{\tau}^1) \geq \pi_{ik}(\boldsymbol{\tau}^0)).$$

[144] This is called the voting criteria (Heckman et al., 1999; Heckman and Vytlacil, 2007).

Another distributional policy parameter that is often of practical interest is the (unconditional) quantile treatment effect for quantile $u \in (0, 1)$, which is defined as

$$QTW_i^u(\boldsymbol{\tau}^0, \boldsymbol{\tau}^1) \coloneqq F^{-1}_{\Pi(\boldsymbol{\tau}^1)}(u) - F^{-1}_{\Pi(\boldsymbol{\tau}^0)}(u),$$

where $F^{-1}_{\Pi(\boldsymbol{\tau})}(\cdot)$ stands for the inverse of the probability distribution of π^*_{ik} under policy regime $\boldsymbol{\tau}$.

See Heckman et al. (1999) for an extensive catalog of distributional treatment effects. It is immediate to show that these distributional criteria are identified when Theorem 4.1 holds.

D.4 Changing Subsidies to Multiple Sectors (Universal Intervention)

In the main text, I restrict attention to the case where only a subsidy to a single sector is manipulated. In practice, however, subsidies to other sectors are also more or less subject to revisions, regardless of whether they are purposefully targeted. Thus, it is practically very important to accommodate polices that intervene in multiple sectors at once — policies with universal coverage. For ease of exposition, suppose that there are only two sectors. Consider a policy reform from $\boldsymbol{\tau}^0 \coloneqq (\tau_1^0, \tau_2^0)$ to $\boldsymbol{\tau}^1 \coloneqq (\tau_1^1, \tau_2^1)$. Here, I assume that both τ_1 and τ_2 satisfy the "within the support" condition of the form of Assumption 4.2.

The object of interest can be written as

$$\begin{aligned} \Delta Y(\boldsymbol{\tau}^0, \boldsymbol{\tau}^1) &\coloneqq \sum_{i=1}^{N} Y_i((\tau_1^1, \tau_2^1)) - \sum_{i=1}^{N} Y_i((\tau_1^0, \tau_2^0)) \\ &= \underbrace{\sum_{i=1}^{N} Y_i((\tau_1^1, \tau_2^1)) - \sum_{i=1}^{N} Y_i((\tau_1^1, \tau_2^0))}_{\text{one-sector problem (the shift from } \tau_2^0 \text{ to } \tau_2^1)} + \underbrace{\sum_{i=1}^{N} Y_i((\tau_1^1, \tau_2^0)) - \sum_{i=1}^{N} Y_i((\tau_1^0, \tau_2^0))}_{\text{one-sector problem (the shift from } \tau_1^0 \text{ to } \tau_1^1)}. \end{aligned}$$

The first term indicates the causal effect of moving from a counterfactual policy regime (τ_1^1, τ_2^0) to another counterfactual policy regime (τ_1^1, τ_2^1). This is nothing but the causal effect of changing only τ_2 from τ_2^0 to τ_2^1 while keeping τ_1 fixed at τ_1^1, which is identified by the analysis of this paper. The second term represents the causal effect of moving from the current policy regime (τ_1^0, τ_2^0) to a counterfactual policy regime (τ_1^1, τ_2^0), which is also identified by the analysis of this paper. Again, this is the causal effect of changing only τ_1 from τ_1^0 to τ_1^1 with τ_2 fixed at τ_2^0. That is, a multiple-subsidy problem can be broken down to multiple one-subsidy problems, each of which is independently identified by the method of this paper.

This observation marks a remarkable distinction between the empirical treatment effect literature and my framework. In my framework, policy interventions that affect all units (i.e., universal treatments) can be well defined and identified, while the effects of such treatments are not generally identifiable in the treatment effect paradigm.

D.5 Optimal Policy Design

Definition. My model can be used to formulate an optimal policy design problem: For a fixed sector n

$$\tau_n^{1*} \in \arg\max_{\tau_n^1} \quad \Delta Y(\tau_n^0, \tau_n^1) \qquad s.t. \qquad \mathcal{C}(\boldsymbol{\tau}^0, \boldsymbol{\tau}^1) \geq \mathbf{0}, \tag{126}$$

where $\mathcal{C}(\boldsymbol{\tau}^0, \boldsymbol{\tau}^1) \geq \mathbf{0}$ represents a set (vector) of constraints faced by the policymaker. This embodies, for example, political economy considerations about equality and fairness among agents (e.g., sectors, firms, and households).

One can extend this formulation to choose which sector to be targeted:[145]

$$\tau_{n^*}^{1}{}^* \in \arg\max_{n \in \mathcal{N}} \left\{ \arg\max_{\tau_n^1} \quad \Delta Y(\tau_n^0, \tau_n^1) \qquad s.t. \qquad \mathcal{C}(\boldsymbol{\tau}^0, \boldsymbol{\tau}^1) \geq \mathbf{0} \right\}.$$

[145]The formlulation can further be extended to choose which set of sectors to be targeted at the cost of notational burden.

E Estimation

Given that the firm-level revenue functions and share regressions are nonparametrically identified (Appendix C), I employ polynomial regressions as a vehicle to nonparametrically estimate these functions. The degrees of polynomials are chosen adaptively on the basis of the root-mean-squared errors (RMSE).

E.1 Firm-Level Quantities and Prices

To estimate $\tilde{\phi}_i(\cdot)$ in Step 1 of Lemma C.4, I consider a polynomial regression specification. For instance, the approximation by a second-order polynomial takes the form of

$$\tilde{r}_{ik} = b_{i,0} + b_{i,1}\tilde{\ell}_{ik} + b_{i,2}\tilde{m}_{ik} + b_{i,3}\tilde{\ell}_{ik}^2 + b_{i,4}\tilde{m}_{ik}^2 + b_{i,5}\tilde{\ell}_{ik}\tilde{m}_{ik} + \tilde{\eta}_{ik} = \tilde{x}_{ik}\boldsymbol{b}_i + \tilde{\eta}_{ik}, \tag{127}$$

where $\tilde{x}_{ik} := [\tilde{\ell}_{ik}, \tilde{m}_{ik}, \tilde{\ell}_{ik}^2, \tilde{m}_{ik}^2, \tilde{\ell}_{ik}\tilde{m}_{ik}]^T$, and $\mathbf{b}_i := [b_{i,0}, b_{i,1}, b_{i,2}, b_{i,3}, b_{i,4}, b_{i,5}]^T$, where T denotes the transpose of a vector. Letting $\hat{\mathbf{b}}_i$ be the Ordinary Least Squares (OLS) estimator, the fitted value of the log-revenue $\tilde{r}_{ik}$ is $\hat{\tilde{\phi}}_i(\tilde{x}_{ik}) := \tilde{x}_{ik}\hat{\mathbf{b}}_i$. Note here that the OLS estimates $\hat{\mathbf{b}}_i$ are obtained under the constraints suggested by Assumption C.2. Moreover, given the estimator $\hat{\mathbf{b}}_i$, the specification (127) naturally gives rise to the estimator for the first-order partial derivatives of $\tilde{\phi}_i(\cdot)$ with respect to $\tilde{\ell}_{ik}$ and $\tilde{m}_{ik}$:

$$\widehat{\frac{\partial\tilde{\phi}_i}{\partial\tilde{\ell}_{ik}}}(\tilde{\ell}_{ik}, \tilde{m}_{ik}) := \hat{b}_{i,1} + 2\hat{b}_{i,3}\tilde{\ell}_{ik} + \hat{b}_{i,5}\tilde{m}_{ik}$$
$$\widehat{\frac{\partial\tilde{\phi}_i}{\partial\tilde{m}_{ik}}}(\tilde{\ell}_{ik}, \tilde{m}_{ik}) := \hat{b}_{i,2} + 2\hat{b}_{i,4}\tilde{m}_{ik} + \hat{b}_{i,5}\tilde{\ell}_{ik}.$$

E.2 Second-Order Derivatives of Firm-Level Production Function

To construct a nonparametric estimator for the derivatives of the firm-level production functions, I consider approximating (111) by polynomials and solve the following minimization problem as proposed in Gandhi et al. (2019) — for example, the case of a second-order polynomial approximation solves

$$\hat{\boldsymbol{\zeta}} \in \arg\min_{\boldsymbol{\zeta}^\circ} \sum_{k=1}^{N_i} \left\{ \tilde{s}_{ik}^{\ell,\tilde{\mu}} - \ln\left\{\zeta_{i,0}^\circ + \zeta_{i,1}^\circ\tilde{\ell}_{ik} + \zeta_{i,2}^\circ\tilde{m}_{ik} + \zeta_{i,3}^\circ\tilde{\ell}_{ik}^2 + \zeta_{i,4}^\circ\tilde{m}_{ik}^2 + \zeta_{i,5}^\circ\tilde{\ell}_{ik}\tilde{m}_{ik}\right\} \right\}^2.$$

Note that this optimization is subject to the implications of Euler's theorem for homogeneous functions. Specifically, I impose equality constraints for the first- and second-order partial derivatives.

E.3 Adaptive Choice of Degrees of Polynomials

In estimating these functions, I fit polynomial regressions of degree one and two.[146] For each of these, the root-mean-squared error (RMSE) is calculated. I then choose the polynomial degree with the lowest MSE as the optimal degree. Throughout this adaptive choice, the effective sample size stays well above the number of parameters of the polynomials.[147]

[146]This setting is sufficient because my analysis only needs the first-order derivative of the revenue function and the first- and second-order derivatives of the production function. Note that the function recovered by the share regression is already a derivative of the production function. Allowing for a potentially higher degree of polynomials requires considerable computational cost and may even undermine the accuracy.

[147]The effective sample size is the sample size (listed in Table 3) multiplied by the number of snapshots (i.e., the number of years), which is 12 in my dataset.

F Monte Carlo Simulations

This section presents Monte Carlo simulations to examine the finite-sample properties of my nonparametric estimation approach described in Section 4. For the ease of exposition, I focus on estimating $\left.\frac{dY_i(s)}{ds}\right|_{s=\boldsymbol{\tau}}$ at $\boldsymbol{\tau} = \boldsymbol{\tau}^0$, given in (16).

F.1 Simulation Design

I assume that there are only two sectors in the economy (i.e., $\mathscr{N} = \{1, 2\}$), each of which is populated by an identical set of $N_i = 20$ (for all $i \in \mathscr{N}$) firms. I consider two cases in terms of the interdependencies of these two sectors. One case assumes away production networks, while the other admits a production network across sectors. This means that the adjacency matrix in the former case is equivalent to an identity matrix (i.e., $\boldsymbol{\Omega} = \mathbf{I}$). For the latter, I assume that Sectors 1 and 2 are symmetric in the input-output linkages with the following adjacency matrix:

$$\boldsymbol{\Omega} = \begin{bmatrix} 0.8 & 0.2 \\ 0.2 & 0.8 \end{bmatrix}.$$

For both cases, I further compare two patterns in terms of the type of market competition. On the one hand, I consider an economy in which firms in each sector engage in oligopolistic competition. On the other hand, I also investigate the case in which firms operate monopolistically. Throughout this section, I focus on the impacts of increasing only the subsidy to Sector 1 (i.e., $n = 1$).

Using a parametric model described below, I first generate simulation data for firm-level revenues, labor and material inputs, productivities, prices, and quantities, as well as other aggregate variables. These are used as a status quo environment. Next, to obtain the effects of marginally increasing the subsidy, I follow the theoretical results (derived in Appendix A). The values calculated in this way are referred to as true policy effects. Then, I also compute the policy effects using my approach (described in Appendices C and E). The resulting estimates are called estimated policy effects. To make my estimation problem as close to reality as possible, the estimated policy effects are calculated without directly using the realizations of firm-level productivities, prices, and quantities, as these are not observed in the real data (see Section 3).

The simulation data are generated under various subsidy levels. Specifically, I repeat the simulation eleven times with $\tau_1 = \tau_2 = 0.000, 0.002, 0.004, \ldots, 0.02$ (i.e., $\mathscr{T}_i = [0.000, 0.002, 0.004, \ldots, 0.020]$ for all $i \in \mathscr{N}$). In this way, I obtain a sequence of snapshots of the same economy (i.e., repeated cross-sectional data).[148] In my framework, these snapshots can be concatenated into a single cross-sectional dataset.[149] This allows me to construct data with the effective number of firms $\check{N}_i = 20, 100, 200$ for all $i \in \mathscr{N}$.

The number of Monte Carlo simulations is set to 1000. I concentrate on the consistency of my nonparametric estimator, which is implied by the continuity of my model and the plug-in nature of my estimator. Although my framework can in principle be used for statistical inference, formally taking the estimates to statistical hypothesis testing requires *i*) deriving an asymptotic distribution, and *ii*) deriving analytical expressions for the relevant moments of the asymptotic distribution (e.g., standard errors), or alternatively developing a new bootstrap procedure that is appropriate in my setup (i.e., accounting for the underlying structural model of firms' strategic interactions, and sectoral production networks as well as general equilibrium feedback). These exceed the scope of this paper and are thus left for future work.

I consider two scenarios for the current policy regimes, namely, Scenarios A and B. In Scenario A, the values for the current policies are all set equal to zero (i.e., $\tau_i = 0.000$ for all $i \in \mathscr{N}$). Scenario B assumes that there are nonzero pre-existing policies: In this scenario, I set $\tau_i = 0.020$ for all $i \in \mathscr{N}$.

F.1.1 Model

The parametric functional-form assumptions used in this section closely follow Grassi (2017). To motivate the econometric approach for policy evaluation, however, the setup of this section deviates from his by assuming that the firm-level production functions are given by a translog aggregator, not by a Cobb-Douglas aggregator (see Proposition C.13).

The sectoral aggregator is assumed to be a constant elasticity of substitution (CES) production function:

$$Q_i = \left(\sum_{k=1}^{N_i} \delta q_{ik}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}},$$

where σ is elasticity of substitution and δ stands for a demand shifter. The corresponding price index is given by $P_i = \left(\sum_{k=1}^{N_i} \delta^{\sigma} p_{ik}{}^{1-\sigma} \right)^{\frac{\sigma}{1-\sigma}}$.

In each sector i, firm k transforms labor ℓ_{ik} and material m_{ik} into output quantity q_{ik}

[148] This sequence can most naturally be understood as being indexed by time — for example, the subsidy level is 0.000 in year 1, 0.002 in year 2, 0.004 in year 3, and so forth.

[149] This is somewhat similar in spirit to pooled cross-sections. In my case, the observable aggregate variables are used as a snapshot-specific dummy or a control variable.

using a translog production function:

$$\log(q_{ik}) = \log(z_{ik}) - \alpha\log(\ell_{ik}) - (1-\alpha)\log(m_{ik}) - \theta\alpha(1-\alpha)(\log(\ell_{ik}) - \log(m_{ik}))^2,$$

where z_{ik} represents the firm's productivity, α indicates the output elasticity of labor input, and θ is a parameter modulating the substitutability between labor and material inputs. In view of Proposition C.13, this specification motivates econometric approach for the policy evaluation. Following Assumption B.3, material input is composed of sectoral intermediate goods $\{m_{ik,j}\}_{j\in\mathscr{N}}$ according to the following Cobb-Douglas aggregator:

$$m_{ik} = \prod_{j=1}^{N} m_{ik,j}^{\gamma_{i,j}},$$

where $\gamma_{i,j}$ corresponds to the input share of sector j's intermediate good, reflecting the production network $\mathbf{\Omega}$.

For the types of market competition, I consider monopolistic and oligopolistic competition, in line with Section 5.

Oligopolistic competition. When firms engage in Cournot competition in the output market, the Cournot-Nash equilibrium prices satisfy the following system of equations: For each sector $i \in \mathscr{N}$,

$$\begin{aligned} p^*_{ik} &= \frac{\sigma}{(1-\sigma)(1-s^*_{ik})} mc^*_{ik} \\ s^*_{ik} &= \delta^{\sigma}\left(\frac{p^*_{ik}}{P^*_i}\right), \end{aligned}$$

where $mc^*_{ik} = z_{ik}^{-1}\alpha^{-\alpha}(1-\alpha)^{1-\alpha}W^{*\alpha}P_i^{M*1-\alpha}$ and s^*_{ik} is the firm's equilibrium (revenue-based) market share.[150] The associated optimal input choices are

$$\begin{aligned} \ell^*_{ik} &= z_{ik}^{-1}\left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha}\left(\frac{P_i^{M*}}{W^*}\right)^{1-\alpha} q^*_{ik} \\ m^*_{ik} &= z_{ik}^{-1}\left(\frac{\alpha}{1-\alpha}\right)^{-\alpha}\left(\frac{P_i^{M*}}{W^*}\right)^{-\alpha} q^*_{ik}, \end{aligned}$$

with the optimal quantity satisfying $q^*_{ik} = \left(\frac{p^*_{ik}}{P^*_i}\right)Q^*_i$.

[150] See Atkeson and Burstein (2008), Grassi (2017), and Gaubert and Itskhoki (2020) for details.

Monopolistic competition. For each sector $i \in \mathscr{N}$, the optimal pricing for a monopolistic firm k is given by

$$p_{ik}^{*} = \frac{\sigma}{\sigma - 1} mc_{ik}^{*}.$$

The input problem is identical to the oligopolistic case.

F.1.2 Parameter Values

Parameter values are chosen in such a way that a Cournot-Nash equilibrium is well-defined. First, *ex ante* identical firms draw their heterogeneous productivities from a log normal distribution: $\ln z_{ik} \sim \mathcal{N}(0, 0.1)$. Then, the firms play a game multiple times with varying levels of the policy variables, as illustrated above. In order to keep my setup as close to the literature (e.g., Atkeson and Burstein, 2008; Grassi, 2017) as possible, I set $\sigma = 10$ (i.e., firms' products are substitutes), $\alpha = 0.6$, and $\theta = 0.01$.[151] For normalization purpose, I set $\delta = (1/N_i)^{1/\sigma}$ for all $i \in \{1, 2\}$.

The econometrician (or the policymaker) has access to firm-level revenue, labor, and material inputs, as well as aggregate variables, but no access to firm-level productivities, prices, and quantities. In line with my framework, the observed revenue is contaminated by measurement error: $\ln \eta_{ik} \sim \mathcal{N}(0, 0.005)$.[152] Lastly, I fix the wage rate at $W^{*} = 1$ throughout the simulation study, meaning that I focus on a partial equilibrium exercise.

To reduce potential noise from nonparametric estimation, I follow the convention in the literature (e.g., Gandhi et al., 2019) in postulating that the econometrician knows that the revenue functions and the share regression functions are well approximated by the first-order polynomial.[153]

F.2 Results

The finite-sample performance of the proposed estimator is evaluated in terms of mean, bias, and root-mean-squared errors.

Tables 4 and 5 compare the simulation results for the responsiveness of the economy-wide GDP across different types of competition for Scenarios A and B, respectively. The tables

[151] This means that the firm-level production function is very similar to a Cobb-Douglas production function. This choice alleviates the concern of misspecification in polynomial regression, while motivating the use of the econometric approach.

[152] The measurement error is assumed to enter in a linear, additive fashion in logarithms, i.e., $\ln r_{ik} = \ln \bar{r}_{ik} + \ln \eta_{ik}$, where r_{ik} and $\bar{r}_{ik}$ are observable and true (simulated) revenue, respectively. It is also assumed that $E[\ln \eta_{ik} \mid \tilde{\ell}_{ik}, \tilde{m}_{ik}] = 0$. See Appendix C.3.2.

[153] In empirical illustration, I use an adaptive procedure to choose the appropriate degrees of polynomials. See Appendix E.2.

report the true policy effects and statistics of the estimated policy effects, such as mean, bias (accompanied by the percentage in absolute terms), and root-mean-squared error (RMSE) for both monopolistic and oligopolistic cases.

In Table 4, for each combination of market competition and a production network, the average of the point estimates approaches the true policy effect as the effective number of firms ($\check{N}_i$) increases. This is associated with the shrinking bias and RMSE. Taken together, the results displayed in this table support that the law of large numbers is certainly at work. Analogous patterns hold for Table 5.

Comparing across Tables 4 and 5 for each combination of market competition and a production network, the responsiveness of GDP varies substantially, depending on the level of the status quo subsidy. For instance, under oligopolistic competition in a networked economy, the policy effect at $\tau_1 = \tau_2 = 0.020$ is approximately 4.15% lower than that at the no-preexisting policy state, highlighting the considerable dependency of the responsiveness of GDP on the underlying policy regime. This means that the common practice of setting $\boldsymbol{\tau}^0 = \mathbf{0}$ (e.g., Liu, 2019; Baqaee and Farhi, 2022) may not generate an accurate prediction, as it masks the dependency on the underlying policy regime (see Appendix C.11).

Table 4: Results: Theoretical and Estimated Policy Effects (Scenario A)

Competition	True	$\check{N}_i$	Estimates		
			Mean	Bias (%)	RMSE
Oligopolistic	-154.80	20	-88.40	66.40 (42.89%)	66.70
		100	-152.50	2.31 (1.49%)	28.81
		200	-153.29	1.51 (0.98%)	10.71
Monopolistic	-521.15	20	-536.31	-15.16 (2.91%)	15.16
		100	-519.81	1.34 (0.26%)	6.77
		200	-520.94	0.21 (0.04%)	2.08

(i) without a production network

Competition	True	$\check{N}_i$	Estimates		
			Mean	Bias (%)	RMSE
Oligopolistic	-154.02	20	-128.61	25.41 (16.50%)	48.41
		100	-158.10	-4.07 (2.64%)	31.94
		200	-154.17	-0.15 (0.10%)	9.41
Monopolistic	-521.93	20	-534.32	-12.39 (2.37%)	12.41
		100	-520.69	1.24 (0.24%)	6.94
		200	-521.80	0.13 (0.02%)	2.11

(ii) with the production network

Note: This table evaluates the performance of the proposed estimator in terms of the mean, bias, and root-mean-squared error. The round brackets following the biases indicate the absolute biases relative to the true policy effects in percentage (%).

Table 5: Results: Theoretical and Estimated Policy Effects (Scenario B)

Competition	True	$\check{N}_i$	Estimates		
			Mean	Bias (%)	RMSE
Oligopolistic	-161.22	20	-91.02	70.20 (43.54%)	70.35
		100	-163.71	-2.49 (1.55%)	28.95
		200	-160.39	0.83 (0.51%)	9.95
Monopolistic	-531.05	20	-546.97	-15.92 (3.00%)	15.92
		100	-530.01	1.04 (0.20%)	6.86
		200	-530.91	0.14 (0.03%)	2.11

(i) without a production network

Competition	True	$\check{N}_i$	Estimates		
			Mean	Bias (%)	RMSE
Oligopolistic	-160.41	20	-131.30	29.10 (18.14%)	87.78
		100	-163.36	-2.95 (1.84%)	29.68
		200	-160.52	-0.12 (0.07%)	9.41
Monopolistic	-531.84	20	-544.86	-13.02 (2.45%)	13.04
		100	-531.20	0.64 (0.12%)	6.45
		200	-531.77	0.08 (0.01%)	2.07

(ii) with the production network

Note: This table evaluates the performance of the proposed estimator in terms of the mean, bias, and root-mean-squared error. The round brackets following the biases indicate the absolute biases relative to the true policy effects in percentage (%).

G Empirical Illustration

G.1 Details of CHIPS and Science Act of 2022

This subsection details the CHIPS and Science Act of 2022. I first provide the institutional background and explain how and why my framework — an approach featuring a production network and oligopolistic competition — is plausible to analyze this policy episode. Then, I turn to the details of the CHIPS Act.

G.1.1 Institutional Background

When the former president Joe Biden took office back in 2021, he launched a series of policies — the so-called Bidenomics — with the following three key objectives: *i*) "making smart public investments in America," *ii*) "empoweing and educating workers to grow the middle class," and *iii*) "promoting competition to lower costs and help entrepreneurs and small businesses thrive" (White House, 2023a).

The first objective was symbolized by (*i*-*a*) the Bipartisan Infrastructure Law, (*i*-*b*) the CHIPS and Science Act, and (*i*-*c*) the Inflation Reduction Act (White House, 2024a). One of the recurring themes of these three acts was to strengthen supply chains (White House, 2021d, 2022a,b). The Biden administration recognizes the importance of supply chains in ensuring economic prosperity and national security and places the resilience of supply chains at the center of its policy implementation (White House, 2021a, 2024b).

The third objective is based on the administration's understanding that lack of competition is prevalent: "For decades, corporate consolidation has been accelerating. In over 75% of U.S. industries, a smaller number of large companies now control more of the business than they did twenty years ago (White House, 2021b)."[154] The White House deemed this imperfectly competitive market structure as the driving force for higher prices for households, lower wages for workers, and delayed innovation and economic growth. To tackle this problem, the government put a range of incentives into motion (White House, 2021b).

In light of these, it stands to reason to assume that the policymaker in my empirical analysis views the production network and oligopolistic competition as the key features of the economy.

[154] The second objective is not directly relevant to this paper. Readers requiring a detailed account are referred to White House (2021c).

G.1.2 CHIPS and Science Act of 2022

CHIPS stands for Creating Helpful Incentives to Produce Semiconductors (White House, 2022a). This act was passed into law in 2022 and aims to invest nearly $53 billion in the U.S. semiconductor manufacturing, research and development, and workforce (White House, 2023b). This policy also includes a 25% tax credit for manufacturing investment, which is projected to provide up to $24.25 billion for the next 10 years (Congressional Budget Office, 2022).

G.2 Main Results

G.2.1 Robustness

To explore the robustness of my estimation procedure, I run the same algorithm for different choices of the number of bins (i.e., $\bar{v}$ in (24a)). Given that the results in the main text are based on the choice $\bar{v} = 100$, this subsection examines the variability of the estimates with respect to increasing and decreasing the number of bins. Specifically, I consider two additional cases: $\bar{v} = 90$ and $\bar{v} = 110$. Table 6 shows the estimates of the policy effect $\widehat{\Delta Y}(\tau_n^0, \tau_n^1)$ for these cases. Clearly, the estimates remain unchanged relative to my main results (Table 1). The robustness is further illuminated by comparing Figures 2 and 4, each of which depicts the trajectories of the responsiveness of GDP over the course of the policy reform. Overall, the estimates remain unaltered both qualitatively and quantitatively across different choices of the number of bins.

Table 6: The estimates of the object of interest

(i) $\bar{v} = 90$

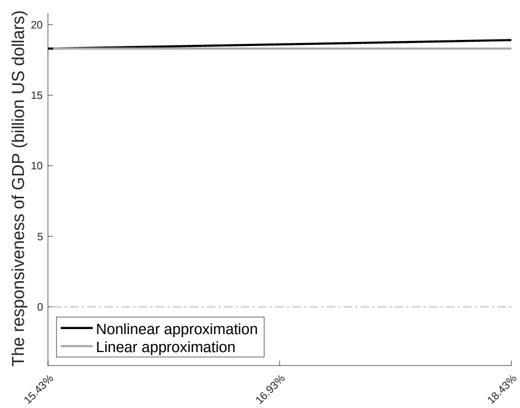
(billion U.S. dollars)	Monopolistic competition	Oligopolistic competition
Estimates based on (24a)	0.5581	-0.0378
Estimates based on (24b)	0.5491	-0.0369

(ii) $\bar{v} = 110$

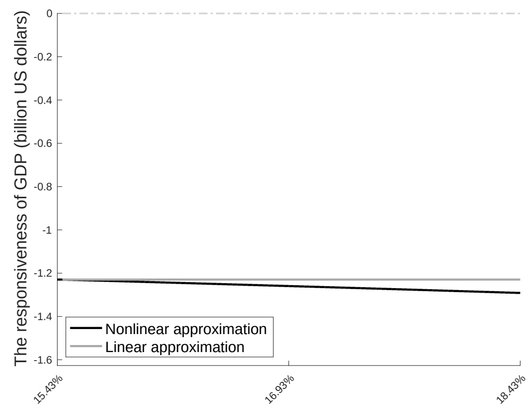
(billion U.S. dollars)	Monopolistic competition	Oligopolistic competition
Estimates based on (24a)	0.5581	-0.0378
Estimates based on (24b)	0.5491	-0.0369

Note: This table compares the estimates for the object of interest (14) based on the benchmark and my method. The estimates are measured in billions of U.S. dollars.

(i) $\bar{v} = 90$

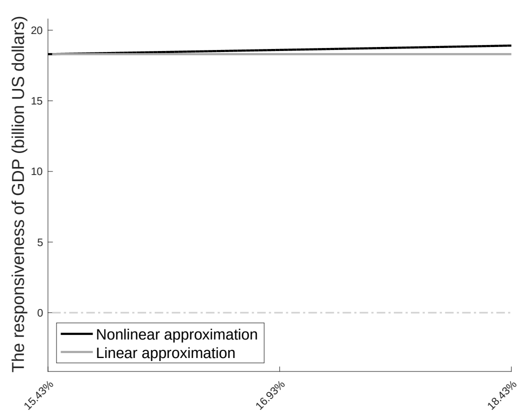


(a) Monopolistic Competition

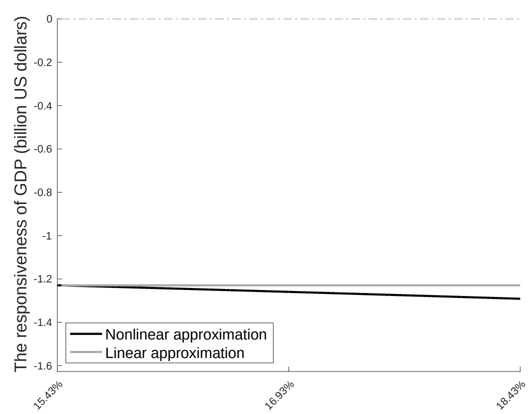


(b) Oligopolistic Competition

(ii) $\bar{v} = 110$



(a) Monopolistic Competition



(b) Oligopolistic Competition

Note: This figure illustrates the estimates of the total derivative of (economy-wide) GDP with respect to the semiconductor subsidy between 15.43% and 18.43%. Panel (a) shows the result for the case of monopolistic competition, and panel (b) displays the result for the case of oligopolistic competition. The solid black line represents the estimates based on the nonlinear approximation (24a). The solid dark grey line indicates the estimates based on the linear approximation (24b). The dash-dotted light grey line stands for the horizontal axis at zero.

Figure 4: The total derivative of Y with respect to τ_n

G.3 Mechanism

G.3.1 Responsiveness of Sectoral GDP

Tables 7 and 8 report the detailed results of the empirical illustration for monopolistic and oligopolistic competition, respectively. These tables break down the responsiveness of sectoral GDP into four components, as explained in Section 5.2, and display the estimates at the status quo policy regime in descending order of the total effect.

Table 7: Responsiveness of Sectoral GDP: Monopolistic Competition (in Billions of U.S. Dollars)

Industry	Total Effect	Effects on Revenue		Effects on Material Cost	
		p.effect	q.effect	w.effect	s.effect
Professional services	3.89	-2.40	9.26	1.70	-4.67
Wholesale trade	3.55	-0.82	5.06	0.67	-1.36
Retail Trade	2.41	-0.96	4.02	1.05	-1.71
Construction	2.27	-0.40	3.03	0.23	-0.59
Information	0.95	-0.78	2.19	0.45	-0.91
Motor vehicles, bodies and trailers, and parts	0.93	-0.16	1.20	0.12	-0.22
Computer and electronic products	0.80	-1.47	3.36	0.19	-1.27
Transportation	0.66	-0.18	1.34	0.12	-0.61
Accommodation and food services	0.55	-0.48	1.33	0.27	-0.59
Administrative and waste management	0.54	-0.23	0.88	0.14	-0.26
Food and beverage and tobacco products	0.35	-0.21	0.66	0.11	-0.21
Other transportation equipment	0.22	-0.07	0.35	0.05	-0.11
Educational services	0.21	-0.07	0.35	0.06	-0.13
Machinery	0.18	-0.07	0.28	0.05	-0.08
Chemical products	0.15	-0.10	0.28	0.04	-0.07
Petroleum and coal products	0.12	-0.04	0.17	0.01	-0.01
Fabricated metal products	0.11	-0.04	0.17	0.03	-0.05
Oil and gas extraction and mining	0.11	-0.05	0.17	0.03	-0.04
Paper products and related services	0.09	-0.03	0.14	0.02	-0.04
Plastics and rubber products	0.06	-0.02	0.09	0.01	-0.02
Electrical equipment, appliances, and components	0.05	-0.01	0.06	0.01	-0.01
Miscellaneous manufacturing	0.03	-0.02	0.05	0.01	-0.01
Primary metals	0.03	-0.01	0.04	0.00	-0.01
Wood and nonmetallic mineral products	0.02	-0.01	0.03	0.00	-0.01
Furniture and related products	0.01	-0.00	0.01	0.00	-0.00
Textile mills and apparel products	0.01	-0.00	0.01	0.00	-0.00
Total	18.30				

Note: This table reports the estimates of the total effects (i.e., the marginal change in sectoral GDP in the order of a billion dollars) for the case of monopolistic competition. The industries are arranged in descending order in terms of the total effects, which are in turn broken down into the effects on revenue and material input costs. They are further decomposed into four effects according to (25), namely, *p.effect* stands for the price effects, *q.effect* the quantity effects, *w.effect* the wealth effects, and *s.effect* the switching effects. Note that the first column in each panel indicates names of industries based on the segmentation given in Table B.2.

Table 8: Responsiveness of Sectoral GDP: Oligopolistic Competition (in Billions of U.S. Dollars)

Industry	Total Effect	Effects on Revenue		Effects on Material Cost	
		p.effect	q.effect	w.effect	s.effect
Transportation	0.00	0.05	-0.05	-0.00	0.00
Miscellaneous manufacturing	0.00	0.00	-0.00	0.00	-0.00
Petroleum and coal products	0.00	0.04	-0.04	-0.00	0.00
Electrical equipment, appliances, and components	0.00	-0.00	0.00	0.00	-0.00
Textile mills and apparel products	0.00	0.00	-0.00	0.00	-0.00
Furniture and related products	-0.00	0.00	-0.00	0.00	-0.00
Primary metals	-0.00	0.00	-0.00	0.00	-0.00
Wood and nonmetallic mineral products	-0.00	0.00	-0.00	0.00	-0.00
Plastics and rubber products	-0.00	0.00	-0.00	0.00	-0.00
Oil and gas extraction and mining	-0.00	0.02	-0.02	-0.00	-0.00
Fabricated metal products	-0.00	0.00	-0.00	0.00	-0.01
Chemical products	-0.00	0.03	-0.03	0.01	-0.01
Machinery	-0.00	0.00	-0.00	0.01	-0.01
Paper products and related services	-0.00	0.00	-0.00	0.00	-0.01
Educational services	-0.00	0.00	-0.00	0.01	-0.01
Food and beverage and tobacco products	-0.00	0.06	-0.06	0.00	-0.01
Motor vehicles, bodies and trailers, and parts	-0.01	0.00	-0.00	0.02	-0.02
Administrative and waste management	-0.01	0.02	-0.02	0.01	-0.02
Other transportation equipment	-0.01	-0.01	0.01	0.01	-0.02
Accommodation and food services	-0.02	0.05	-0.05	0.00	-0.02
Construction	-0.03	0.13	-0.13	0.01	-0.04
Retail Trade	-0.04	0.09	-0.09	0.07	-0.11
Information	-0.05	-0.02	0.02	0.08	-0.13
Wholesale trade	-0.09	0.26	-0.26	0.08	-0.17
Professional services	-0.26	-0.07	0.07	0.21	-0.47
Computer and electronic products	-0.71	-1.27	1.27	0.06	-0.77
Total	-1.23				

Note: This table reports the estimates of the total effects (i.e., the marginal change in sectoral GDP in the order of a billion dollars) for the case of oligopolistic competition. The industries are arranged in descending order in terms of the total effects, which are in turn broken down into the effects on revenue and material input costs. They are further decomposed into four effects according to (25), namely, *p.effect* stands for the price effects, *q.effect* the quantity effects, *w.effect* the wealth effects, and *s.effect* the switching effects. Note that the first column in each panel indicates names of industries based on the segmentation given in Table B.2.

G.3.2 Macro and Micro Complementarities

Tables 9 and 10 display the full results for changes in the sectoral price and material cost indices, along with estimates for the macro and micro complementarities. In these tables, the industries are arranged in the order consistent with Tables 7 and 8. Table 9 summarizes the results for monopolistic competition, while Table 10 shows those for oligopolistic competition.

Table 9: Changes in Sectoral Output Price and Material Cost Indices: Monopolistic Competition

Industry	h_i^L	$h_{i,n}^M$	$\frac{dP_i^{M*}}{d\tau_n}$	$\bar{\lambda}_{i\cdot}^L$	$\bar{\lambda}_{i\cdot}^M$	$\frac{dP_i^*}{d\tau_n}$
Professional services	19.41	0.16	-2.95	1.14	0.18	-0.58
Wholesale trade	15.96	0.11	-2.17	2.55	0.07	-0.23
Retail Trade	16.60	0.10	-1.97	1.36	0.13	-0.31
Construction	7.49	0.04	-0.88	2.27	0.16	-0.22
Information	12.22	0.13	-2.34	0.82	0.08	-0.21
Motor vehicles, bodies and trailers, and parts	6.54	0.06	-1.06	0.77	0.15	-0.19
Computer and electronic products	4.07	1.23	-18.20	0.13	0.04	-0.78
Transportation	16.86	0.05	-1.24	1.63	0.11	-0.20
Accommodation and food services	14.04	0.06	-1.40	0.65	0.11	-0.18
Administrative and waste management	18.79	0.11	-2.26	0.71	0.06	-0.16
Food and beverage and tobacco products	5.88	0.02	-0.56	0.56	0.09	-0.07
Other transportation equipment	5.52	0.09	-1.51	0.31	0.08	-0.13
Educational services	16.92	0.11	-2.25	0.29	0.04	-0.11
Machinery	6.85	0.07	-1.26	0.34	0.06	-0.08
Chemical products	3.69	0.02	-0.47	0.47	0.06	-0.05
Petroleum and coal products	1.61	0.00	-0.10	0.53	0.03	-0.02
Fabricated metal products	3.85	0.03	-0.54	0.26	0.08	-0.05
Oil and gas extraction and mining	6.59	0.02	-0.57	0.38	0.05	-0.04
Paper products and related services	8.33	0.06	-1.13	0.30	0.05	-0.06
Plastics and rubber products	6.05	0.04	-0.79	0.20	0.05	-0.04
Electrical equipment, appliances, and components	4.45	0.06	-0.98	0.13	0.04	-0.05
Miscellaneous manufacturing	10.65	0.11	-1.91	0.16	0.01	-0.03
Primary metals	2.06	0.01	-0.20	0.25	0.05	-0.02
Wood and nonmetallic mineral products	4.55	0.03	-0.57	0.16	0.03	-0.02
Furniture and related products	5.54	0.04	-0.76	0.09	0.02	-0.02
Textile mills and apparel products	4.42	0.03	-0.62	0.07	0.02	-0.02

Note: This table displays the estimates for the macro and micro complementarities for those industries listed in Table 7. The subscript n on the variables denotes the targeted industry, i.e., the computer and electronic product industry.

Table 10: The Changes in Sectoral Price Indices and Material Cost Indices: Oligopolistic Competition

Industry	h_i^L	$h_{i,n}^M$	$\frac{dP_i^{M*}}{d\tau_n}$	$\bar{\lambda}_{i\cdot}^L$	$\bar{\lambda}_{i\cdot}^M$	$\frac{dP_i^*}{d\tau_n}$
Transportation	1.47	0.00	0.02	0.17	0.02	0.01
Miscellaneous manufacturing	1.35	0.03	-0.34	0.03	0.00	0.00
Petroleum and coal products	0.26	0.00	0.01	0.13	0.01	0.01
Electrical equipment, appliances, and components	0.60	0.02	-0.20	0.02	0.01	-0.00
Textile mills and apparel products	0.55	0.01	-0.08	0.01	0.01	0.00
Furniture and related products	0.68	0.01	-0.10	0.01	0.00	0.00
Primary metals	0.34	0.00	-0.01	0.06	0.01	0.00
Wood and nonmetallic mineral products	0.63	0.01	-0.06	0.05	0.01	0.00
Plastics and rubber products	0.93	0.01	-0.09	0.04	0.01	0.00
Oil and gas extraction and mining	0.93	0.00	0.01	0.10	0.02	0.00
Fabricated metal products	0.58	0.01	-0.08	0.07	0.02	0.00
Chemical products	0.71	0.01	-0.05	0.17	0.02	0.01
Machinery	0.85	0.02	-0.22	0.08	0.01	0.00
Paper products and related services	0.95	0.01	-0.14	0.05	0.01	0.00
Educational services	1.93	0.02	-0.20	0.06	0.01	0.00
Food and beverage and tobacco products	0.85	0.00	-0.02	0.18	0.03	0.01
Motor vehicles, bodies and trailers, and parts	0.59	0.01	-0.15	0.10	0.02	0.00
Administrative and waste management	1.92	0.02	-0.15	0.15	0.02	0.00
Other transportation equipment	0.59	0.02	-0.30	0.06	0.02	-0.00
Accommodation and food services	1.71	0.01	-0.02	0.16	0.03	0.01
Construction	0.96	0.01	-0.04	0.26	0.03	0.01
Retail Trade	1.57	0.01	-0.13	0.29	0.03	0.01
Information	1.39	0.03	-0.40	0.23	0.03	-0.00
Wholesale trade	1.50	0.02	-0.27	0.38	0.01	0.01
Professional services	1.98	0.03	-0.37	0.31	0.05	-0.00
Computer and electronic products	0.58	1.08	-15.90	0.06	0.02	-0.30

Note: This table displays the estimates for the macro and micro complementarities for those industries listed in Table 8. The subscript n on the variables denotes the targeted industry, i.e., the computer and electronic product industry.

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