Econometric Evaluation of Industrial Policies in Macroeconomic Models of Strategic Interactions and Production Networks*

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Abstract

This paper studies the econometric evaluation of industrial policies — policies targeted at particular industries — when industries are linked through production networks and firms in each industry engage in strategic interactions. I develop a general equilibrium model with these two features to define a causal policy effect as a *ceteris paribus* difference in outcome variables across different policy regimes. The key mechanism of my model is that when firm-level production functions exhibit constant returns to scale, policy effects are mediated by changes in firms' marginal profits not only through adjustments of their own actions but also via those of competitors' actions (i.e., strategic complementarities), and that both of these changes are compounded by the production network. To identify such policy effects, I develop a new procedure that first characterizes them in terms of sector-level variables and firm-level variables — firm-level sufficient statistics, and then recovers these comparative statics with the aid of the control function approach of the industrial organization literature. Using my framework, I examine the causal impact of one part of the U.S. CHIPS and Science Act of 2022 on GDP. My estimation predicts that accounting for firms' strategic interactions even flips the sign of the policy effect, highlighting the policy relevance of strategic interactions in the presence of a production network.

Keywords: Policy evaluations, Industrial policies, Strategic interactions, Production networks, Identification

JEL Codes: E61, E65, F13, F41, L13, L16

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1 Introduction

Over the past few decades, industrial policies — policies that are purposefully targeted at particular industries — have been at the forefront of economic policy debates in a range of contexts.[1] In recent years, U.S. tariffs, primarily on imports from China, were raised by about 14 percentage points to an average of almost 16.6%.[2] In addition, the CHIPS and Science Act of 2022 aims to make nearly $53 billion of investment in the semiconductor industry.[3] Of great importance for policymakers are questions as to how much financial support should be provided to which industries. How large will the causal effects of subsidizing particular industries on an economy's well-being be?

This paper develops a framework that can be used to answer this type of policy question in macroeconomics, building on the econometric policy evaluation literature (e.g., Heckman and Vytlacil 2005, 2007). The policy-invariant data generating process entertained in this paper is disciplined by an economic model that arises from two strands of the recent literature. The first one is the literature exploring oligopolistic competition models to successfully analyze market concentration and firms' markups in ranging categories of products.[4,5] Moreover, oligopolistic competition has also proved to be plausible in explaining a number of salient macroeconomic empirical regularities — for instance, an incomplete pass-through of a price shock (Atkeson and Burstein 2008) and market power (De Loecker et al. 2020, 2021). The other strand is the literature studying the role of production networks in macroeconomic outcomes — for instance, business cycle (Horvath 1998, 2000), aggregate fluctuations (Acemoglu et al. 2012), and misallocation (Baqaee and Farhi 2020). While the existing policy analysis looks at these features separately,[6] the policy implications of their joint existence are left unexplored. Hence, this paper investigates the causal effects of industrial policies in a macroeconomic model with these two features.

What are the points of using an economic model in causal policy analysis? There has been no

[1] For a recent review of industrial policies, see Rodrik (2008), Juhász et al. (2023), and Juhász and Steinwender (2023).

[2] See Fajgelbaum et al. (2020).

[3] See Appendix G.1 for the detail.

[4] A short list of prominent examples includes, among many others, automobiles (Berry et al. 1995), ready-to-eat cereal (Nevo 2001), aircraft (Benkard 2004), and cement (Ryan 2012).

[5] The primary focus of this paper is on understanding the "effects of the causes," a distinct task from investigating the "causes of the effects" (Holland 1986; Heckman 2005, 2008). For the latter, the modeling choice of this paper is motivated by the voluminous literature documenting the empirical salience of a sectoral production network and firms' strategic interactions in each sector, as explained in this paragraph.

[6] See Liu (2019) and Lashkaripour and Lugovskyy (2023) for industrial policies in an economy with a production network, and Gaubert et al. (2021) for the effects of tariffs in an oligopolistic environment.

consensus on the definitions of causal effects and causality.[7] While the empirical treatment-effect approach has recently been rolling in the macroeconomic literature, it cannot generally provide an answer to macroeconomic policy questions at least for two reasons.[8] First, this methodology is typically established under the two premises: i) units being studied are randomly split into those that are exposed to an intervention (the treatment group) and those that are not (the control group), and ii) there are no interferences between these two groups (Rubin 1980). This setup, however, precludes the firms' strategic interactions, peer effects through a production network, and general equilibrium feedback, all of which are at the heart of the macroeconomic policy analysis.[9] Moreover, this paradigm may not be compatible with macroeconomic policy questions because policymakers may want to manipulate policy variables virtually for all units at once, in which case everyone in the population is "treated" — a universal treatment.[10] Second, the reduced-form treatment-effect estimates cannot generally be transported to a different policy environment, thereby being unable to inform policymakers of the policy effects before the actual implementation — *ex ante* policy evaluations. With the aid of an economic model, the policy parameter put forth in this paper circumvents these shortcomings while retaining a causal interpretation as a *ceteris paribus* difference in outcomes across different policy regimes.[11]

In order to define a causal policy parameter, I first develop a general equilibrium model of a multisector economy with a sectoral production network featuring firms' oligopolistic competition in each sector. The causal policy effect is then defined as the change in GDP due to an industrial policy with other things being equal, i.e., a *ceteris paribus* causal effect (Marshall 1890). The key

[7] See Granger (1969) and Sims (1972) for the case of time-series economic analysis. Hoover (2001) discusses various other concepts of causality in macroeconomics. A parallel line of research is the graphical approach in computer science (e.g., Pearl 2009). Also, Cartwright (2004) provides a review from the philosopher's standpoints.

[8] There can be many other reasons for this. It is essential to emphasize that the notion of "randomization" is not *necessary* for *defining* a causal policy effect; it is only *useful* for *identifying* it. See Heckman and Vytlacil (2007) and Deaton (2010) for discussion. See also Lane (2020) and Juhász et al. (2023) for a review of empirical studies of industrial policies.

[9] In Section 2.7, I make the case that in the presence of a production network and firms' strategic interactions, even if a policy is targeted at a particular industry, its effect propagates along the production network while being amplified or weakened by the firms' strategic interactions in each sector. Moreover, this insight opens a door for the policymaker to leverage these interaction effects in designing optimal policies (see, e.g., Ballester et al. 2006; Calvó-Armengol et al. 2009).

[10] To streamline the exposition, the main text of this paper focuses on an extreme scenario of an industrial policy, wherein only a single sector experiences a policy change, in the main text. Universal treatments — the other edge of the spectrum — can also be considered in my framework, as discussed in Appendix D.4.

[11] *Ceteris paribus* causal effects are one of the most widely accepted notions of causal effects in economics ever since Alfred Marshall (Marshall 1890). It is worth stressing that treatment effects are a special case of this class of causal effects. My paper puts forth an alternative to treatment effects, which is another special case of *ceteris paribus* causal effects.

mechanism of my model is that when firms' production functions exhibit constant returns to scale, the production network compounds not only the responses of firms' marginal profits with respect to their own choices but also those with respect to competitors' (i.e., strategic complementarities), with the latter being absent in monopolistic models. To further study the empirical relevance of this mechanism, I take my model to real-world data. Identifying the policy effect, however, is challenging because in strategic interaction models, individual firms have the potential to exert a nonnegligible influence over sectoral outcomes; thus, the policy parameter cannot be characterized by aggregate variables alone. This invalidates the aggregate sufficient statistics approach, a method increasingly used in recent macroeconomics and international trade literature.[12] This paper exploits widely used firm-level data and proposes a new sequential procedure that identifies the policy effect in terms of the individual firms' responses, which I call *firm-level sufficient statistics*. This identification approach is constructive, so that a nonparametric estimator for the policy effect can be obtained by reading the procedure in reverse.[13] I then consider one part of the U.S. CHIPS and Science Act — corresponding to an additional subsidy on the semiconductor industry — and compare the estimate based on oligopolistic competition to that based on monopolistic competition. I find that accommodating firms' strategic behaviors reverses the sign on the estimate for the policy effect from positive to negative, with the magnitude roughly the same. This result echoes the policy relevance of (not) accounting for strategic competition in the presence of a production network.

My model builds on Liu (2019) to study a general equilibrium multisector model of a production network by assuming that each sector is populated by a finite number of heterogeneous oligopolistic firms, thereby firm-level markups being endogenously variable. The government helps firms to purchase sectoral intermediate goods through an ad-valorem subsidy specific to the purchaser sector. The policy effect is defined as the *ceteris paribus* change in GDP due to a shift in the level of the sector-specific subsidy (i.e., an industrial policy). I demonstrate that the policy effect is characterized by sectoral comovements (or pass-through), which depend on sectoral measures of market competitiveness compounding through the production network across sectors. The sectoral competitiveness measure comprises not only the responsiveness of firms' marginal profits with respect

[12]See, for example, Arkolakis et al. (2012), Adão et al. (2017), Arkolakis et al. (2019), and Adão et al. (2020) for applications in the context of macroeconomics. See Chetty (2009) and Kleven (2021) for a general idea of the sufficient statistics approach. This idea is also known as *Marschak's Maxim* in the econometric policy evaluation literature (Heckman 2005, 2008, 2010; Heckman and Vytlacil 2007).

[13]See Matzkin (2013) for constructive identification and nonparametric estimation.

to their own choices but also those with respect to competitors' (i.e., strategic complementarities). The size and sign of this measure hinge on the specification of the market competition and have the potential to significantly change or even revert the sectoral comovement, which may in turn alter the policy effect. This observation points to the practical importance of jointly accommodating a production network and firms' strategic interactions, a feature that has attracted little to no attention in the existing literature. This moreover motivates the identification of the policy effect under a minimal set of assumptions, so that the policy analysis can remain agnostic about the configuration of the market competition, which is generally unknown *a priori* to the policymaker.

The identification analysis of this paper first rewrites the causal policy effect in terms of sector- and firm-level comparative statics. To recover the latter, I then adopt techniques from the literature on production function identification and estimation (e.g., Ackerberg et al. 2015; Gandhi et al. 2019). This requires three sets of additional assumptions. The first assumption restricts the firm-level production function to exhibit Hicks-neutral productivity. The second set of assumptions is concerned with the sectoral aggregator: it takes the form of a homothetic demand system with a single aggregator (HSA; Matsuyama and Ushchev 2017), and the single aggregator is exchangeable in its argument. Under this specification of the sectoral aggregator, I show that the firms' equilibrium choices depend on competitors' productivities only through some aggregates. The last set of assumptions, combined with the first two sets, ensures that this equilibrium quantity function is "invertible" in the firm's own productivity. Nevertheless, I further demonstrate that these assumptions are flexible enough to accommodate the specifications commonly used in the macroeconomics literature. This identification analysis is constructive, so that a nonparametric estimator for the policy effect can be obtained by reading these procedures in reverse order.

My framework differs from the conventional structural approach for counterfactual predictions in macroeconomics in four important ways. For instance, policy analysis in the computational general equilibrium models typically proceeds in five steps: (i) specify models in detail, which often involves a large number of parameters; (ii) preset some parameter values on the basis of prior or external knowledge (e.g., parameter estimates from the preceding research); (iii) simulate (or calibrate) the model to match the data in terms of some criteria of researcher's choice, yielding values for the remaining parameters with abstracting away from any random variation in the data generating process; (iv) conditioning on the obtained parameter values, simulate again the model under a counterfactual state; and (v) compare outcomes generated by these two simulations. In contrast, (i)' my approach specifies the model primitives only up to classes of functions, and recovers only a limited number of comparative statics, thereby the subsequent empirical analysis being more robust against misspecification and less computationally burdensome. (ii)' Estimation in my framework does not require any external information, and thus can be performed in a self-contained fashion, obviating the arbitrariness inherent to the parameter preselection.[14] (iii)' Loss functions in my estimation naturally arise from the preceding identification argument, which eliminates the arbitrariness in the choice of the estimation criteria. (iv)' My approach is designed to directly recover the causal effect in a single procedure with admitting sampling variation.[15]

Finally, in order to quantify the empirical relevance of firms' strategic forces compounding through the production network, I bring my model to the U.S. firm-level data and evaluate the economic impacts of the CHIPS and Science Act, which selectively promotes the semiconductor industry and was enacted in 2022. I consider a hypothetical policy experiment of shifting the ad-valorem subsidy on the computer and electronic products industry from the 2021 level, which is 15.03%, to an alternative level of 16.03% — equivalent to $0.48 billion. The estimate accounting for strategic interactions as well as the production network predicts that GDP falls by $2.17 billion, while the estimate based on monopolistic competition under the production network suggests an increase of $19.34 billion. Comparing these two estimates underlines the policy relevance of correctly specifying market competition in the presence of a production network.[16]

To better understand the mechanism behind this, I further analyze the responsiveness of GDP at the 2021 subsidy with an industry-level breakdown. I show that the comovements of the sectoral variables are substantially weakened by the firm's strategic forces accruing through the production network. This observation is further investigated by the decomposition of the aggregate fiscal multiplier into the network multiplier and sectoral fiscal multiplier. In my estimation, the network multiplier reduces from 22.39 in the monopolistic case to 1.40 in the oligopolistic case. Likewise, the

[14] The advantage of this feature becomes particularly acute when the model under consideration has never previously been studied in the literature, as is the case with this paper.

[15] This provides a ground for statistical hypothesis testing pertaining to the causal effect.

[16] Although my model is developed without reference to any particular functional-form assumptions, and thus its implications apply fairly generally, the subsequent empirical analysis is constrained by the data limitation and additional identification assumptions, as is the case with any empirical analysis. In light of this, my empirical estimates may not necessarily be an accurate gauge of the "actual" policy effects. Rather, the empirical illustration of this paper is tailored to examine the quantitative relevance of the wedge in policy effects, created by jointly accommodating firms' strategic interactions and a production network.

sectoral fiscal multiplier varies from 3.74 under monopolistic competition to -3.07 under oligopolistic competition. These two estimates jointly account for the difference in the aggregate fiscal multiplier, which implies that the accumulated firm's strategic forces have a dampening effect on the policy spillovers along the production network.[17]

1.1 Related literature

This paper contributes to four strands of the literature. First, the framework put forth in this paper is directly related to the literature on *ex ante* counterfactual predictions of economic shocks (e.g., trade costs, productivity), such as Arkolakis et al. (2012), Melitz and Redding (2015), Adão et al. (2017), Feenstra (2018), and Adão et al. (2020). My framework, though, marks a distinction in two ways. First, the preceding papers are based on perfectly competitive or monopolistic firms, whereas my paper explicitly accounts for firms' strategic interactions. Second, the existing literature is mostly concerned with directly expressing an aggregate outcome in terms of aggregate variables — aggregate sufficient statistics. In contrast, my approach first decomposes the policy parameter into firm-level variables — firm-level sufficient statistics, and identifies these variables from the observables, which in turn recovers the policy parameter.

Second, this paper advances the literature on industrial policies on both theoretical and empirical grounds. The theory of optimal industrial policy in a multisector environment is explored in Itskhoki and Moll (2019) and Liu (2019) for exogenous market distortions; in Lashkaripour and Lugovskyy (2023) for endogenous but constant markups; and in Bartelme et al. (2021) for endogenously varying market distortions. In my model, the market distortions arise from oligopolistic competition and thus can endogenously vary according to the strategic interactions. On the empirical front, my paper intersects with the treatment effect literature. Among many others, Criscuolo et al. (2019) discuss the "reduced-form" causal effects of an industrial policy.[18] The causal interpretation of their policy parameter, however, is limited to those units that have experienced

[17] It should be remarked that whether the accumulated firm's strategic forces have a dampening or amplifying effect is essentially an empirical matter, as elaborated in Section 2.7. Thus, the result of this paper should not be taken as a guarantee that the firm's strategic interactions always weaken the policy spillovers, a caution drawn in Section 5.1. See also footnote 16.

[18] A rapidly expanding body of literature has deployed natural or quasi-experiments to study the causal effects of industrial policies. For example, Juhász (2018) and Lane (2021) exploit, respectively, the Napoleonic blockade against Britain afforded to French cotton spinners and President Park's assassination to define their causal effects. For a more thorough review, see Lane (2020) and Juhász et al. (2023).

(exogenous) changes in the eligibility of receiving the policy. From the perspective of a policy-maker who considers the well-being of a society as a whole, such a locally tailored notion of "causal effect" might not be of central interest. In the spirit of the econometric policy evaluation literature (e.g., Heckman and Vytlacil 2007), this paper studies an alternative policy parameter that is both economically interesting (i.e., inclusive of strategic interactions, peer effects through production networks and general equilibrium feedback) and causal in the sense of Marshall (1890).[19] In a similar vein, Rotemberg (2019) investigates the aggregate effects, taking into account the general equilibrium effects, and Sraer and Thesmar (2019) derive formulas that are able to counterfactually expand firm-level treatment effects to the aggregate level. Their methodologies are, however, essentially *ex post*, whereas my framework can be used for *ex ante* policy evaluations. Furthermore, the identification approach of this paper supplements the econometric policy evaluation literature by exploiting variations in firms' productivities, instead of those in policy variables.

Third, this paper contributes to the literature documenting the empirical relevance of endogenous firms' markups, such as oligopolistic competition and non-constant-elasticity-of-substitution demand function (e.g., Atkeson and Burstein 2008; Amiti et al. 2014; Edmond et al. 2015; Arkolakis et al. 2019; Gaubert and Itskhoki 2020; De Loecker et al. 2021; Azar and Vives 2021). I connect this line of research to the macroeconomic literature on production networks (Baqaee and Farhi 2020, 2022; Bigio and La'O 2020).[20] Specifically, I show that the sectoral comovements are traced out by the combination of the within-sector interactions summarizing firms' strategic complementarities (what I refer to as *micro complementarities*) and the between-sector interactions compounding the micro complementarities along the production network (what I call *macro complementarities*).[21] These features are absent in the existing literature on industrial policies under monopolistic competition, such as Liu (2019) and Lashkaripour and Lugovskyy (2023). Grassi (2017) also studies the case of oligopolistic competition, but his focus is on positive analysis under a parametric specification of production and demand functions. My paper is concerned with evaluating the policy

[19] The policy parameter proposed in this paper is inspired by the policy-relevant treatment effects (Heckman and Vytlacil 2001, 2005, 2007). See Section 2.6.

[20] These works are principally interested in characterizing welfare loss due to misallocation in the presence of production networks: they start from an efficient economy (i.e., they assume away from an initial state of market distortions) and then focus on the consequence of adding a policy as a source of distortion. My paper admits market distortions in the initial state of the economy, including the policy itself, and then investigates a welfare-improving policy prescription.

[21] These terminologies draw from Klenow and Willis (2016) and Alvarez et al. (2023).

effects with a minimal set of parametric assumptions.

Lastly, outside the domain of the macroeconomics literature, my method is tightly linked to the industrial organization literature on the identification of firms' production functions. In particular, the existing work (e.g., Olley and Pakes 1996; Levinsohn and Petrin 2003) has customarily assumed perfect competition (e.g., Ackerberg et al. 2015; Gandhi et al. 2019) or monopolistic competition (e.g., Kasahara and Sugita 2020). My paper applies these approaches to the case of strategic interactions by adapting the notion of sufficient statistics for competitors' decisions and productivities. There have been recent studies that adopt analogous approaches, such as Blum et al. (2023), Ackerberg and De Loecker (2024), Doraszelski and Jaumandreu (2024).[22] Their methodologies are established under the premise that firm-level prices and/or quantities are observable, and recover the entire shapes of the production function. In my framework, in contrast, revenue is the only available firm-level outcome variable, and only the equilibrium values of the responsivenesses of the production and demand functions are recovered.

2 Model

The goal of this section is to define a causal policy parameter that *i*) internalizes firms' strategic interactions, peer effects through a production network, and general equilibrium effects; *ii*) compares aggregate variables between the baseline (e.g., status quo) environment and an alternative policy regime; and *iii*) can be used for *ex ante* predictions.

To define such a parameter, this section spells out a general equilibrium closed-economy multisector model of oligopolistic competition among heterogeneous firms under a sectoral production network. The model is akin to Liu (2019), who considers the optimal policy in the presence of a production network when there are exogenous market distortions. I depart from his setup by replacing the exogenous wedges with endogenously variable firms' markups. In my model, the markups can arise from oligopolistic competition among a finite number of heterogeneous firms and the non-CES specification of the residual inverse demand functions faced by the firms.[23]

[22] Doraszelski and Jaumandreu (2019), Brand (2020), and Bond et al. (2021) draw attention to the risk of simply applying the standard control function approach to the case of oligopolistic competition, but they do not provide a methodology to deal with the strategic interactions in recovering the firm's production function.

[23] Arkolakis et al. (2019) consider a model of variable markups under monopolistic competition with a flexible class of non-CES demand functions. My paper adds an additional source of endogenous markups, strategic interactions.

It is postulated that as a way to neutralize the market distortions induced by the endogenous markups, the government manipulates sector-specific policy instruments $\boldsymbol{\tau} := \{\tau_i\}_{i=1}^N$, where τ_i is understood as an ad-valorem subsidy on sector i's purchase of sectoral intermediate goods if it is positive, and a tax otherwise.[24,25] I restrict my attention to the short-run policy effects, abstracting away from the firms' entry and exit decisions (extensive margins).[26]

The model is static and there is no uncertainty. The economy consists of a representative household, a government, and N production sectors, indexed by $i \in \mathbf{N} := \{1, \ldots, N\}$. Each sector i is populated by a finite number N_i of heterogeneous oligopolistic firms, indexed by $k \in \mathbf{N}_i := \{1, \ldots, N_i\}$, each of which produces a single horizontally differentiated good. There is a sectoral aggregator that aggregates the firms' products in the same sector into a single intermediate good. Sectoral goods are further combined to produce a final consumption good. Both the final and sectoral aggregators operate in perfectly competitive markets.

Firm-level production uses labor and sectoral intermediate goods as inputs. The transaction of sectoral goods by firms shapes the input-output linkages, denoted by $\Omega := [\omega_{i,j}]_{i,j \in \mathbf{N}}$ with $\omega_{i,j}$ being the share of sector j's intermediate good in sector i's expenditure for inputs.[27]

2.1 Market Distortions and Industrial Policy

Let $\boldsymbol{\tau}^0$ denote the policy regime currently in place. Suppose that the policymaker wishes to learn how much GDP would increase or decrease by moving to an alternative policy regime $\boldsymbol{\tau}^1$. That is, the current policy $\boldsymbol{\tau}^0$ might not yet be optimized but rather can be a part of the market distortions, and the policymaker is looking for a way to improve GDP.[28] In particular, the policymaker is interested in changing only the subsidy on sector n while keeping the subsidies on the other sectors

[24] I abstract from other policy measures such as technology adoption, direct price regulation, and antitrust law.

[25] While I focus on subsidies for the purchase of sectoral intermediate goods that are specific to purchasing sectors, the subsequent analysis naturally extends to the case of sector-input-specific subsidies (including labor-input-specific subsidies), as considered in Liu (2019).

[26] This simplifying assumption is often posited in the literature (e.g., Mayer et al. 2021; Wang and Werning 2022). The short-run scope can be rationalized by acknowledging that firms' entry and exit decisions generally invoke a considerable amount of cost and time. Technically, accommodating the endogenous choice of entry and exit requires another layer of the fixed-point problem concerning the free-entry condition, which in general is very hard to solve. In particular, given that the number of firms in my setup is finite, it is not even possible to consider differentiation of the free-entry condition. Extending the theory to a long-run analysis is left for future work.

[27] Analogously, I write $\boldsymbol{\omega}_L := [\omega_{i,L}]_{i=1}^N$ with $\omega_{i,L}$ indicating the labor share in sector i's cost.

[28] A similar setup is considered in Bigio and La'O (2020).

(i.e., an industrial policy on sector n).[29] Thus, the policy parameter is defined as the change in GDP due to a policy reform from τ_n^0 to τ_n^1, which is denoted by $\Delta Y(\tau_n^0, \tau_n^1)$.

To grant this policy parameter a causal interpretation, I impose the following assumptions.

Assumption 2.1 (Policy Invariance)**.** *Throughout the policy reform from $\boldsymbol{\tau}^0$ to $\boldsymbol{\tau}^1$, (i) the index set for sectors $\mathbf{N}$, (ii) the index set for firms in each sector $\mathbf{N}_i$, (iii) each sectoral aggregator, (iv) every firm-level production function in each sector, and (v) the shape of the input-output linkages $\boldsymbol{\omega}_L$ and Ω do not change.*

Assumption 2.1 (i) is consistent with the focus of this study on ad-valorem subsidies, excluding other competition interventions. Invariance condition (ii) assumes away from endogenous entry and exit in response to the policy change, which is implied by the short-run scope of this paper. Conditions (iii) and (iv) jointly mean that the policy reform does not alter the firms' operating environments, which in turn rules out both direct and indirect impacts of the policy reform on firms' productivities.[30] Part (v) states that the input-output linkages $\boldsymbol{\omega}_L$ and Ω do not reshape in reaction to the policy reform. This again accords with the scope of my analysis and also resonates with the existing literature that assumes the production network to be stable over a period of time (e.g., Baqaee and Farhi 2020).

2.2 Household

Consider a representative household that consumes a final consumption good, inelastically supplies labor across sectors. The household owns all firms so that it receives firms' profits as dividends. The household derives utility only from consumption of the final good, with the utility function being the standard.

Assumption 2.2 (Utility Function)**.** *The consumer's utility function is strictly monotonic and continuously differentiable in the final consumption good.*

Assumption 2.2 means that there exists a one-to-one mapping between the utility level and consumption of the final good. Based on this preference, the household chooses the utility-maximizing

[29] That is, $\tau_n^0 \neq \tau_n^1$ and $\tau_{n'}^0 = \tau_{n'}^1$ for all $n' \neq n$. In the example of the CHIPS Act, sector n corresponds to the semiconductor industry.

[30] See Bartelsman and Doms (2000) and Syverson (2011).

quantity of the final consumption good subject to the binding budget constraint:

$$C = WL + \Pi - T, \tag{1}$$

where Π is firm's total profit, and T indicates the tax payment to the government in the form of a lump-sum transfer. I let the price index of the final consumption good be the numeraire.

2.3 Technologies

Economy-wide and sectoral aggregations. The economy-wide aggregator collects sectoral intermediate goods to produce a final consumption good Y using the production function $\mathcal{F} : \mathbb{R}_+^N \to \mathbb{R}_+$, that is,

$$Y = \mathcal{F}(\{X_i\}_{i \in \mathbf{N}}), \tag{2}$$

where X_i represents sector i's intermediate good used for the production of the final consumption good. In each sector $i \in \mathbf{N}$, firm-level products are aggregated into a single sectoral good Q_i according to

$$Q_i = F_i(\{q_{ik}\}_{k \in \mathbf{N}_i}), \tag{3}$$

where $F_i : \mathbb{R}_+^{N_i} \to \mathbb{R}_+$ represents the sector-specific aggregator that collects firms' products in sector i and q_{ik} denotes the quantity of firm k's product.[31] Both the economy-wide and sectoral aggregators operate in perfectly competitive markets under the following standard assumptions.

Assumption 2.3 (Economy-Wide and Sectoral Aggregators)**.** *(i) The economy-wide aggregation function $\mathcal{F}(\cdot)$ is increasing and concave in each of its arguments. (ii) For each $i \in \mathbf{N}$, the sectoral aggregator $F_i(\cdot)$ is a) twice continuously differentiable and b) increasing and concave in each of its arguments.*

Each sectoral aggregator solves the cost-minimization problem, which delivers the price index of sector i's good P_i. A sectoral aggregator serves two purposes. First, it is a useful modeling device

[31] To economize on notation, I use the same notation q_{ik} to mean the demand for firm k's good and firm k's output quantity. This is innocuous as the sectoral aggregator is the sole buyer of firms' output.

that allows me to unite firms' differentiated goods into a single homogeneous good (Bigio and La'O 2020; La'O and Tahbaz-Salehi 2022).[32] This helps isolate the firm's input choices from the strategic considerations. Second, from the perspective of an individual firm, the sectoral aggregator acts as a "demand function" through which the strategic interactions between firms are mediated.

Firm-level production. The firm-level production process combines labor and material inputs, where the latter is a composite of sectoral intermediate goods along the production network. It is assumed that all inputs are variable (i.e., firms do not incur fixed costs). To focus on the short-run behavior, I do not model the firms' entry decisions; instead, I assume that each sector is populated by an exogenously fixed number of firms that are heterogeneous in productivities.

In the output market of each sector, firms engage in a Cournot competition of complete information, while they are perfectly competitive in the input markets. Thus, each firm first chooses its output quantity so as to maximize its profits in the Cournot-quantity competition, followed by input decisions based on cost-minimization problems under the constraint of output quantity.

The production technology for firm k in sector i is described by

$$q_{ik} = f_i(\ell_{ik}, m_{ik}; z_{ik}) \qquad with \qquad m_{ik} = \mathcal{G}_i(\{m_{ik,j}\}_{j \in \mathbf{N}}), \tag{4}$$

where q_{ik}, ℓ_{ik}, and m_{ik} denote, respectively, the quantity of gross output, labor input, and material input, z_{ik} is firm-specific productivity, $m_{ik,j}$ represents the input demand for sector j's intermediate good, and $f_i : \mathbb{R}_+^2 \to \mathbb{R}_+$ and $\mathcal{G}_i : \mathbb{R}_+^N \to \mathbb{R}_+$ indicate, respectively, the firm-level production technology and material aggregator, both of which are specific to the sector.[33] Note that $\mathcal{G}_i(\cdot)$ reflects the input-output linkages Ω. Notice moreover that both $f_i(\cdot)$ and $\mathcal{G}_i(\cdot)$ are only traced by sector index i, meaning that firms in the same sector i have access to the same production technologies up to the idiosyncratic heterogeneous productivity z_{ik}.[34]

[32] The economic content of this aggregation is that every buyer of goods from sector i purchases the same bundle of goods produced by the firms in that sector (Liu 2019).

[33] I abstract away capital accumulation in order to stick to a static environment. When bringing my model to the data, I interpret the firm's productivity z_{ik} as its overall production capacity, including capital assets. See Appendix B.3.5.

[34] This also implies that producer-side heterogeneity pertaining to product differentiation (e.g., quality) is encoded in the productivity term z_{ik}. In my setup, differentiated goods are produced by heterogeneous firms, so that the level at which product differentiation is defined is the same as that at which firm heterogeneity is defined. Thus, the notion of firm coincides with that of variety.

Assumption 2.4 (Firm-Level Production Functions). *For each sector $i \in \mathbf{N}$, both aggregators $f_i(\cdot)$ and $\mathcal{G}_i(\cdot)$ (i) display constant returns to scale, (ii) are twice continuously differentiable in all arguments, (iii) are increasing and concave in each of its arguments, and (iv) satisfy $f_i(0,0) = 0$ and $\mathcal{G}_i(\mathbf{0}) = 0$. Moreover, (v) for each firm $k \in \mathbf{N}_i$ in sector i, it holds that $\left(\frac{\partial f_i(\cdot)}{\partial \ell_{ik}}\right)^2 \frac{\partial^2 f_i(\cdot)}{\partial m_{ik}^2} + \left(\frac{\partial f_i(\cdot)}{\partial m_{ik}}\right)^2 \frac{\partial^2 f_i(\cdot)}{\partial \ell_{ik}^2} - 2\frac{\partial f_i(\cdot)}{\partial \ell_{ik}} \frac{\partial f_i(\cdot)}{\partial m_{ik}} \frac{\partial^2 f_i(\cdot)}{\partial \ell_{ik} \partial m_{ik}} < 0$ for all $(\ell_{ik}, m_{ik}) \in \mathbb{R}_+^2$.*

Assumptions 2.4 (i) – (iv) jointly state that the aggregators $f_i(\cdot)$ and $\mathcal{G}_i(\cdot)$ are neoclassical, an assumption employed in Bigio and La'O (2020).[35] Assumption (v) guarantees an interior solution for the firm's cost minimization problem.

Importantly, when a firm decides the quantity of output, it also takes into account its input decisions in a forward-looking way. Thus, the firm's decision problem proceeds backward in effect. First, taking the quantities of output and material input and sectoral price indices as given, the firm's optimal demand for sectoral intermediate goods is given by

$$\{m_{ik,j}^*\}_{j \in \mathbf{N}} \in \underset{\{m_{ik,j}\}_{j \in \mathbf{N}}}{\arg\min} \quad \sum_{j=1}^{N} (1 - \tau_i) P_j m_{ik,j} \qquad s.t. \quad \mathcal{G}_i(\{m_{ik,j}\}_{j \in \mathbf{N}}) \geq \bar{m}_{ik}, \tag{5}$$

where $m_{ik,j}^*$ denotes the optimal level of purchase of sector j's good, and $\bar{m}_{ik}$ indicates the level of material input corresponding to a given quantity of output. Note that the associated unit cost condition defines the cost index of material input P_i^M gross of the policy $\boldsymbol{\tau}$.

Second, taking the output quantity and input prices as given, the optimal input quantities for firm k in sector i are given by

$$\{\ell_{ik}^*, m_{ik}^*\} \quad \in \quad \underset{\ell_{ik}}{\arg\min} \left\{ \min_{m_{ik} | \ell_{ik}} \quad W\ell_{ik} + P_i^M m_{ik} \qquad \text{s.t.} \quad f_i(\ell_{ik}, m_{ik}; z_{ik}) \geq \bar{q}_{ik} \right\}, \tag{6}$$

where W denotes the wage[36] and $\bar{q}_{ik}$ is a given level of output quantity.[37] Implicit in this expression is the timing assumption that every firm chooses its labor input prior to material input. An

[35] Although Assumption 2.4 (i) might appear to be restrictive at first glance, a number of applied studies have found that the constant-returns-to-scale (CRS) production function serves as a good approximation (e.g., Basu and Fernald 1997; Syverson 2004; Foster et al. 2008; Bloom et al. 2012). In fact, the CRS production functions are customarily assumed by recent works on firm-level macroeconomic models — for example, Atkeson and Burstein (2008) in an oligopolistic competition model of international trade and Baqaee and Farhi (2022) in a multi-country model of international trade in the presence of production networks.

[36] Since the labor force is assumed to be frictionlessly mobile across sectors, the wage W is common for all sectors.

[37] Input decisions (5) and (6) are separated purely for expositional purposes. These two problems could be collapsed.

economic intuition behind this is that labor is easier to obtain compared to material.[38] This assumption is imposed only for the purpose of econometric analysis (see, e.g., Gandhi et al. 2019), and the quantitative implication remains the same even if it is replaced by a simultaneous choice of labor and material inputs (Ackerberg et al. 2015).

Third, taking the competitors' quantity choices and aggregate variables as given, firm k in sector i chooses the quantity of output $q_{ik} \in \mathscr{S}_i := \mathbb{R}_+ \cup \{+\infty\}$ to maximize its profit.[39] Let $\pi_{ik} : \mathscr{S}_i \times \mathscr{S}_i^{N_i-1} \to \mathbb{R}$ represent firm k's profit function that maps its own quantity choice q_{ik} and competitors' choices $\mathbf{q}_{i,-k} := \{q_{ik'}\}_{k' \in \mathbf{N}_i \setminus \{k\}}$ to the profit under the information set $\mathcal{I}_i$:

$$\mathcal{I}_i := \{Y, \{X_j\}_{j\in\mathbf{N}}, \{Q_j\}_{j\in\mathbf{N}\setminus\{i\}}, W, P_i^M, \{z_{ik}\}_{k\in\mathbf{N}_i}, \boldsymbol{\omega}_L, \Omega, \boldsymbol{\tau}\}.$$

The construction of $\mathcal{I}_i$ reflects the fact that when firms in sector i make quantity decisions, they take these aggregate variables as fixed while internalizing the possibility of the sectoral aggregate quantity Q_i and the associated price index P_i varying as a result of their own decisions.[40] Note that the sectoral cost index for material input P_i^M is taken as given. All sectoral price indices $\{P_j\}_{j\in\mathbf{N}}$ are determined to be consistent with all sectoral cost indices for material input $\{P_j^M\}_{j\in\mathbf{N}}$ in the aggregate equilibrium.[41] The inclusion of the firms' productivities $\{z_{ik}\}_{k\in\mathbf{N}_i}$ partly embodies the complete information structure of the strategic interaction. For each $i \in \mathbf{N}$, the Cournot-Nash equilibrium quantities $\mathbf{q}_i^* := \{q_{ik}^*\}_{k\in\mathbf{N}_i}$ must satisfy the following system of equations:[42] for each

[38] Since my model is static, and assumes away from firm's endogenous entry and exit, my model can be interpreted as a long-run approximation, in which every firm behaves just like a "continuing" firm. For such firms, labor input is as easy as maintaining the existing employment relationship.

[39] The firm's profit here is defined as revenue minus variable costs.

[40] Note that, as seen in (12), government spending G can be dropped under (1), (8), and (9).

[41] It might seem to be natural to consider a situation where firms recognize their impacts on input prices as well as output prices. In such a case, firms' strategic interactions prevail across sectors through input uses along the production network. This entails two additional theoretical complications: i) all firms engage in a single very large strategic competition across sectors, and ii) firms have oligopsony power in the input markets (e.g., Berger et al. 2022). The causal mechanism of this paper, on the other hand, is motivated by existing research that points to the prevalence of i)' within-sector strategic interactions and ii)' oligopolistic competition in the output markets. To keep the focus of the analysis consistent with the motivating literature, I maintain the sectoral aggregator (3), which effectively safeguards the input markets against the firms' strategic forces. Exploring the case of oligopsony across sectors is left for future work.

[42] The existence of Cournot-Nash equilibria in each sector immediately follows from the Debreu-Glicksberg-Fan theorem (Debreu 1952; Fan 1952; Glicksberg 1952).

$k \in \mathbf{N}_i$,

$$q_{ik}^{*} \quad \in \quad \arg\max_{q} \quad \pi_{ik}(q, \mathbf{q}_{i,-k}^{*}; \mathcal{I}_i). \tag{7}$$

In what follows, the dependence on the information set $\mathcal{I}_i$ is made implicit, and it is understood as being absorbed by the sector i subscript.[43]

2.4 Government

The government sets the level of subsidies $\boldsymbol{\tau}$ under the balanced budget. Government expenditures consist of two components. First, the government purchases the final consumption good, which can be conceived as public spending G. The second element refers to the total policy expenditure S_i in sector i. The residual between these two expenditures is charged to the representative consumer in the form of a lump-sum tax T. Hence, the government's budget constraint is

$$G + \sum_{i=1}^{N} S_i = T \qquad \textit{where} \qquad S_i := \sum_{k=1}^{N_i} \sum_{j=1}^{N} \tau_i P_j m_{ik,j}. \tag{8}$$

2.5 Equilibria

2.5.1 Market Clearing

The market clearing conditions are standard:

$$\text{[Final consumption good]} \qquad Y = C + G \tag{9}$$

$$\text{[Sectoral intermediate goods]} \qquad Q_j = X_j + \sum_{i=1}^{N} \sum_{k=1}^{N_i} m_{ik,j} \qquad \forall j \in \mathbf{N} \tag{10}$$

$$\text{[Labor]} \qquad L = \sum_{i=1}^{N} \sum_{k=1}^{N_i} \ell_{ik} \tag{11}$$

The resource constraints (9) and (10) hold, respectively, because the final consumption good is either consumed by the household or purchased by the government, and because the sectoral intermediate goods are used either for producing the final consumption good or as input in an individual firm's

[43] Strictly speaking, each step of the firm's decision is based on different information sets. For instance, the information set at the time of input decision should be $\mathcal{I}_i' := \mathcal{I}_i \cup \{q_{ik'}^{*}\}_{k'=1}^{N_i}$. The i index should thus be understood as conditioning on the appropriate information set.

production.[44] In the labor market clearing condition (11), labor L is assumed to be inelastically supplied, fully employed, and frictionlessly mobile across sectors and firms. Lastly, substituting (1) and (8) into (9), it follows that

$$Y = WL + \Pi - \sum_{i=1}^{N} S_i, \tag{12}$$

which is nothing but the income accounting identity of GDP.

2.5.2 Equilibria Defined

I assume that subsidies $\boldsymbol{\tau}$ are externally determined (by the government).[45] Under Assumption 2.1, the numbers of sectors and firms, firms' productivities, and the network structures are invariant to a policy shift, while other aggregate variables, together with firm-level variables, are endogenously determined in equilibrium. Defining the equilibria in this model amounts to finding a fixed point in these endogenous variables. I use the symbol $*$ to denote the equilibrium values.

Definition 2.1 (General Equilibria). *Given the realization of firms' productivities $\{\{z_{ik}\}_{k\in\mathbf{N}_i}\}_{i\in\mathbf{N}}$, sector-specific subsidies $\boldsymbol{\tau}$, and the input-output linkages $\boldsymbol{\omega}_L$ and Ω, the general equilibria of this model are defined as fixed points that solve the following problems:*

Sectoral equilibria: *For each sector i, given the information set $\mathcal{I}_i$, the solution to the quantity-setting game* (7) *yields a vector of sectoral Cournot-Nash equilibrium quantities $\{q^*_{ik}\}_{k\in\mathbf{N}_i}$, followed by the cost-minimization problems* (5) *and* (6) *to derive the optimal labor and material inputs $\{\ell^*_{ik}, m^*_{ik}\}_{k\in\mathbf{N}_i}$, and input demand for sectoral intermediate goods $\{\{m^*_{ik,j}\}_{j\in\mathbf{N}}\}_{k\in\mathbf{N}_i}$.*

Aggregate equilibria: *Given a collection of sectoral equilibrium quantities $\{q^*_{ik}, \ell^*_{ik}, m^*_{ik}, \{m^*_{ik,j}\}_{j\in\mathbf{N}}\}_{i,k}$, an aggregate equilibrium is referenced by the set of aggregate quantities $\{Y^*, \{X^*_j, Q^*_j\}_{j\in\mathbf{N}}\}$ together with the set of aggregate prices $\{W^*, \{P^*_j\}_{j\in\mathbf{N}}\}$, such that i) the household maximizes its utility subject to* (1)*, ii) the market clearing conditions for composite intermediate goods* (10) *and labor* (11) *are satisfied, and iii) the income accounting identity* (12) *holds.*

[44] The market clearing condition for individual firms' products is straightforward, as firm-level products are only used by the sectoral aggregator. Thus, it is already implicitly applied in the exposition.

[45] I abstract from issues of endogenous policies, such as considered in Grossman and Helpman (1994).

2.6 The Object of Interest

Recall from Section 2.1 that the policymaker hopes to learn how much GDP would change due to the policy reform from τ_n^0 to τ_n^1. Let $Y^{\boldsymbol{\tau}}$ be the country's GDP in equilibrium under policy regime $\boldsymbol{\tau}$. From (11) and (12), it follows that

$$Y^{\tau} = \sum_{i=1}^{N} Y_i(\boldsymbol{\tau}) \qquad where \qquad Y_i(\boldsymbol{\tau}) := \sum_{k=1}^{N_i} \left(W^* \ell_{ik}^* + \pi_{ik}^* - \sum_{j=1}^{N} \tau_i P_j^* m_{ik,j}^* \right), \tag{13}$$

where π_{ik} stands for firm k's profit. In (13), $Y_i(\boldsymbol{\tau})$ can be viewed as sectoral i's GDP.

Now the object of interest $\Delta Y(\tau_n^0, \tau_n^1)$ is defined as

$$\Delta Y(\tau_n^0, \tau_n^1) := \sum_{i=1}^{N} Y_i(\boldsymbol{\tau}^1) - \sum_{i=1}^{N} Y_i(\boldsymbol{\tau}^0). \tag{14}$$

While a variety of "causal effects" of an industrial policy have been proposed in the empirical treatment-effect literature, they do not necessarily speak to policy-relevant questions such as those considered in this paper.[46] The policy parameter (14) directly compares the country's GDP under $\boldsymbol{\tau}^0$ to that under $\boldsymbol{\tau}^1$ and thus answers the important macroeconomic question. A virtue of this parameter is that under Assumption 2.1, it represents an *intensive-margin causal effect* of the policy reform in the sense of a *ceteris paribus* change in an outcome variable across different policy regimes (Marshall 1890).[47] In the same spirit as the policy-relevant treatment effect (Heckman and Vytlacil 2001, 2005, 2007),[48] the target parameter (14) pertains to *ex ante* evaluation of causal effects of universal treatments with internalizing firms' strategic interactions, network spillovers, and the general equilibrium feedback effect, each of which is typically assumed away in the treatment effect literature.[49]

Remark 2.1. *While I confine attention to the causal effect of an industrial policy on GDP, my model can be used to define various other (both aggregate and distributional) causal parameters*

[46] See Lane (2020) and Juhász et al. (2023).

[47] For the long-run analysis, wherein the firm's endogenous entry and exit are allowed, the *extensive-margin causal effect* can be defined analogously (Appendix D.2).

[48] Similar notions of "causal effects" are also *defined* under the premise of randomized control trials, e.g., overall treatment effects (Halloran and Struchiner 1991; Hudgens and Halloran 2008) and global treatment effects (Munro et al. 2023).

[49] There have been recent advancements in the treatment effect literature to accommodate these elements (see, e.g., Rotemberg (2019) and Sraer and Thesmar (2019)). However, no existing work accounts for all of these elements simultaneously.

(Appendix D.3), to analyze changing subsidies to multiple sectors (Appendix D.4), and to formulate an optimal policy problem (Appendix D.5).

2.7 Properties of the Policy Parameter $\Delta Y(\tau_n^0, \tau_n^1)$

Under Assumptions 2.1, the object of interest (14) is differentiable over the domain of definition of the model[50] and thus is equivalently rewritten as

$$\Delta Y(\tau_n^0, \tau_n^1) = \sum_{i=1}^{N} \int_{\tau_n^0}^{\tau_n^1} \frac{dY_i(\cdot)}{d\tau_n} d\tau_n, ^{51} \tag{15}$$

where

$$\left. \frac{dY_i(s)}{ds} \right|_{s=\boldsymbol{\tau}} = \sum_{k=1}^{N_i} \left\{ \frac{dp_{ik}^*}{d\tau_n} q_{ik}^* + p_{ik}^* \frac{dq_{ik}^*}{d\tau_n} - \sum_{j=1}^{N} \left(\frac{dP_j^*}{d\tau_n} m_{ik,j}^* + P_j^* \frac{dm_{ik,j}^*}{d\tau_n} \right) \right\}.^{52} \tag{16}$$

In the reminder of this section, I investigate the determination of the comparative statics in (16) using a simplified version of the model, while a full description is delegated to Appendix A.

2.7.1 Macro and Micro Complementarities

To highlight how the firms' strategic interactions interact with the production network across sectors, I focus on the comparative statics of firm-level and sectoral prices as well as material input cost, namely, $\frac{dp_{ik}^*}{d\tau_n}$, $\frac{dP_i^*}{d\tau_n}$, and $\frac{dP_i^{M*}}{d\tau_n}$. The following proposition characterizes the structural equation between these comparative statics. For the sake of simplicity, I assume away from the general equilibrium effects, i.e., wage is invariant to a policy change.

Proposition 2.1 (Partial Equilibrium)**.** *Suppose that the economy is in partial equilibrium, so that* $\frac{dW^*}{d\tau_n} = 0$*. Let* $\wp_{ik}(\mathbf{q})$ *be the residual inverse demand function faced by firm* k *in sector* i*. Then, it holds that (i)* $\frac{dP_i^{M*}}{d\tau_n} = \sum_{j=1}^{N} \frac{\partial \mathcal{P}_i^M(\cdot)^*}{\partial P_j} \bar{\lambda}_{j\cdot}^M \frac{dP_j^*}{d\tau_n} + \frac{\partial \mathcal{P}_i^M(\cdot)^*}{\partial \tau_n} \mathbb{1}_{\{n=i\}}$*; (ii)* $\frac{dP_i^*}{d\tau_n} = \sum_{k'=1}^{N_i} \frac{\partial \mathcal{P}_i(\cdot)^*}{\partial p_{ik'}} \frac{dp_{ik'}^*}{d\tau_n}$*; and (iii)* $\frac{dp_{ik}^*}{d\tau_n} = \left(\sum_{k'=1}^{N_i} \frac{\partial \wp_{ik}(\cdot)^*}{\partial q_{ik'}} \bar{\lambda}_{ik'}^M \right) \frac{dP_i^{M*}}{d\tau_n}$*, where* $\mathcal{P}_i(\cdot)$ *and* $\mathcal{P}_i^M(\cdot)$ *are functions such that* $P_i^* = \mathcal{P}_i(\{p_{ik'}\}_{k'=1}^{N_i})$

[50] The domain of definition is not necessarily the same as the empirical support of data. This is discussed in Section 4.

[51] Note that subsidies to other sectors $\{\tau_j\}_{j \neq n}$ are fixed constant throughout the integral, so that $Y_i(\cdot)$ can effectively be treated as a univariate function of τ_n. In light of this, I write $\frac{dY_i(\cdot)}{d\tau_n} = \frac{\partial Y_i(\cdot)}{\partial \tau_n}$.

[52] With a slight abuse of notation, for an equality $V^* = V(s)$, I write $\left. \frac{dV(s)}{ds} \right|_{s=\boldsymbol{\tau}} = \frac{dV^*}{d\tau_n}$.

and $P_i^{M*} = \mathcal{P}_i^M(\{P_j^*\}_{j=1}^N, \tau_i)$*, respectively; and* $\bar{\lambda}_{ik}^M$ *represents a theoretically well-defined coefficient defined in Appendix A.1.*

Proof. See Appendix A.1. □

The coefficient $\bar{\lambda}_{ik}^M$ represents the firm k's contribution to the sector's overall strategic complementarity.[53] In part (i) of Proposition 2.1, the response of the material cost index is decomposed into two parts: The first term of the right hand side captures the indirect effects of the policy reform coming through changes in intermediate good prices, while the second term indicates the direct effect of the policy change. Part (ii) shows how the individual firms' responses are aggregated into the change in the sectoral variable. Since in oligopolistic competition, $\frac{\partial \mathcal{P}_i(\cdot)^*}{\partial p_{ik'}}$ is generally non-zero, this expression opens up the possibility of an individual firm consisting of a non-negligible fraction of the aggregate response, in line with the literature on "granularity" (e.g., Gabaix 2011). This insight manifests itself as an identification problem, as expanded in Section 4. In addition to such a "micro-to-macro" perspective, part (iii) offers an "macro-to-micro" viewpoint as well, namely, how much of the change in the aggregate variable translates into the firm-level responses, in the spirit of the incomplete pass-through literature (e.g., Atkeson and Burstein 2008).

Equations in Proposition 2.1 can further be solved to obtain the following "reduced-form" expressions for (i) and (ii): (i)' $\frac{dP_i^{M*}}{d\tau_n} = h_{i,n}^M \frac{\partial \mathcal{P}_n^M(\cdot)^*}{\partial \tau_n}$ and (ii)" $\frac{dP_i^*}{d\tau_n} = \bar{\lambda}_{i\cdot}^M \frac{dP_i^{M*}}{d\tau_n}$. The coefficient $h_{i,n}^M$ in (i)' represents the pass-through of the direct impact of the policy change to the change in the targeted sector's material cost index P_n^{M*}, and is given by the (i,n) entry of the matrix $(I-\Gamma)^{-1}$ where $\Gamma := \left[\frac{\partial \mathcal{P}_i^M(\cdot)^*}{\partial P_j} \bar{\lambda}_{j\cdot}^M\right]_{i,j=1}^N$ with $\bar{\lambda}_{j\cdot}^M$ being a weighted average of all $\bar{\lambda}_{jk}^M$ in the same sector j.[54] By construction, $\bar{\lambda}_{j\cdot}^M$ can be conceived as a sector-level measure of firms' strategic complementarities, and as shown in (ii)', it dictates a pass-through from the material cost index to output price index. Notice that $\mathcal{P}_i^M(\cdot)$ involves the information about the production network carried over from the aggregator $\mathcal{G}_i(\cdot)$, and so are its partial derivatives $\frac{\partial \mathcal{P}_i^M(\cdot)^*}{\partial P_j}$. While $\frac{\partial \mathcal{P}_n^M(\cdot)^*}{\partial \tau_n}$

[53]This coefficient is defined as a ratio whose denominator takes the form of a linear combination of the responsivenesses of all firms' marginal profits with respect to all firms, and whose numerator is given by another linear combination of the responsivenesses of all firms' marginal profits in response to all firms' quantities but for firm k's quantity. The denominator can be regarded as a measure of sector's overall strategic complementarity. Since the numerator does not involve the firm k's quantity adjustment, this coefficient backs out the extent to which firm k affects the sectoral measure of strategic complementarity. See Appendix A.1 for the detail.

[54]It is assumed that $(I-\Gamma)^{-1}$ exists. The weight for $\bar{\lambda}_{ik}^M$ is proportional to the share of firm k's product in sectoral aggregate Q_i.

allows for the interpretation as the "initial policy effect," the coefficients $\{h_{j,n}^M\}_{j=1}^N$, together with $\{\bar{\lambda}_{j\cdot}^M\}_{j=1}^N$, dictate the comovement patterns of the sectoral price and material cost indices.

For instance, when $n \neq i$, the coefficient $h_{i,n}^M$ is given by

$$\underbrace{\bar{\lambda}_{n\cdot}^M \frac{\partial \mathcal{P}_i^M(\cdot)^*}{\partial P_n}}_{dP_n^M \to dP_n \to dP_i^M} + \sum_{j=1}^N \underbrace{\bar{\lambda}_{n\cdot}^M \frac{\partial \mathcal{P}_j^M(\cdot)^*}{\partial P_n} \bar{\lambda}_{j\cdot}^M \frac{\partial \mathcal{P}_i^M(\cdot)^*}{\partial P_j}}_{dP_n^M \to dP_n \to dP_j^M \to dP_j \to dP_i^M} + \sum_{j=1}^N \sum_{j'=1}^N \underbrace{\bar{\lambda}_{n\cdot}^M \frac{\partial \mathcal{P}_j^M(\cdot)^*}{\partial P_n} \bar{\lambda}_{j\cdot}^M \frac{\partial \mathcal{P}_{j'}^M(\cdot)^*}{\partial P_j} \bar{\lambda}_{j'\cdot}^M \frac{\partial \mathcal{P}_i^M(\cdot)^*}{\partial P_{j'}}}_{dP_n^M \to dP_n \to dP_j^M \to dP_j \to dP_{j'}^M \to dP_{j'} \to dP_i^M} + \ldots . \tag{17}$$

It is evident in (17) that $h_{i,n}^M$ designates the indirect effects due to changes in other sectors' price indices accumulated through the production network. In each term, $\bar{\lambda}_{j\cdot}^M$ designates the pass-through from the material cost index to the output price index within sector j, while $\frac{\partial \mathcal{P}_{j'}^M(\cdot)^*}{\partial P_j}$ represents the change in the sector j''s material cost index caused by the change in the sector j's output price index. For instance, the first term represents a feedback effect coming through the purchase of intermediate goods from the own sector. The second and third terms capture the feedback effects coming through multiple rounds of input purchases by other sectors.[55] Each round of the indirect effects is augmented by the source sectors' overall strategic complementarities $\{\bar{\lambda}_{j\cdot}^M\}_{j=1}^N$. Intuitively, $h_{i,n}^M$ compounds the degree of sector-level strategic complementarities along the production network. I refer to $\{\bar{\lambda}_{j\cdot}^M\}_{j=1}^N$ as the *micro complementarities* and $\{h_{j,n}^M\}_{j=1}^N$ as the *macro complementarities*.[56]

Clearly, different specifications of market competition or a production network lead to different values of the micro and macro complementarities.[57] Put another way, different specifications may result in different or even opposite policy conclusions. To fix ideas, I now explore these two pass-through coefficients using a special case of the model above, namely, duopoly in a familiar-looking parametric environment. This exercise distills the motivation for the empirical policy evaluation under a minimal set of identification assumptions.

[55] The second term measures the feedback effects in terms of triads, whereas the third term does so in terms of tetrads.

[56] Even in the absence of strategic competition, such as in monopolistic competition, micro complementarities generally do not vanish because $\{\bar{\lambda}_{jk}\}_{k=1}^{N_i}$ involve the responsiveness of firms' marginal profits with respect to their own quantity adjustments, which are not necessarily zero. See Example A.3 in Appendix A.

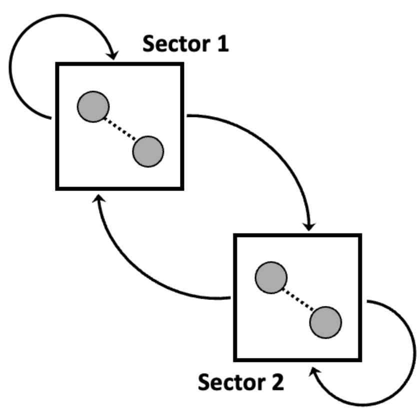
[57] Especially in the absence of a production network, $\frac{\partial \mathcal{P}_i^M(\cdot)^*}{\partial P_n}$ equals zero if $i = n$, and zero otherwise.

2.7.2 An Illustrative Example: Two Sectors and Two Firms

Suppose that the economy consists of two sectors, i.e., $\mathbf{N} = \{1, 2\}$. Each sector is populated by two firms, i.e., $\mathbf{N}_i = \{1, 2\}$ for all $i \in \mathbf{N}$. Without loss of generality, firm 1 is assumed to be more productive than firm 2, i.e., $z_{i1} > z_{i2}$. In each sector, firms engage in strategic competition over quantity in the output market (i.e., Cournot duopoly). Consider an industrial policy targeted at sector 1, i.e., $n = 1$.

The economy-wide aggregator $\mathcal{F}(\cdot)$ is given by a Cobb-Douglas production function. The sectoral aggregator $F_i(\cdot)$ takes the form of a constant elasticity of substitution (CES) production function with an elasticity of substitution $\sigma_i > 1$ (i.e., firms' products are substitutes). Each individual firm produces a differentiated good using a Cobb-Douglas production function $f_i(\cdot)$ with Hicks-neutral productivity z_{ik}. The material aggregator $\mathcal{G}_i(\cdot)$ is once again given by a Cobb-Douglas production function, with the input share of sector j's intermediate good $\gamma_{i,j}$ reflecting the production network Ω. It is assumed that $\gamma_{i,j} > 0$ for all $i, j \in \mathbf{N}$, so that every firm purchases positive quantities of intermediate goods from both sectors 1 and 2 (see Figure 1). The associated unit cost condition determines the material cost index: $P_i^{M*} = \prod_{j \in \mathbf{N}} \frac{1}{\gamma_{i,j}^{\gamma_{i,j}}} \left\{(1 - \tau_j) P_j^*\right\}^{\gamma_{i,j}}$, thereby yielding $\frac{\partial \mathcal{P}_i^M(\cdot)^*}{\partial P_j} = \gamma_{i,j} \frac{P_i^{M*}}{P_j^*}$ and $\frac{\partial \mathcal{P}_i^M(\cdot)^*}{\partial \tau_n} = -\frac{P_i^{M*}}{1-\tau_i} \mathbb{1}_{\{n=i\}}$.

Figure 1: Duopoly in Two-Sector Economy



Notes: This figure illustrates the two-sector economy studied in Section 2.7.2. Black square borders stand for sectors. Two gray circles entrenched in each of the square represent duopoly firms with dotted lines indicating strategic interactions between them. Circular arrows designate input purchases along the production network. For example, the circular arrow from sector 1 to 2 means the purchase of sector 1's intermediate goods by firms in sector 2.

Micro complementarity. In Appendix A.4, I show that in equilibrium, firm 1's quantity choice is a strategic complement to firm 2's choice, whereas firm 2's choice is a strategic substitute for firm 1's choice. I further demonstrate that if firm 2's product is a "relatively strong" strategic substitute, then the sectoral measure of strategic complementarity $\bar{\lambda}^M_{i\cdot}$ is positive.[58] The contrapositive of this claim suggests that a negative micro complementarity is evidence of firm 2's being a "relatively modest" strategic substitute.

Macro complementarity. In this two-sector economy, (17) reduces to

$$h^M_{2,1} = \bar{\lambda}^M_{1\cdot}\left(\gamma_{2,1}\frac{P_2^{M*}}{P_1^*} + \gamma_{1,1}\frac{P_1^{M*}}{P_1^*}\bar{\lambda}^M_{1\cdot}\gamma_{2,1}\frac{P_2^{M*}}{P_1^*} + \gamma_{2,1}\frac{P_2^{M*}}{P_1^*}\bar{\lambda}^M_{2\cdot}\gamma_{2,2}\frac{P_2^{M*}}{P_2^*} + \dots\right). \tag{18}$$

Intuitively, the $\bar{\lambda}^M_{1\cdot}$ in front of the round bracket indicates the "initial" response of the sector 1's sectoral price index to the "initial" change in the material cost index. The first term inside the bracket measures the shift in the sector 2's material cost index due to the direct purchase from sector 1 (the circular arrow from sector 1 to 2 in Figure 1). The second keeps track of sector 1's good that is first used by firms in sector 2 before purchased by firms in sector 2's (the circular arrow from sector 1 to 1, followed by the one from sector 2 to 1 in Figure 1), while the third records sector 1's good that is used to produce sector 2's good, which in turn is used as another input by sector 2 (the circular arrow from sector 1 to 2, followed by the one from sector 2 to 2 in Figure 1).

Here, suppose for a moment that firm 2 in each sector is only a "relatively strong" strategic substitute, so that $\bar{\lambda}^M_{1\cdot} > 0$ and $\bar{\lambda}^M_{2\cdot} > 0$. In this case, it is immediate to see $h^M_{2,1} > 0$, i.e., a positive macro complementarity. By contrast, suppose instead that firm 2 in each sector is a "relatively modest" strategic substitute, and thus $\bar{\lambda}^M_{1\cdot} < 0$ and $\bar{\lambda}^M_{2\cdot} < 0$. In this case, the sign of the macro complementarity $h^M_{2,1}$ becomes ambiguous, as the first term inside the round bracket of (18) takes a positive value while the second and third terms are negative. Likewise, the remaining terms also exhibit switching of the sign. Hence, the sign and magnitude of $h^M_{2,1}$ are essentially an empirical matter.

[58] Firm 2's product is said to be a "relatively strongly" strategic substitute if $\frac{\partial \frac{\partial \pi_{i2}(\cdot)^*}{\partial q_{i2}}}{\partial q_{i1}} \Big/ \frac{\partial \frac{\partial \pi_{i1}(\cdot)^*}{\partial q_{i1}}}{\partial q_{i1}} \in \left(\frac{z_{i1}}{z_{i2}}, \infty\right)$, and a "relatively modest" strategic substitute otherwise. See Appendix A.4 for the detail.

Key implications. The observation drawn here is of direct policy relevance as it means that even if a policy is targeted at a particular sector, the effects can propagate along the production network; and moreover, such propagations are mediated (amplified, weakened, or even reverted) by the firms' strategic interactions in each sector. This insight brings about two implications for empirical policy evaluation. First, to accurately evaluate the policy parameter $\Delta Y(\tau_n^0, \tau_n^1)$ warrants the joint consideration of the production network and firms' strategic interactions. Second, the identification of $\Delta Y(\tau_n^0, \tau_n^1)$ should be accomplished under a minimal set of assumptions about the underlying market environment, so that the analysis can remain agnostic about the configurations of the policy effect spillovers. These observations motivate my semi-parametric approach for identification and estimation (Section 4), and the subsequent empirical analysis (Section 5). To prepare a ground, the next section describes data available to the policymaker.

3 Data

This section briefly describes the dataset used in my empirical analysis and the procedures by which I construct the empirical counterparts to the variables in my model.[59] My dataset spans between 2007 and 2021, but I do not exploit its time-series feature; rather, I regard it as a collection of snapshots of the same economy with varying levels of subsidies. In this way, I can construct "repeated samples." Consistent with the static nature of the model, the firm-level functions (e.g., technology, demand) are posited to be, conditional on an array of sector-level and aggregate variables, the same across these snapshots.[60] I assume that the observations are generated from an equilibrium (see Assumption 4.1).

3.1 Wage and Price Indices

Data on wage and labor hours worked are taken from the U.S. Bureau of Labor Statistics (BLS) through the Federal Reserve Bank of St. Louis (FRED) at an annual frequency. Consistent with my conceptual framework, I use the average hourly earnings of all employees as my data counterpart for the wage W^*.[61] I obtain data on sectoral price index P_i^* from the GDP by industry data at

[59] The details are provided in Appendix B.

[60] This aligns with the approach adopted by Ackerberg and De Loecker (2024).

[61] Recall that labor is assumed to be frictionlessly mobile across sectors, which implies that the wage is the same everywhere in the economy.

the Bureau of Economic Analysis (BEA), wherein the industries in the BEA data are used as the empirical counterparts of sectors in my framework.

3.2 Input-Output Tables

I adopt the annual U.S. input-output data from the BEA. The data contain industrial output and input for 66 industries and cover the period from 1995 to 2021. Following Baqaee and Farhi (2020), I omit the government, noncomparable imports, and second-hand scrap industries. I also follow Bigio and La'O (2020) in dropping finance, insurance, real estate, rental and leasing (FIRE) industries. I further follow Gutiérrez and Philippon (2017) in segmenting the industries into coarser categories, leaving me with 32 industries.

Each input-output account comes with two distinct tables, namely, the use and supply tables. The use table reports the amounts of commodities used by each industry as intermediate inputs and by final user, and the value added by each industry. The value-added section of the use table includes compensation of employees and taxes on products less subsidies for each purchaser industry. Each cell in the supply table indicates the amount of each commodity produced by each industry.

To transform the use table into an industry-by-industry format, I make the following assumption: each product has its own specific sales structure, irrespective of the industry where it is produced (Assumption B.1). Here, the sales structure refers to the shares of the respective intermediate and final users in the sales of a commodity. Under this assumption, I can convert the commodity-by-industry use table to the industry-by-industry table, thereby conforming to my conceptual model of the production network Ω (see Appendix B.2.1 for details).[62] The transformed input-output table can further be used to back out data for $\boldsymbol{\tau}$ as a value-added net subsidy, which is understood as an amalgamate of sales and input subsidies.

3.3 Compustat Data

The dataset for firm-level variables is Compustat, which is assembled by S&P and provided by Wharton Research Data Services (WRDS). The Compustat data record information about firm-

[62]Using the compensation of employees, I can also construct data for $\boldsymbol{\omega}_L$. Throughout the transformation, the value-added section of the use table remains intact.

level financial statements, such as sales, input expenditure, capital stock information, and detailed industry activity classifications, from 1950 to 2021. From this data, in conjunction with the data on aggregate variables, I first construct measurements for firm-level labor and material inputs as well as revenue. I follow De Loecker et al. (2020) in eliminating outliers.

Since the dataset does not offer a further breakdown of material input, I need to apportion the expenditure on material input to generate separate information about the demand for sectoral intermediate goods. This requires an explicit functional-form assumption on the material input aggregator $\mathcal{G}_i(\cdot)$ in (4). In this paper, I employ a Cobb-Douglas production function:

$$m_{ik} = \prod_{j=1}^{N} m_{ik,j}^{\gamma_{i,j}}, \tag{19}$$

where $m_{ik,j}$ is sector j's intermediate good demanded by firm k in sector i and $\gamma_{i,j}$ denotes the input share of sector j's intermediate good with $\sum_{j=1}^{N} \gamma_{i,j} = 1$. A virtue of this specification is that the production network across sectoral intermediate goods $\{\omega_{i,j}\}_{j\in\mathbf{N}}$ is directly reflected in the output elasticity parameters $\{\gamma_{i,j}\}_{j\in\mathbf{N}}$, which are constant.[63] This property is plausible in light of the particular focus of this paper on the short-run effects of the policies (see Assumption 2.1).[64] Under this specification, the input demand for sector j's good $m^*_{ik,j}$ is given by

$$m^*_{ik,j} = \gamma_{i,j} \frac{P_i^{M*}}{(1-\tau_i)P_j^*} m^*_{ik}, \tag{20}$$

where $P_i^{M*} m^*_{ik}$ indicates the expenditure on material input gross of subsidies, which can be obtained in the data (see Fact B.5).

I admit the possibility that the data on firm-level revenues are subject to measurement errors.[65] Importantly, the Compustat data do not provide information about output quantity and price. To

[63] The Cobb-Douglas production function has traditionally been used in a wide range of the macroeconomics literature — for example, the real business cycle theory (Long and Plosser 1983; Horvath 1998, 2000) and international trade (Caliendo and Parro 2015; Grassi 2017; Bigio and La'O 2020). The recent literature has emphasized the importance of an endogenous input-output structure of the economy and employed a CES aggregator (e.g., Atalay 2017; Baqaee and Farhi 2019; Caliendo et al. 2022).

[64] In principle, the functional form assumption (19) is necessitated in order to compensate for the shortcoming of the dataset at hand. In general, this assumption could be relaxed to the extent that the information about demand for sectoral intermediate goods are recovered. Moreover, this assumption could even be completely dispensed if the econometrician (or the policymaker) has access to detailed data on firm-to-firm trade, such as the Belgium data (Dhyne et al. 2021), the Chilean data (Huneeus 2020) and the Japanese data (Bernard et al. 2019).

[65] I assume additive separability in terms of log variables.

recover these variables from the error-contaminated revenue data, I leverage a methodology that has recently been developed in the industrial organization literature (see Section 4.2).

4 Identification and Estimation

This section discusses identification of the object of interest (14) based on the model laid out in Section 2 and the dataset described in Section 3. The identification results are constructive, which naturally validates the use of nonparametric plug-in estimators.

To simplify the identification analysis, I make two sets of assumptions. First, in order to sidestep the concern about the multiplicity of equilibria, I impose assumptions on the equilibrium selection probability. Second, I focus on a situation where the policymaker is only interested in changing the policy within the historically observed support. Let $\mathscr{T} := \times_{i=1}^{N} \mathscr{T}_i$ where $\mathscr{T}_i \subseteq \mathbb{R}$ represents the observed support of τ_i.

Assumption 4.1 (Equilibrium Selection). *(i) The observations in the data are generated from a single equilibrium. (ii) The equilibrium that is played does not change over the course of the policy reform.*

Assumption 4.2 (Support Condition). $[\tau_n^0, \tau_n^1] \subseteq \mathscr{T}_n$

Assumption 4.1 (i) states that the equilibrium selection probability is degenerated to a single equilibrium, and the condition (ii) means that it is this single equilibrium that will be chosen in the policy counterfactuals.[66] Assumption 4.1 is widely used in the literature of discrete choice models (Aguirregabiria and Mira 2010).[67] Assumption 4.2 excludes the scenario that the new policy is such a policy that has never been implemented before. Assumptions 4.1 and 4.2 could jointly be relaxed at the expense of additional assumptions, as studied by Canen and Song (2022).[68]

To solve the evaluation problem, it is essential to distinguish the policymaker's (or the observing econometrician's) information set from the agent's information set.[69] Four remarks are in order.

[66] The latter is embodied in Assumptions A.1 and A.2.

[67] Notice that Assumption 4.1 only restricts the equilibrium selection probability and does not exclude the possibility of multiple equilibria per se.

[68] See the discussions in Sections 5 and 6.

[69] It is tacitly assumed that as far as the information set is concerned, the government, which is an agent of the model, is identical to the econometrician outside the model.

First, the former includes $\boldsymbol{\tau}^1$, reflecting the premise that the policy variables can be externally manipulated by the policymaker. Second, the firm's productivity z_{ik} is not known to the policymaker, while firms are assumed to have complete information (Section 2). Third, the firm's equilibrium revenue r^*_{ik} is not available to the policymaker; and the observed firm's revenue r_{ik} is contaminated by a measurement error. Lastly, the firm's equilibrium output price p^*_{ik} and quantity q^*_{ik} are not included in the policymaker's information set due to the limitation of the data (Section 3).

4.1 Identification Strategy

My identification argument builds on (15) and aims to identify the integrand $\frac{dY_i(s)}{ds}$ for all $s \in [\boldsymbol{\tau}^0, \boldsymbol{\tau}^1]$. The existing approach to recover (16) is to characterize its left-hand side in terms of aggregate variables that are directly observed in the data (e.g., Arkolakis et al. 2012, 2019; Adão et al. 2020). Their aggregation results crucially hinge on the modeling assumption of a mass of continuum of firms. Under this assumption, individual firms are infinitesimally small and thus inconsequential to the aggregate variables owing to the law of large numbers (Gaubert and Itskhoki 2020). By contrast, my framework embraces only a finite number of firms, in which case firm-level idiosyncrasies are not washed away even in the aggregate. My approach is rather to recover each of the firm-level responses on the right-hand side of (16). In doing so, I apply the control function approach that has been developed in the industrial organization literature. As a by-product, the characterization result of this paper does not rely on the approximation of (16) around the economy with no pre-existing policies (i.e., $\boldsymbol{\tau}^0 = \mathbf{0}$), a simplification employed in the existing literature.[70]

Remark 4.1. *(i) The idea behind my identification strategy resembles the exact hat algebra (Dekle et al. 2007, 2008), a method that is routinely used to generate a counterfactual prediction in the literature (e.g., Caliendo and Parro 2015; Adão et al. 2017, 2020).*[71] *My approach is distinct in two ways, however. First, the exact hat algebra is not principally concerned with the identification and estimation of the comparative statics; it only calculates the comparative statics taking model parameters as known (Dingel and Tintelnot 2023). My paper provides a unified framework for the identification and estimation of both "model parameters" and the comparative statics. Second, the*

[70] While useful as an approximation around the equilibrium in response to a small shock, the common practice of setting $\boldsymbol{\tau}^0 = \mathbf{0}$ (e.g., Liu 2019; Baqaee and Farhi 2022) is rarely feasible in empirical research because in most realistic cases it is that $\mathbf{0} \notin \mathscr{T}$.

[71] See Costinot and Rodríguez-Clare (2014) for an outline of the method.

presumption of exact hat algebra is that all endogenous equilibrium variables are observable. This requirement, however, is not fulfilled in my case as firm-level quantity q^*_{ik} and price p^*_{ik} are not available in the data (see Section 3). In Section 4.2, I provide a path forward to move on in the presence of these unobservable endogenous variables. (ii) The left-hand side of (16) *alone may be of limited practical relevance because it only measures the impact of an infinitesimally small policy change around $\boldsymbol{\tau}^0$ (e.g., Caliendo and Parro 2015). My target parameter* (14), *in contrast, can be used to analyze a large policy reform from $\boldsymbol{\tau}^0$ to $\boldsymbol{\tau}^1$.*[72]

4.2 Identification

To recover (16) requires the identification of firm-level price and quantity, and comparative statics, with the latter further calling for the identification of derivatives of firm-level inverse demand and production functions. Notice, however, that *a*) firm-level quantity and price are not observed in my dataset (see Section 3), and *b*) derivatives of the firm-level production and inverse demand functions are not known by definition (see Section 2). To keep track of these variables from the policymaker's viewpoint, I leverage the techniques of the industrial organization literature by imposing three sets of additional assumptions.

First, I assume that the firm-level production function exhibits Hicks-neutral productivity. Let $\mathscr{L}_i$ and $\mathscr{M}_i$, respectively, denote the observed supports of labor and material inputs.

Assumption 4.3 (Hicks-Neutral Productivity)**.** *In each sector $i \in \mathbf{N}$ and each firm $k \in \mathbf{N}_i$, $q_{ik} = z_{ik} g_i(\ell_{ik}, m_{ik})$, where $g_i : \mathscr{L}_i \times \mathscr{M}_i \to \mathscr{S}_i$ is a sector-specific production technology.*

This assumption is routinely employed in the macroeconomics literature (e.g., Baqaee and Farhi 2020; Bigio and La'O 2020). Notably, this assumption, together with the specification (19), includes the nested Cobb-Douglas production function of the kind studied in Bigio and La'O (2020).

Second, in order to make the model amenable to empirical analysis while maintaining flexibility, I restrict the sectoral aggregator to take the form of a *homothetic demand system with a single aggregator* (HSA; Matsuyama and Ushchev 2017).

[72] In a related vein, Baqaee and Farhi (2022) investigate the consequences of discrete changes in distortions. Assuming away from any distortions in the initial state of the economy, they provide a second-order approximation for the responses of real GDP and welfare. Accordingly, the discrete changes in their characterization need to be small enough to make the second-order approximation sufficiently good. By contrast, this paper derives an exact formula that is valid for discrete changes of arbitrary size (as long as they are in the historically observed support) from the current policy regime that may not necessarily be efficient. See also Kleven (2021) for a discussion.

Assumption 4.4 (HSA Inverse Demand Function)**.** *In each sector $i \in \mathbf{N}$, (i) the sectoral aggregator $F_i(\cdot)$ exhibits an HSA inverse demand function; that is, the inverse demand function faced by firm $k \in \mathbf{N}_i$ is given by*

$$p_{ik} = \frac{\Phi_i}{q_{ik}} \Psi_i\left(\frac{q_{ik}}{A_i(\mathbf{q}_i)}\right) \qquad \textit{with} \qquad \sum_{k'=1}^{N_i} \Psi_i\left(\frac{q_{ik'}}{A_i(\mathbf{q}_i)}\right) = 1, \tag{21}$$

where Φ_i is a constant indicating the expenditure by sector i's aggregator, $\Psi_i(\cdot)$ represents the share of firm k's good in the expenditure of sector i's aggregator, and $A_i(\mathbf{q}_i)$ denotes the aggregate quantity index capturing interactions between firms' choices with $\mathbf{q}_i := \{q_{ik'}\}_{k' \in \mathbf{N}_i}$; and (ii) the quantity index $A_i(\cdot)$ in (21) *is exchangeable in $(q_{i1}, \cdots, q_{iN_i})$.*[73]

From an individual firm's perspective, the quantity index $A_i(\mathbf{q}_i)$ in (21) summarizes the firm's interactions in sector i, and this is the only channel through which other firms' choices matter to the firm's own decision.[74] The exchangeability assumption (ii) is imposed to apply the method developed in Kasahara and Sugita (2020) to my context.[75] While in oligopolistic competition, the firm's equilibrium quantity generally depends on the competitors' productivities as well as its own — a feature absent in Kasahara and Sugita (2020),[76] this assumption helps account for the competitors' productivities: Under Assumption 4.4, it holds that for each $i \in \mathbf{N}$, there exists a constant $M_i \in \mathbb{N}$ such that there exist some continuous functions $\mathcal{H}_{i,1}, \ldots, \mathcal{H}_{i,M_i} : \mathscr{Z}_i^{\mathbf{N}_i} \to \mathbb{R}$ and

[73]A function $h(x_1, \ldots, x_n)$ is said to be exchangeable (or permutation invariant) in $(x_1, \ldots, x_n)$ if $h(x_1, \ldots, x_n) = h(x_{\varsigma(1)}, \ldots, x_{\varsigma(n)})$ for all ς, where $\varsigma := (\varsigma(1), \ldots, \varsigma(n))$ is a permutation of $(1, \ldots, n)$. See Kallenberg (2005) and de Finetti (2017) for the concept of exchangeability.

[74]Intuitively, instead of keeping track of every single one of other firms' choices, the firm only needs to look at this aggregate quantity.

[75]It has long been recognized that the use of the quantity measure of revenue data — revenue data deflated by price index — as a proxy for quantity data induces an omitted price bias (Klette and Griliches 1996) and masks the demand-side heterogeneity encoded in firm-specific price variables. See, for example, Klette and Griliches (1996), Doraszelski and Jaumandreu (2019), Flynn et al. (2019), Bond et al. (2021), Kirov et al. (2022), and Kasahara and Sugita (2020) for the details.

[76]The host of the literature on the identification of production functions assumes away from strategic interactions. For example, in the context of the control function approach, Ackerberg et al. (2015) and Gandhi et al. (2019) assume perfectly competitive markets, and Kasahara and Sugita (2020) focus on monopolistic competition. Doraszelski and Jaumandreu (2019) and Brand (2020) point out that the canonical scalar unobservability assumption eliminates the possibility of strategic interactions and examine the extent to which the estimates are biased if the standard approach is mistakenly used. Matzkin (2008) considers the identification of a system of equations permitting strategic interactions, but requires linear separability in excluded regressors, which may not be supported on theoretical grounds in my context.

$\chi_i : \mathscr{Z}_i \times \mathbb{R}^{M_i} \to \mathbb{R}_+$ such that

$$q_{ik}^* = \chi_i(z_{ik}; \mathcal{H}_{i,1}(\mathbf{z}_i), \ldots, \mathcal{H}_{i,M_i}(\mathbf{z}_i)), \tag{22}$$

where $\mathcal{H}_{i,m}(\mathbf{z}_i)$ is exchangeable in $(z_{i1}, \ldots, z_{iN_i})$ for all $m \in \{1, \ldots, M_i\}$. This result suggests that the firm's equilibrium quantity depends on other firms' productivities only through some aggregates, each of which is common to all firms. The equation (84) admits an interpretation analogous to the quantity index $A_i(\cdot)$ in Assumption 4.4, i.e., the aggregate productivities $\{\mathcal{H}_{i,m}(\mathbf{z}_i)\}_{m=1}^{M_i}$ are "sufficient statistics" for the competitors' productivities.[77] An intuition is that instead of interacting one another, each firm only needs to interact with these aggregate productivities, as they act as a "translator" of the strategic interaction in the market. These aggregates can most naturally be understood as measures of the overall competitiveness of the market, and can be viewed as versions of the conventional measure of competitiveness, such as the Herfindahl-Hirschman Index (HHI). They are, though, distinct in that the latter is usually observed in data, while the former is by definition not known to the econometrician.[78]

Remark 4.2. *(i) Assumption 4.4 is slightly stronger than the original definition by Matsuyama and Ushchev (2017), and abstracts from unobservable demand-side heterogeneity in the sectoral aggregator $F_i(\cdot)$. This assumption is adopted only to simplify identification and estimation, and can be relaxed at the cost of an additional technicality. See Kasahara and Sugita (2023). (ii) In the production function context, Blum et al. (2023), Ackerberg and De Loecker (2024) and Doraszelski and Jaumandreu (2024) consider demand functions similar in spirit to* (21). *The identification results of Ackerberg and De Loecker (2024) and Doraszelski and Jaumandreu (2024) require that their terms corresponding to $A_i(\mathbf{q}_i)$ be observable, while this paper, as well as Blum et al. (2023), do not.*

The HSA specification (21) is broad enough to accommodate a wide variety of aggregators, including those that are commonly used in the international trade literature — for example, the constant elasticity of substitution (CES), and the flexible class of non-CES homothetic aggregators

[77] The aggregate productivities do not need to be observed by the econometrician. The only thing that she needs to know is that the competitor's productivity is summarized by some sector-specific aggregates.

[78] Yet, note that owing to the completeness of the information structure, the values of these aggregate productivities are known to all firms in the same sector at the time of decision making.

explored in Kimball (1995), Burstein and Gopinath (2014), and Arkolakis et al. (2019).[79]

Example 4.1 (CES aggregator)**.** *For each sector $i \in \mathbf{N}$, consider the CES aggregator: $F_i(\{q_{ik}\}_{k\in\mathbf{N}_i}) := \big(\sum_{k=1}^{N_i} \delta_i^{\sigma} q_{ik}^{\frac{\sigma-1}{\sigma}}\big)^{\frac{\sigma}{\sigma-1}}$, where σ represents the elasticity of substitution specific to the sector, and δ_i is a demand shifter specific to sector i.[80] Associated with this is the residual inverse demand curve faced by firm k: $p_{ik} = \frac{\Phi_i}{q_{ik}} \frac{\delta_i q_{ik}^{\frac{\sigma-1}{\sigma}}}{\sum_{k'=1}^{N_i} \delta_i q_{ik'}^{\frac{\sigma-1}{\sigma}}}$. Assumption 4.4 is then satisfied by setting $\Psi_i(x;\mathcal{I}_i) := \delta_i B_0^{\frac{\sigma-1}{\sigma}} x^{\frac{\sigma-1}{\sigma}}$ with $A_i(\mathbf{q}_i) = \frac{1}{B_0}\sum_{k'=1}^{N_i} \delta_i q_{ik'}^{\frac{\sigma-1}{\sigma}}$, where B_0 is a normalization constant.*

The last set of assumptions, together with Assumption 4.3, guarantees that the equilibrium quantity function $\chi_i(\cdot)$ is "invertible" in the firm's productivity z_{ik}.

Assumption 4.5. *For each $i \in \mathbf{N}$, the function $\chi_i(\cdot)$ in* (84) *satisfies the following properties. (i) $\frac{\chi_i(z_{ik};\cdot)}{z_{ik}} \neq \frac{\chi_i(z_{ik'};\cdot)}{z_{ik'}}$ for all $k, k' \in \mathbf{N}_i$. (ii) $\chi_i(\cdot)$ is strictly monotone in its first argument.*

Part (i), coupled with Assumption 4.3, ensures that variation in the firms' productivities is reflected in the difference in their input choices. Part (ii) pertains to the partial derivative of $\chi_i(\cdot)$ with respect to the firm's own productivity, keeping the aggregate productivities fixed. Note that Assumption 4.5 directly refers to the equilibrium configuration. Formally examining this requires the detailed knowledge about the sectoral aggregator and firm-level production function, which goes against the goal of this paper — an analysis with minimal assumptions. Nevertheless, there is reason to believe that part (i) is plausible because $\chi_i(\cdot)$ is given as a solution to a system of (possibly) highly nonlinear equations, and that part (ii) is the case with a strictly increasing $\chi_i(\cdot)$ because with the market competitiveness being constant, productive firms are more likely to have higher market shares, producing more goods.

Taken together with (6), it follows from Assumptions 4.3 – 4.5 that there exists a continuous function $\mathcal{M}_i : \mathscr{L}_i \times \mathscr{M}_i \times \mathbb{R}^{M_i} \to \mathscr{Z}_i$ such that

$$z_{ik} = \mathcal{M}_i(\ell^*_{ik}, m^*_{ik}; \mathcal{H}_{i,1}(\mathbf{z}_i), \ldots, \mathcal{H}_{i,M_i}(\mathbf{z}_i)) \tag{23}$$

[79] A short list of other examples includes the symmetric translog (Feenstra and Weinstein 2017), the constant response demand (Mrázová and Neary 2017, 2019). See Matsuyama and Ushchev (2017), Kasahara and Sugita (2020), and Matsuyama (2023) for other examples.

[80] The CES aggregator is routinely assumed in the bulk of the macroeconomics literature on international pricing (Atkeson and Burstein 2008; Amiti et al. 2014; Gaubert and Itskhoki 2020).

for all $k \in \mathbf{N}_i$. In light of this, the expression (84) and Assumption 4.5 jointly correspond to the scalar unobservability assumption and the strict monotonicity assumption of the existing literature (e.g., Olley and Pakes 1996; Levinsohn and Petrin 2003; Ackerberg et al. 2015). The expression (23) allows the econometrician to control for unobservable productivity in terms of observable labor and material inputs.

Remark 4.3. *(i) To recover the firm's production function over the entire empirical support, the literature typically goes to further assume that the firm's productivity follows a Markov process (e.g., Ackerberg et al. 2015; Gandhi et al. 2019). In contrast, my analysis is only concerned with identifying the equilibrium values of the relevant functions and variables (see Section 4.1), thereby abstracting from the stochastic process of the firm's productivity. This is plausible in view of the fact that the economic model of my framework is static in nature, and thus my empirical analysis does not exploit the time-series feature of the data (see Section 3). (ii) Plugging* (23) *into* (4)*, the firm's production function can be written in a way that does not depend on competitors' variables.*[81] *This observation is combined with the repeated sample paradigm (see Section 3) to restore identification of firm-level variables under the "large n" asymptotics (see Ackerberg and De Loecker (2024)).*

Assumptions 4.3 – 4.5 permit a variety of specifications for both sector- and firm-level production functions. Continuing Example 4.1, I demonstrate that these assumptions are satisfied in a model widely used in the macroeconomics and international trade literature.

Example 4.2 (CES Sectoral Aggregator and Cobb-Douglas Production Function)**.** *Assume that for each $i \in \mathbf{N}$, $F_i(\{q_{ik'}\}_{k'=1}^{N_i}) := (\sum_{k'=1}^{N_i} \delta_i^{\sigma} q_{ik'}^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}}$ and $f_i(\ell_{ik}, m_{ik}; z_{ik}) := z_{ik}\ell_{ik}^{\alpha} m_{ik}^{1-\alpha}$. To make my claim as transparent as possible, I focus on the case of three firms ($N_i = 3$) and $\sigma = 2$. In this case, the Cournot-Nash equilibrium quantity is given by $q_{ik}^* = \left(\frac{A_i^*\Phi_i}{2A_i^{*2}mc_{ik}+\Phi_i}\right)^2$, where the equilibrium value of the quantity index A_i^* takes the form of a function of $\mathcal{H}_{i,1}(\{z_{ik}\}_{k=1}^3) := z_{i1}^{-1} + z_{i2}^{-1} + z_{i3}^{-1}$ and $\mathcal{H}_{i,2}(\{z_{ik}\}_{k=1}^3) := z_{i1}z_{i2}z_{i3}$. Here, $mc_{ik} := z_{ik}^{-1}mc_i$ stands for the firm k's marginal cost. This conforms to the expression* (84)*, and satisfies Assumption 4.5.*

Taking this expression as given, the input decision is constrained by the production possibility frontier at output level q_{ik}^*: $z_{ik}{\ell_{ik}}^{\alpha}{m_{ik}}^{1-\alpha} = \left(\frac{\Phi_i A_i^*}{2mc_{ik}A_i^{*2}+\Phi_i}\right)^2$ (see the inner optimization of (6)*).*

[81] The competitors' productivity matters only through aggregate productivities, which are effectively absorbed by the sectoral index.

Upon solving this for z_{ik}, it is immediate to see that in equilibrium there exists a function $\mathcal{M}_i(\cdot)$ such that $z_{ik} = \mathcal{M}_i(\ell^*_{ik}, m^*_{ik}, \mathcal{H}_{i,1}(\{z_{ik}\}_{k=1}^3), \mathcal{H}_{i,2}(\{z_{ik}\}_{k=1}^3)$, yielding the expression (23).[82]

Under Assumptions 4.3 – 4.5, I follow Kasahara and Sugita (2020) to identify the equilibrium values of the firm-level quantities and prices, and those of the derivatives of the residual inverse demand functions. Moreover, with the CRS property (Assumption 2.4) and the Hicks-neutral productivity (Assumption 4.3) in hand, I can apply the method developed in Gandhi et al. (2019) to recover the equilibrium values of the first- and second-order derivatives of the production functions.

With additional regularity conditions,[83] I therefore obtain the following theorem.

Theorem 4.1 (Identification of the Object of Interest)**.** *Suppose that Assumptions 4.1 – 4.5, C.2 and C.3 hold. Then, the object of interest* (14) *is identified from the observables.*

Proof. See Appendix C.7. □

Remark 4.4. *(i) Under the same set of assumptions as Theorem 4.1, various other (both aggregate and distributional) causal parameters (Appendix D.3) and the effects of changing subsidies to multiple sectors (Appendix D.4) can also be identified. (ii) A version of Theorem 4.1 remains valid under monopolistic competition with the solution concept appropriately modified (Appendix C.7).*

4.2.1 Systematic Patterns Induced by Identification Assumptions

The identification assumptions induce several important patterns in the recovered firm-level responses, which in turn affects the policy parameter $\Delta Y(\tau_n^0, \tau_n^1)$.

Proposition 4.1. *Suppose that Assumptions 2.4, 4.3 and 4.4 hold. Then, for each $i \in \mathbf{N}$, (i) $\frac{dq^*_{ik}/d\tau_n}{q^*_{ik}} = \bar{c}_i$; (ii) $\frac{dp^*_{ik}/d\tau_n}{p^*_{ik}} = -\bar{c}_i$; (iii) $\frac{dp^*_{ik}}{d\tau_n} q^*_{ik} + p^*_{ik} \frac{dq^*_{ik}}{d\tau_n} = 0$ for all $k \in \mathbf{N}_i$, where $\bar{c}_i$ is a constant common to all firms in the same sector.*

Proof. See Appendix C.8. □

Parts (i) and (ii) of this proposition state that the elasticities of firm-level quantity and price with respect to subsidy do not vary across firms. Part (iii) is an immediate consequence of the first two,

[82] See Appendix C.1.1 for the detail.

[83] These regularity conditions consist of three parts, namely, *a*) the strict exogeneity of the measurement error on firm-level revenues, *b*) continuous differentiability of the revenue function in terms of labor and material inputs, and *c*) normalization of both the firm's production function and sectoral aggregator.

and refers to the responsiveness of the firm-level revenue: The price effect exactly cancels out the quantity effects. This also means that the first two terms inside the curly bracket in (16) (i.e., the revenue effects) vanish, leaving the cost effects (see Section 5.2).[84]

To facilitate interpretation, it is useful to look at Proposition 4.1 in terms of the elasticity of price with respect to quantity: for each $i \in \mathbf{N}$, $\frac{dp^*_{ik}/p^*_{ik}}{dq^*_{ik}/q^*_{ik}} = -1$, for all $k \in \mathbf{N}_i$.[85] This expression entails two observations. First, the elasticity being the same across firms in the same sector is a natural consequence of Assumption 4.4. Second, unitary elasticity suggests that the sectoral demand, coupled with the strategic interactions, is "strong enough" to affect the price level in a way that exactly offsets the effect of a change in quantity demanded, keeping the sectoral aggregator's total expenditure unchanged. In contrast, the demand in monopolistic competition (i.e., in the absence of strategic forces) is inelastic due to the firm's market power (see Appendix C.8.4).

4.3 Estimation

Since the identification results demonstrated above are constructive, I build on the analogy principle to obtain a nonparametric estimator for the policy effect (14).[86] I first nonparametrically estimate the values of the firm-level quantity and price, and the first- and second-order derivatives of the firm's production function. Guided by the theory, I then combine these to derive the nonparametric estimator for (14). Given that the object of interest is continuous with respect to the exogenous variables, the resulting estimator is consistent. The accuracy of my estimator is verified through a numerical simulation in Appendix F.

As stated in Section 3, I acknowledge the possibility that the data on firm-level revenues are contaminated by measurement errors. To purge the measurement errors, my estimation of the firm-level quantity and price follows the convention of the industrial organization literature in applying a polynomial regression. In estimating the firm's production elasticities, I follow the specification suggested in Gandhi et al. (2019). See Appendix E for the details.

[84]While this is an artifact of the functional form assumptions, it is worth emphasizing that these assumptions include the specifications commonly employed in the existing literature, as seen in Example 4.1.

[85]The fact that the constant $\bar{c}_i$ depends on the macro and micro complementarities offers an alternative view, namely, the complementarities are determined in a way that the quantity elasticities become common across firms in the same sector.

[86]My approach takes a stance on econometric estimation rather than calibration. See Hansen and Heckman (1996) and Dawkins et al. (2001) for an extensive discussion about the methodological difference between calibration and econometric estimation. See also Matzkin (2013) for nonparametric estimation.

Compared to the calibration-type approach, my estimation procedure has two practical advantages. First, it does not require any external information (e.g., parameter estimates from the preceding research) and thus can be performed in a self-contained fashion. This feature obviates the need for conducting a "robustness check" with respect to the pre-specified values of some parameters (see Section 5.1.1).[87] Second, while the canonical calibration method is merely a benchmarking exercise, my approach prepares the ground for statistical hypothesis testing of model predictions, thereby allowing for the accumulation of knowledge in the hypothetico-deductive way.[88]

5 Empirical Application: CHIPS and Science Act of 2022

In this section, I study the empirical relevance of the joint existence of a production network and firms' strategic interactions by taking my model to the real-world data described in Section 3. As a policy narrative, I investigate the recent episode of the CHIPS and Science Act (CHIPS), which was passed into law in 2022 and aims to invest nearly \$53 billion in the U.S. semiconductor manufacturing, research and development, and workforce. This policy also includes a 25% tax credit for manufacturing investment, which is projected to provide up to \$24.25 billion for the next 10 years.[89] In my model, this tax credit can be analyzed as an additional subsidy targeted at the computer and electronic product manufacturing industry (Appendix B.2.2), which is indexed by n. Simply dividing the estimated \$24.25 billion by 10 years implies \$2.43 billion per year. This corresponds to raising the subsidy to 22.77%.[90] In my dataset, the historically observed support for a subsidy on this industry is between 3.57% and 16.39%.[91]

However, analyzing the whole part of this policy requires the researcher to send the value of the subsidy to outside the observed support, while my identification result hinges on the "within the observed support" assumption (Assumption 4.2). Extending my analysis to outside the support is

[87] The benefit of this property becomes acute when there are no existing works that align closely to the setup being studied by the researcher, as there is no hope of "borrowing" estimates from other research. This is actually the case with the present paper. Further discussion on this and others can be found in Dawkins et al. (2001).

[88] See Dawkins et al. (2001) for a further discussion about these two methodologies. Cartwright (2007) and Deaton and Cartwright (2018) compare the econometric policy analysis and statistical causal inference methods (such as randomized control trials) from a philosophical viewpoint. Moreover, Heckman and Vytlacil (2007) emphasize the merits of using economic models to accumulate knowledge across studies.

[89] See Appendix G.1 for the detail of the CHIPS and Science Act of 2022.

[90] The total amount of value-added tax in 2021 is \$8.44 billion, and the total value of material input (before tax and subsidy are applied) is \$47.74 billion. Hence, $(8.44 + 2.43)/47.74 \times 100 = 22.77\%$. See Appendix B.2.2.

[91] In the dataset, the semiconductor subsidy ranged from 3.57% in 2007 to 16.39% in 2019. In terms of the notation in Section 4, it is represented as $\mathscr{T}_n = [0.0357, 0.1639]$.

possible at the cost of additional assumptions, as explored in Canen and Song (2022). But this goes beyond the scope of this paper and is left for future work. Instead, the exercise of this section focuses on a part of the CHIPS subsidy. Specifically, I consider a hypothetical policy scenario of increasing the subsidy on the semiconductor industry from the 2021 level of 15.03% to an alternative ratio of 16.03% — equivalent to \$0.48 billion.[92] This accounts for approximately one-eighth of the per-year tax credit.[93] Note that this policy scenario satisfies Assumption 4.2. It is assumed that the semiconductor industry is the only industry that is directly targeted during this policy reform.

The goal of this section is to discuss the empirical relevance of the joint existence of a production network and firms' strategic interactions by first estimating the change in GDP due to this counterfactual industrial policy and then analyzing the mechanism behind the estimated policy effect. In Section 5.1, I first calculate the estimate of the policy effect (14). To shed light on the policy relevance of accounting for strategic interactions, I carry out the estimation for both monopolistic and oligopolistic cases.[94] In Section 5.2, I take advantage of the structural construction of my framework to provide a breakdown of the gains and losses of the policy reform into sector-level price and quantity effects. To understand the determination of these effects, I further explore the comovement of sectoral price and material cost indices.

5.1 The Policy Effect: Change in GDP

Based on (15), I estimate the change in GDP due to the policy reform from $\tau_n^0 = 0.1503$ to $\tau_n^0 = 0.1603$. An advantage of my approach is that the responsiveness of GDP can be traced out as a (possibly nonlinear) function of the subsidy over $[\tau_n^0, \tau_n^1]$. For computation purposes, I divide this interval evenly into a fixed number of segments and calculate the estimate according to

$$\widehat{\Delta Y}(\tau_n^0, \tau_n^1) \approx \sum_{v=0}^{\bar{v}-1} \sum_{i=1}^{N} \widehat{\frac{dY_i(s)}{ds}\bigg|}_{s=\tau_n^0+v\Delta\tau_n} \times \Delta\tau_n, \tag{24a}$$

[92] To make the analysis as close to reality as possible, I set the current policy regime to the latest year available, which is 2021. In terms of the model, this policy reform can be expressed by letting $\tau_n^0 = 0.1503$ and $\tau_n^1 = 0.1603$.

[93] Observe that $\frac{16.03-15.03}{22.77-15.03} = 0.1292$. One way to interpret this policy scenario is that it takes time to put the whole part of the CHIPS Act into effect, and what can be realized in the short run is only a part of it. This view is consistent with the short-run perspective of this paper.

[94] In view of Corollary C.3, these two cases can be analyzed by the same procedure.

where the symbol $\widehat{\ }$ is used to denote an estimator or estimate, and $\Delta\tau_n := \frac{\tau_n^1 - \tau_n^0}{\bar{v}}$ with $\bar{v}$ being the number of bins equally segmenting the interval $[\tau_n^0, \tau_n^1]$.[95] To highlight the consequence of ignoring the nonlinearity, I also estimate the policy effect using the following approximation:

$$\widehat{\Delta Y}(\tau_n^0, \tau_n^1) \approx \sum_{i=1}^{N} \left.\widehat{\frac{dY_i(s)}{ds}}\right|_{s=\tau_n^0} \times (\tau_n^1 - \tau_n^0). \tag{24b}$$

That is, the estimate is computed by assuming that the responsiveness of GDP is constant throughout the course of the policy change at the level of the current policy regime.

Table 1 compares the estimates for the policy effect based on (24a) and (24b) in both cases of monopolistic and oligopolistic competition. Two things stand out about this table. First, the estimate (24a) under oligopolistic competition is markedly different from that under monopolistic competition; the former is about 111 percent lower relative to the latter, flipping the sign from positive to negative.[96] This considerable discrepancy reflects the impact of the policy reform coming through the strategic interactions as studied in Section 2.7, highlighting their empirical relevance. Second, regardless of the type of market competition, the estimates based on (24b) are noticeably different from those based on (24a). This underlines the substantial degree of nonlinearity in the responsiveness of GDP as a function of the subsidy, which is visualized in Figure 2. The nonlinearity essentially arises from the fact that the firms' reactions depend on their quantity and price, as well as their production elasticities, each of which in turn depends on the value of the underlying subsidy.

Three caveats in interpreting the implications of Table 1 should be clarified before proceeding. First, the primary focus of this section is not on accurately gauging the size of the policy effect, but on empirically assessing the significance of the presumed economic mechanism in policy effects. Second, the dataset used in this paper is by no means representative of the universe of U.S. firms.[97] Third, the estimates are obtained by ignoring part of the demand-side heterogeneity (Assumption 4.4). With these caveats firmly in mind, it is important not to misconstrue Table 1 as a generic

[95] In this analysis, I set $\bar{v} = 20$.

[96] One may wonder if there is a chance that further increasing the subsidy by, say, 2% eventually reverts the policy effect to being positive. However, my identification result builds on Assumption 4.2, which restricts an alternative policy to stay within the observed support of the policy variable. Establishing the identification for a policy that sends the policy variable to outside the observed support in general requires additional invariance conditions, as studied by Canen and Song (2022).

[97] In fact, the Compustat data are not representative of the universe of U.S. firms, and moreover the dataset goes through multiple steps of outlier and missing data elimination (see Appendix B).

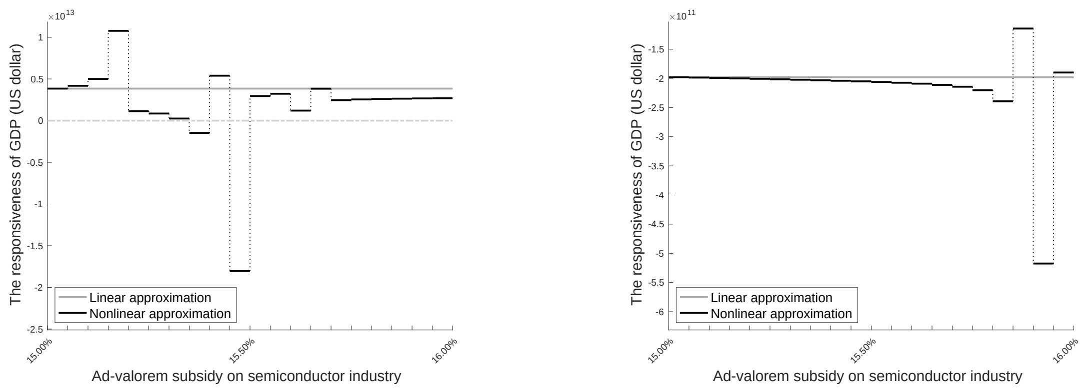
endorsement of the (in)effectiveness of industrial policy; rather, it should be understood as empirical evidence in support of the policy relevance of the firms' strategic forces accruing through the production network, a property illuminated in Section 2.7.

Table 1: The estimated policy effect under different market structures

(billion U.S. dollars)	Monopolistic competition	Oligopolistic competition
Estimates based on (24a)	19.34	-2.17
Estimates based on (24b)	38.39	-1.98

Note: This table compares the estimates for the object of interest (14) based on the benchmark and my method. The estimates are measured in billions of U.S. dollars.

Figure 2: The total derivative of Y with respect to τ_n



(a) Monopolistic Competition (b) Oligopolistic Competition

Note: This figure illustrates the estimates of the total derivative of (economy-wide) GDP with respect to the semiconductor subsidy between $\tau_n = 15.03\%$ and 16.03%. Panel (a) shows the result for the case of monopolistic competition and panel (b) for the case of oligopolistic competition. The solid black line represents the estimates based on the nonlinear approximation (24a). The solid medium grey line indicates the estimates based on the linear approximation (24b). The dash-dotted light grey line stands for zero. Hence, the part surrounded by the light grey line and back line above it measures the total increment of GDP over the course of the policy reform, while the other part gives the total decrement of GDP. The difference between these two areas delivers the estimated value of the policy effect according to (24a). Similarly, the area surrounded by the light grey line and medium grey line gives the estimated value of the policy effect according to (24b).

5.1.1 Robustness

In general, there are three types of "robustnesses" that require some care, namely, *i*) robustness with respect to the choices of pre-specified parameter values, *ii*) robustness with respect to the criteria for data construction and cleaning, and *iii*) robustness with respect to the choices of truncation and turning parameters in the estimators. For the first case, as discussed in Section 4.3, my approach does not presuppose any external information, thereby being free from any concern of this type. Second, the dataset used in my analysis goes through several steps of outliers and missing data elimination. These manipulations are rationalized by the assumptions imposed on the model (see Appendix B). Relaxing the criteria for these steps runs the risk of misspecification, which is of great interest in its own right and exceeds the scope of this paper. The third type, in my case, pertains to *iii-a*) the choice of degree of polynomials in estimating the firm-level revenue function and share regressions, and *iii-b*) the choice of the number of bins ($\bar{v}$ in (24a)). In my estimation algorithm, the former is chosen adaptively, leaving the latter as the only computation parameter that needs to be given before the implementation.[98] In calculating the main results, it is set equal to 20. Robustness checks with respect to this choice are conducted and illustrated in Appendix G.2.1. Overall, the results are both quantitatively and qualitatively unaffected.

5.2 Mechanism

To study the mechanism behind the results obtained in Section 5.1, I investigate the determination of the integrand of (15) (i.e., the responsiveness of sectoral GDP).

5.2.1 Responsiveness of sectoral GDP

Design. I anchor my interpretation of the responsiveness of sectoral GDP around (16):

$$\frac{dY_i(s)}{ds}\bigg|_{s=\tau_n} = \underbrace{\sum_{k=1}^{N_i} \frac{dp^*_{ik}}{d\tau_n} q^*_{ik}}_{\text{price effect}} + \underbrace{\sum_{k=1}^{N_i} p^*_{ik} \frac{dq^*_{ik}}{d\tau_n}}_{\text{quantity effect}} + \Bigg\{ \underbrace{\Bigg(-\sum_{k=1}^{N_i}\sum_{j=1}^{N} \frac{dP^*_j}{d\tau_n} m^*_{ik,j} \Bigg)}_{\text{wealth effect}} + \underbrace{\Bigg(-\sum_{k=1}^{N_i}\sum_{j=1}^{N} P^*_j \frac{dm^*_{ik,j}}{d\tau_n} \Bigg)}_{\text{switching effect}} \Bigg\}, \tag{25}$$

which states that the marginal effect of a policy change consists of changes in revenue and expenditure on material input net of subsidies. The former is broken down into price and quantity effects. When a firm produces more of its output, the price effect dictates the loss due to the increased supply in light of the law of demand. Under oligopolistic competition, this downward pressure depends not only on the increase in a firm's own quantity, but also on a change in every other firm's output quantity through the cross-price elasticities of demand. The other component of (25)

[98] Investigating the criteria of these adaptive selections per se is of independent interest, and is left to be explored.

can similarly be decomposed into two parts: the wealth and switching effects. The wealth effects are changes in a firm's "budget" as a result of changes in sectoral price indices. The switching effects are changes in the sectoral composition of the firm's input purchase, holding the price level constant.

Result. The empirical estimates for (25) at $\tau_n = \tau_n^0$ are displayed in Tables 7 and 8. From these tables, it can be seen that the sectoral distributional consequence — which sector wins and which sectors lose — depends on the tension between the two types of price and quantity effects defined in (25). For example, take the computer and electronic products industry, which is one of the top five industries in the case of monopolistic competition, while falling to the least benefited in oligopolistic competition. Under monopolistic competition, the semiconductor firms can exercise market power to increase their output with keeping the decline in the output price relatively mild. The resulting positive revenue effect is large enough to compensate for the increased input costs. When the markets are oligopolistic, the positive quantity effects are exactly offset by the negative price effects (Proposition 4.1), while the positive wealth effects are surpassed by the negative switching effects, leaving the firms with a higher input cost. An intuition is that the semiconductor firms choose to produce more of their output because they expect other industries to decrease their output prices, which in turn will push down the semiconductor firms' input costs; however, other industries end up not lowering their output prices as much as the semiconductor firms have expected.

To explore this intuition, I next focus on the comovements between sectoral price indices.

5.2.2 Macro and Micro Complementarities

Key equations. Here, I derive the general-equilibrium version of Proposition 2.1 (i)' and (ii)', which are given, respectively, by (i)" $\frac{dP_i^{M*}}{d\tau_n} = -h_{i,n}^M \frac{P_n^{M*}}{1-\tau_n} + h_i^L \frac{dW^*}{d\tau_n}$ and (ii)" $\frac{dP_i^*}{d\tau_n} = \bar{\lambda}_{i\cdot}^M \frac{dP_i^{M*}}{d\tau_n} + \bar{\lambda}_{i\cdot}^L \frac{dW^*}{d\tau_n}$, where $h_{i,n}^M$ and $\bar{\lambda}_{i\cdot}^M$ are defined in Section 2.7.1 ($h_{i,n}^L$ and $\bar{\lambda}_{i\cdot}^L$ are analogously defined). Note that $-\frac{P_n^{M*}}{1-\tau_n}$ can be interpreted as the "initial" impact of the policy change, and $\frac{dW^*}{d\tau_n}$ can be written in terms of firm-level elasticities of production and inverse demand functions of all firms across sectors.[99] These two equations jointly envision the comovement of sectoral price and material cost

[99] Provided the identification of the latter, these two equations can thus be viewed as "reduced-form" equations, with which I can proceed as if the material cost indices responded first, followed by the adjustments of the sectoral price indices. Notice, though, that the reduced-form coefficients in the above three equations are already composites

indices. With this relationship in mind, I compute the elasticity of each sectoral price index relative to that of the semiconductor industry.

Result. My estimation suggests that in the case of oligopolistic competition, the other sector's price indices respond only poorly to the semiconductor industry's, while displaying a fair degree of comovement in the monopolistic case (Table 9). This is mostly clearly seen the electrical equipment, appliances, and components industry, an industry adjacent to the semiconductor industry. The elasticity of its price index amounts to about 38% of that of the semiconductor industry under monopolistic competition, but plunge dramatically to 2% in the oligopolistic case. This observation can be attributed to macro and micro complementarities. Overall, both types of complementarities (with respect to both labor and material inputs) are less pronounced in magnitude under oligopolistic competition, compared to the monopolistic case (Tables 10 and 11). This implies that the accumulated strategic forces act as a blockage in the sectoral production network,[100] and thus substantially weaken (if not eliminate) the intersectoral dependence.

Lastly, I look at this result through a back-of-the-envelop calculation of the reduced-form coefficients for the aggregate fiscal multiplier AFM_n at $\boldsymbol{\tau} = \boldsymbol{\tau}^0$:

$$AFM_n := \Big(\sum_{j=1}^{N} \frac{dY_j}{d\tau_n} d\tau_n\Big)\Big/ dS_n = \Big\{\Big(\sum_{j=1}^{N} \frac{dY_j}{d\tau_n} d\tau_n\Big)\Big/ \frac{dY_n}{d\tau_n} d\tau_n\Big\}\Big(\frac{dY_n}{d\tau_n} d\tau_n \Big/ dS_n\Big),$$

where $dS_n := \sum_{k=1}^{N_n} P_n^M m_{nk} d\tau_n$ is the marginal change in the policy expenditure on sector n.[101] The term in the curly bracket in the rightmost of this expression represents the network multiplier, while the other term indicates the sectoral fiscal multiplier.

The estimates for these multipliers are summarized in Table 2. The network multiplier under oligopolistic competition is remarkably dampened relative to the monopolistic case. The sectoral fiscal multiplier in the presence of firm's strategic interactions shows the opposite sign to that in monopolistic competition with the magnitude roughly the same. It is worth stressing once again that these two multipliers are determined by the macro and micro complementarities discussed

of firm-level production and inverse demand functions and thus do not allow for behavioral interpretations; rather, they only represent comovement patterns of the comparative statics.

[100] This aligns with Atkeson and Burstein (2008), who point to the role of firm's strategic interactions in explaining the incomplete pass-through of the cost shock to price.

[101] By the setup, $dS_n > 0$.

above.

Table 2: The network and fiscal multipliers

	Monopolistic competition	Oligopolistic competition
Aggregate fiscal multiplier	83.64	-4.31
Network multiplier	22.39	1.40
Sectoral fiscal multiplier	3.74	-3.07

Note: This table shows the estimates for the aggregate and sectoral fiscal multipliers, as well as the network multiplier, as defined in Section 5.2.2.

6 Conclusions

Industrial policies have been and will continue to be an important policy tool for policymakers to achieve a range of policy goals. This paper studies the causal impact of an industrial policy on an aggregate outcome in the presence of firm-level strategic interactions and sectoral production networks. Following the econometric policy evaluation literature, the causal effect in this paper is defined as a *ceteris paribus* difference in outcome variables across different policy regimes. To formulate this policy parameter, I develop a general equilibrium multisector model of heterogeneous oligopolistic firms with a production network. For the identification, I develop a new, multi-stage identification procedure that first decomposes the policy parameter into sector- and firm-level variables — firm-level sufficient statistics — and then recovers the latter by using the control function approach of the industrial organization literature, which in turn identifies the desired policy parameter. To keep track of the firm's strategic interactions, I restrict the classes of the firm's production and inverse demand functions. I show that these assumptions are general enough to encompass many specifications commonly used in the macroeconomics literature. Moreover, my approach is constructive, so that a nonparametric estimator for the policy effect can be obtained by reading this procedure in reverse without adapting any external information (e.g., parameter estimates from the preceding research).

A key mechanism of my model is that when firm-level production functions exhibit constant returns to scale, policy effects are mediated by the production network that compounds changes in firms' marginal profits not only through adjustments of their own actions but also via those of competitors' actions (i.e., strategic complementarities), with the latter absent in monopolistic competition. This additional wedge in network spillovers manifests itself as the differences in the comovements of sectoral price indices and material cost indices, or pass-through coefficients. In line with this observation, my empirical estimates, based on U.S. firm-level data, suggest that comovement patterns in response to an additional subsidy on the semiconductor industry differ substantially between monopolistic and oligopolistic competition. The resulting policy effect in oligopolistic competition is approximately 111 percent lower than that in monopolistic competition, meaning that the presence of firm's strategic interactions has potential to even revert the policy predictions. This observation echoes the policy relevance of jointly accounting for firm's strategic interactions and a production network.

Interpreting the results displayed in this paper requires some care because they are susceptible to errors to the extent that the Compustat data are incomplete and non-representative and incur substantial imputation.[102] Besides the data limitation, there are three directions for future work. First, since my framework is fairly general, it can straightforwardly be extended to embrace other types of policies such as fiscal and monetary policies and trade policies. Second, this paper abstracts away from the firm's entry and exit problem over the course of policy reform, restricting the scope of analysis to short-run policy effects. Accommodating a long-run perspective inserts an additional layer into my framework, namely, the free-entry condition. Deriving the comparative statics, however, is nontrivial in my setup as the number of firms is finite, and thus the standard notion of derivatives cannot be well-defined. Third, the identification analysis of this paper assumes that the economy features a single equilibrium, the same equilibrium is played over the course of a policy reform, and the policy reform is restricted to be within the historically observed support. These limitations can be simultaneously addressed at the cost of additional assumptions concerning the equilibrium selection probability, as studied in Canen and Song (2022). Lastly, my model is static and thus silent about the policy implications of capital accumulation, which is usually at the center of policy debate. An extension to a dynamic environment requires an explicit consideration of not only the firm's own future choices but also competitors' future choices. This convoluted forward-looking nature opens up another source of multiplicity of equilibria.

[102] See Baqaee and Farhi (2020) and Covarrubias et al. (2020).

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A Comparative Statics

In this section, theoretical results displayed in Section 2 are derived. The goal of this section is to solve for comparative statics — the responsiveness of firm-level and sector-level variables with respect to a change in the policy variable (i.e., sector-specific subsidy). The results of this section express the comparative statics in terms of the endogenous variables in the current equilibrium, the exogenous variables and the policy-invariant functions, each of which are delineated in Section 2. The exposition is streamlined along the firm's decision process.

Remark A.1. *For the sake of econometric analysis, the main text assumes that the quantity of labor input is determined prior to material input, as described in* (6)*. As far as its quantitative implications are concerned, however, this "sequential decision" problem can equally be rewritten as a standard simultaneous decision problem (Ackerberg et al. 2015). For ease of exposition, I thus consider the simultaneous decision formulation throughout this section.*

A.1 Profit Maximization

In each sector $i \in \mathbf{N}$, for the equilibrium wage W^*, the material price index P_i^{M*} and for each firm's optimal quantity q_{ik}^*, there exists a pair of labor and material inputs that satisfies the following one-step profit maximization problem:

$$(\bar{\ell}_{ik}^*, \bar{m}_{ik}^*) \quad \in \quad \underset{\ell_{ik}, m_{ik}}{\arg\max} \left\{ p_{ik}^* q_{ik}^* - (W^* \ell_{ik} + P_i^{M*} m_{ik}) \right\} \qquad s.t. \qquad q_{ik}^* = f_i(\ell_{ik}, m_{ik}; z_{ik}).$$

The first order conditions with respect to labor and material inputs are given, respectively, by:

$$[\ell_{ik}] : mr_{ik}(\cdot)^* \frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}} = W^* \tag{26}$$

$$[m_{ik}] : mr_{ik}(\cdot)^* \frac{\partial f_i(\cdot)^*}{\partial m_{ik}} = P_i^{M*}, \tag{27}$$

where $mr_{ik}(\mathbf{q}_i)$ is the firm k's marginal revenue function, and I denote $mr_{ik}(\cdot)^* := mr_{ik}(\mathbf{q}_i^*)$. Moreover, define $\frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}} := \frac{\partial f_i(\cdot)}{\partial \ell_{ik}}\Big|_{(\ell_{ik}, m_{ik}) = (\bar{\ell}_{ik}^*, \bar{m}_{ik}^*)}$, and $\frac{\partial f_i(\cdot)^*}{\partial m_{ik}} := \frac{\partial f_i(\cdot)}{\partial m_{ik}}\Big|_{(\ell_{ik}, m_{ik}) = (\bar{\ell}_{ik}^*, \bar{m}_{ik}^*)}$. Likewise, $\frac{\partial mr_{ik}(\cdot)^*}{\partial q_{ik}} := \frac{\partial mr_{ik}(\cdot)}{\partial q_{ik'}}\Big|_{\mathbf{q}_i = \mathbf{q}_i^*}$. Taking total derivatives of the both hand sides of (26) and (27) in terms of τ_n yields, respectively,

$$\left(\sum_{k'=1}^{N_i} \frac{\partial mr_{ik}(\cdot)^*}{\partial q_{ik'}} \frac{dq_{ik'}^*}{d\tau_n} \right) \frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}} + mr_{ik}(\cdot)^* \left(\frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik}^2} \frac{d\bar{\ell}_{ik}^*}{d\tau_n} + \frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik} \partial m_{ik}} \frac{d\bar{m}_{ik}^*}{d\tau_n} \right) = \frac{dW^*}{d\tau_n} \tag{28}$$

$$\left(\sum_{k'=1}^{N_i} \frac{\partial mr_{ik}(\cdot)^*}{\partial q_{ik'}} \frac{dq_{ik'}^*}{d\tau_n} \right) \frac{\partial f_i(\cdot)^*}{\partial m_{ik}} + mr_{ik}(\cdot)^* \left(\frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik} m_{ik}} \frac{d\bar{\ell}_{ik}^*}{d\tau_n} + \frac{\partial^2 f_i(\cdot)^*}{\partial m_{ik}^2} \frac{d\bar{m}_{ik}^*}{d\tau_n} \right) = \frac{dP_i^{M*}}{d\tau_n}, \tag{29}$$

where

$$\frac{dq_{ik}^*}{d\tau_n} = \frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}} \frac{d\bar{\ell}_{ik}^*}{d\tau_n} + \frac{\partial f_i(\cdot)^*}{\partial m_{ik}} \frac{d\bar{m}_{ik}^*}{d\tau_n}.$$

Remark A.2. *Here, remember that firms only choose their output quantities through profit maximization, while input decisions are made in a way that minimizes total costs. Thus the "optimal" labor $\bar{\ell}_{ik}^*$ and material inputs $\bar{m}_{ik}^*$ chosen above are not necessarily the same as the ones that are actually chosen by the firm. Rather, $\bar{\ell}_{ik}^*$ and $\bar{m}_{ik}^*$ should be understood as a combination of inputs that only pins down the change in the firm's output quantity, whose corresponding production possibility frontier is in turn used to determine the optimal input choices in the subsequent cost minimization problem (see Remark A.5 in Appendix A.2).*

From (28) and (29), it follows that, in equilibrium,

$$\begin{aligned}
&\left(\sum_{k'=1}^{N_i} \frac{\partial mr_{ik}(\cdot)^*}{\partial q_{ik'}} \frac{dq_{ik'}^*}{d\tau_n}\right)\left(\frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}} \bar{\ell}_{ik}^* + \frac{\partial f_i(\cdot)^*}{\partial m_{ik}} \bar{m}_{ik}^*\right) \\
&+ mr_{ik}(\cdot)^* \left(\frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik}^2} \bar{\ell}_{ik}^* + \frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik} \partial m_{ik}} \bar{m}_{ik}^*\right) \frac{d\bar{\ell}_{ik}^*}{d\tau_n} + mr_{ik}(\cdot)^* \left(\frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik} \partial m_{ik}} \bar{\ell}_{ik}^* + \frac{\partial^2 f_i(\cdot)^*}{\partial m_{ik}^2} \bar{m}_{ik}^*\right) \frac{d\bar{m}_{ik}^*}{d\tau_n} \\
&= \frac{dW^*}{d\tau_n} \bar{\ell}_{ik}^* + \frac{dP_i^{M*}}{d\tau_n} \bar{m}_{ik}^* \\
&\therefore \sum_{k'=1}^{N_i} \frac{\partial mr_{ik}(\cdot)^*}{\partial q_{ik'}} \frac{dq_{ik'}^*}{d\tau_n} = \frac{1}{q_{ik}^*}\left(\frac{dW^*}{d\tau_n} \bar{\ell}_{ik}^* + \frac{dP_i^{M*}}{d\tau_n} \bar{m}_{ik}^*\right), \qquad (30)
\end{aligned}$$

where the implication is a consequence of Assumption 2.4 (i). The expression (30) holds for each firm $k \in \mathbf{N}_i$ in the same sector i, thereby constituting a system of N_i equations:

$$\underbrace{\begin{bmatrix} \frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i1}} & \frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i2}} & \cdots & \frac{\partial mr_{i1}(\cdot)^*}{\partial q_{iN_i}} \\ \frac{\partial mr_{i2}(\cdot)^*}{\partial q_{i1}} & \frac{\partial mr_{i2}(\cdot)^*}{\partial q_{i2}} & \cdots & \frac{\partial mr_{i2}(\cdot)^*}{\partial q_{iN_i}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial mr_{iN_i}(\cdot)^*}{\partial q_{i1}} & \frac{\partial mr_{iN_i}(\cdot)^*}{\partial q_{i2}} & \cdots & \frac{\partial mr_{iN_i}(\cdot)^*}{\partial q_{iN_i}} \end{bmatrix}}_{=:\Lambda_{i,1}} \begin{bmatrix} \frac{dq_{i1}^*}{d\tau_n} \\ \frac{dq_{i2}^*}{d\tau_n} \\ \vdots \\ \frac{dq_{iN_i}^*}{d\tau_n} \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{\bar{\ell}_{i1}^*}{q_{i1}^*} & \frac{\bar{m}_{i1}^*}{q_{i1}^*} \\ \frac{\bar{\ell}_{i2}^*}{q_{i2}^*} & \frac{\bar{m}_{i2}^*}{q_{i2}^*} \\ \vdots & \vdots \\ \frac{\bar{\ell}_{iN_i}^*}{q_{iN_i}^*} & \frac{\bar{m}_{iN_i}^*}{q_{iN_i}^*} \end{bmatrix}}_{=:\Lambda_{i,2}} \begin{bmatrix} \frac{dW^*}{d\tau_n} \\ \frac{dP_i^{M*}}{d\tau_n} \end{bmatrix}. \qquad (31)$$

In order to ensure that this system generates a unique set of firms' quantity changes in response to the change in subsidy, I impose the following regularity condition.

Assumption A.1 (Regularity Condition 1)**.** *For each sector $i \in \mathbf{N}$, the matrix*

$$\Lambda_{i,1} \coloneqq \begin{bmatrix} \frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i1}} & \frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i2}} & \cdots & \frac{\partial mr_{i1}(\cdot)^*}{\partial q_{iN_i}} \\ \frac{\partial mr_{i2}(\cdot)^*}{\partial q_{i1}} & \frac{\partial mr_{i2}(\cdot)^*}{\partial q_{i2}} & \cdots & \frac{\partial mr_{i2}(\cdot)^*}{\partial q_{iN_i}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial mr_{iN_i}(\cdot)^*}{\partial q_{i1}} & \frac{\partial mr_{iN_i}(\cdot)^*}{\partial q_{i2}} & \cdots & \frac{\partial mr_{iN_i}(\cdot)^*}{\partial q_{iN_i}} \end{bmatrix}$$

is nonsingular.

Remark A.3. *Assumption A.1 requires that the column vectors of $\Lambda_{i,1}$ are linearly independent, and guarantees the premultiplying term of the left-hand side of* (31) *is invertible. This assumption trivially holds in monopolistic competition as the matrix $\Lambda_{i,1}$ simplifies to a diagonal matrix.*

Note here that under the setup in Section 2, firms' marginal costs are constant, and thus it holds $\frac{\partial mr_{ik}(\cdot)}{\partial q_{ik'}} = \frac{\partial \frac{\partial \pi_{ik}(\cdot)}{\partial q_{ik}}}{\partial q_{ik'}}$. In light of this, the economic content of Assumption A.1 can be envisioned in terms of firms' strategic complementarities.

Example A.1 (Duopoly)**.** *For simplicity, consider a case of duopoly, wherein firm* 1 *and* 2 *are engaged in quantity competition. It generally holds that $|\frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i1}}| \geq |\frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i2}}|$. But, it is also true that $|\frac{\partial mr_{i2}(\cdot)^*}{\partial q_{i1}}| \leq |\frac{\partial mr_{i2}(\cdot)^*}{\partial q_{i2}}|$. Hence, there is no such a constant that makes the column vectors $\Lambda_{i,1}$ linearly dependent. In this sense, Assumption A.1 excludes a situation where the firm's own strategic complementarity is exactly the same as the competitor's. See also Appendix A.4.2.*

Under Assumption A.1, the system of equations (31) can be solved for $\{\frac{dq^*_{ik}}{d\tau_n}\}_{k=1}^{N_i}$:

$$\begin{bmatrix} \frac{dq^*_{i1}}{d\tau_n} \\ \frac{dq^*_{i2}}{d\tau_n} \\ \vdots \\ \frac{dq^*_{iN_i}}{d\tau_n} \end{bmatrix} = \Lambda_{i,1}^{-1}\Lambda_{i,2} \begin{bmatrix} \frac{dW^*}{d\tau_n} \\ \frac{dP_i^{M*}}{d\tau_n} \end{bmatrix}.$$

In this expression, $\Lambda_{i,1}^{-1}$ captures the strategic interactions between firms in terms of strategic complementarities. Moreover, it can also be seen, from this expression, that $\{\frac{dq^*_{ik}}{d\tau_n}\}_{k=1}^{N_i}$ depends on the levels of firm's current inputs and output through $\Lambda_{i,2}$ as well as the responsiveness of the wage and material cost index.

Letting $\lambda^{-1}_{ik,k'}$ be the (k, k') entry of the matrix $\Lambda_{i,1}^{-1}$, I obtain

$$\begin{aligned} \frac{dq^*_{ik}}{d\tau_n} &= \left(\sum_{k'=1}^{N_i} \lambda^{-1}_{ik,k'} \frac{\bar{\ell}^*_{ik'}}{q^*_{ik'}}\right)\frac{dW^*}{d\tau_n} + \left(\sum_{k'=1}^{N_i} \lambda^{-1}_{ik,k'} \frac{\bar{m}^*_{ik'}}{q^*_{ik'}}\right)\frac{dP_i^{M*}}{d\tau_n} \\ &= \bar{\lambda}^L_{ik}\frac{dW^*}{d\tau_n} + \bar{\lambda}^M_{ik}\frac{dP_i^{M*}}{d\tau_n}, \end{aligned} \tag{32}$$

where $\bar{\lambda}_{ik}^L := \sum_{k'=1}^{N_i} \lambda_{ik,k'}^{-1} \frac{\bar{\ell}_{ik'}^*}{q_{ik'}^*}$ and $\bar{\lambda}_{ik}^M := \sum_{k'=1}^{N_i} \lambda_{ik,k'}^{-1} \frac{\bar{m}_{ik'}^*}{q_{ik'}^*}$ correspond to the kth element of the first and second column of the matrix $\Lambda_{i,1}^{-1}\Lambda_{i,2}$, respectively. In (32), the weighted sums $\bar{\lambda}_{ik}^L$ and $\bar{\lambda}_{ik}^M$, respectively, dictate the comovements between the change in firm-level output quantity and change in wage, and between the change in firm-level output quantity and change in sectoral material cost index.[103]

Notice that while the denominator of $\bar{\lambda}_{ik}^L$ includes all of $\{\frac{\partial mr_{ik}(\cdot)}{\partial q_{ik'}}\}_{k,k' \in \mathbf{N}_i}$, the numerator does not contain the terms $\{\frac{\partial mr_{ik'}(\cdot)}{\partial q_{ik}}\}_{k \in \mathbf{N}_i}$, thereby the ratio $\bar{\lambda}_{ik}^L$ backing out the contribution of changes in q_{ik} to a sectoral measure of strategic complementaritiy given by the denominator.[104] This measure summarizes the extent of influence that firms exert in strategic interactions. The same is true for $\bar{\lambda}_{ik}^M$. These indices are informative about the extent to which the market competition is affected by the change in firm k's quantity, and are similar in spirit to the index of competitor price changes of Amiti et al. (2019).[105] This observation can clearly be seen in the examples of duopoly and monopolistic competition.

Example A.2 (Duopoly)**.** *Continuing the same setup as Example A.1, the inverse matrix $\Lambda_{i,1}^{-1}$ is given by:*

$$\Lambda_{i,1}^{-1} = \frac{1}{det(\Lambda_{i,1})} \begin{bmatrix} \frac{\partial mr_{i2}(\cdot)^*}{\partial q_{i2}} & -\frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i2}} \\ -\frac{\partial mr_{i2}(\cdot)^*}{\partial q_{i1}} & \frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i1}} \end{bmatrix}$$

where $det(\Lambda_{i,1}) = \frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i1}} \frac{\partial mr_{i2}(\cdot)^*}{\partial q_{i2}} - \frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i2}} \frac{\partial mr_{i2}(\cdot)^*}{\partial q_{i1}}$. Note first that the denominator of the right-hand side, i.e., $det(\Lambda_{i,1})$, involves every element of $\Lambda_{i,1}$, and thus can be viewed as a measure of the sector's overall strategic complementarity.[106] *Next, each of the first row of the numerators, i.e., $\frac{\partial mr_{i2}(\cdot)^*}{\partial q_{i2}}$ and $-\frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i2}}$, each of which represents the strategic complementarity with respect to the firm 2's quantity adjustment. Divided by $det(\Lambda_{i,1})$ and summed over columns with the weights, the indices $\bar{\lambda}_{i1}^L$ and $\bar{\lambda}_{i1}^M$ back out the contribution of the firm 1's quantity change to the sector's overall strategic complementarity. See also Appendix A.4.2.*

[103] The weights $\frac{\bar{\ell}_{ik}^*}{q_{ik}^*}$ and $\frac{\bar{m}_{ik}^*}{q_{ik}^*}$ represent measures of the firm k's labor and material productivity, respectively. Note that these weights are not normalized to equal one.

[104] To see this, observe that for a square matrix $\mathcal{O}$, the inverse matrix $\mathcal{O}^{-1}$ is given by $\mathcal{O}^{-1} = \frac{\text{adj}(\mathcal{O})}{|\mathcal{O}|}$, where adj$(\mathcal{O})$ is the adjoint matrix of $\mathcal{O}$, i.e., the transpose of the cofactor matrix. The cofactor matrix C of $\mathcal{O}$ is defined as $C := [c_{a,b}]_{a,b}$, where $c_{a,b} := (-1)^{a+b}|M_{a,b}|$, with $M_{a,b}$ representing the minor matrix of $\mathcal{O}$ that can be created by eliminating the a-th row and b-th column from the matrix $\mathcal{O}$. In my context, the k'-th column of the cofactor matrix of $\Lambda_{i,1}$ excludes $\{\frac{\partial mr_{ik}(\cdot)^*}{\partial q_{ik'}}\}_{k=1}^{N_i}$, all of which are in turn ruled out from the k'-th row of the adjoint matrix. Since the determinant involves the effect of all firms' quantity changes, the weighted sum along each row of $\Lambda_{i,1}^{-1}$ reflects the contribution of the changes in firm k''s output quantity.

[105] While their index compares the firm's contribution to the rest of the market, my indices $\bar{\lambda}_{ik}^L$ and $\bar{\lambda}_{ik}^M$ compares the rest of the market to the entire market, backing out the firm's share.

[106] In general, the determinant of a 2×2 matrix gives the (signed) area of a parallelogram spanned by its column vectors. In the case of $\Lambda_{i,1}$, the column vectors consist in the partial derivatives of firm's marginal revenues with respect to each firm. Thus $det(\Lambda_{i,1})$ is a natural measure that summarizes firms' contributions to the overall strategic complementarity. Without loss of generality, the sign of the determinant can be assumed to be positive, as it can be reversed through swapping some of the column vectors. Rather, it is a mapping of the overall strategic substitutability/complementarity from $(-\infty, \infty)$ to $[0, \infty)$, acting as a normalization constant.

Example A.3 (Monopolistic Competition). *I consider the same setup as Example A.1, but depart by assuming that both firms are monopolistic. In this case,*

$$\Lambda_{i,1}^{-1} = \begin{bmatrix} (\frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i1}})^{-1} & 0 \\ 0 & (\frac{\partial mr_{i2}(\cdot)^*}{\partial q_{i2}})^{-1} \end{bmatrix}.$$

Then, the two measures of the firm 1's contribution to the overall sectoral strategic complementarity are given by $\bar{\lambda}_{i1}^L = (\frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i1}})^{-1}\frac{\bar{\ell}_{i1}^*}{q_{i1}^*}$ *and* $\bar{\lambda}_{i1}^M = (\frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i1}})^{-1}\frac{\bar{m}_{i1}^*}{q_{i1}^*}$*, both of which are typically negative.*[107] *Provided that both* $\bar{\lambda}_{i1}^L$ *and* $\bar{\lambda}_{i1}^M$ *are negative,* (32) *implies that when the wage and material cost index become higher in reaction to a policy change, firm 1 decreases its output quantity. An analogous argument applies to firm 2. When the firms are oligopolistic as in Example A.2, the signs of* $\bar{\lambda}_{i1}^L$ *and* $\bar{\lambda}_{i1}^M$ *are ambiguous because they involve strategic complementarities. See Appendix A.4*

In equilibrium, the sectoral price index associated with the sectoral aggregator (3) satisfies the following unit cost condition: for each $i = 1, \ldots, N$,

$$P_i^* \quad = \quad \min_{\{e_{ik}\}_{i=1}^{N}} \sum_{k=1}^{N_i} p_{ik}^* e_{ik} \qquad s.t. \qquad F_i(\{e_{ik}\}_{k=1}^{N_i}) \geq 1, \tag{33}$$

where p_{ik}^* is the price of a product set by firm k in sector i. By solving this, it follows that there exists a mapping $\mathcal{P}_i : \mathscr{S}_i^{N_i} \to \mathbb{R}_+$ such that

$$P_i^* = \mathcal{P}_i(\mathbf{q}_i^*). \tag{34}$$

Totally differentiating (34) yields

$$\frac{dP_i^*}{d\tau_n} = \sum_{k'=1}^{N_i} \frac{\partial \mathcal{P}_i(\cdot)^*}{\partial q_{ik'}} \frac{dq_{ik'}^*}{d\tau_n}, \tag{35}$$

where $\frac{\partial \mathcal{P}_i(\cdot)^*}{\partial q_{ik'}} := \frac{\partial \mathcal{P}_i(\cdot)}{\partial q_{ik'}}\Big|_{\mathbf{q}_i=\mathbf{q}_i^*}$

Remark A.4. *(i) Associated with* (33) *is the (residual) inverse demand function* $\wp_{ik}(\cdot)$*, i.e.,* $p_{ik} = \wp_{ik}(\mathbf{q}_i^*)$*. By the chain rule, it holds that*

$$\frac{dp_{ik}^*}{d\tau_n} = \sum_{k'=1}^{N_i} \frac{\partial \wp_{ik}(\cdot)^*}{\partial q_{ik'}} \frac{dq_{ik'}^*}{d\tau_n}, \tag{36}$$

[107] Precisely, the sign depends on the demand side parameters. For instance, when the sectoral aggregator takes the form of a CES production function as in Example 4.1, these indices are negative as long as $\sigma_i > 2$.

where $\frac{\partial \wp_{ik}(\cdot)^*}{\partial q_{ik'}} := \frac{\partial \wp_{ik}(\cdot)}{\partial q_{ik'}}\Big|_{\mathbf{q}_i=\mathbf{q}_i^*}$. *Substituting* (32) *for* $\frac{dq^*_{ik'}}{d\tau_n}$ *leads to*

$$\frac{dp^*_{ik}}{d\tau_n} = \left(\sum_{k'=1}^{N_i} \frac{\partial \wp_{ik}(\cdot)^*}{\partial q_{ik'}} \bar{\lambda}^L_{ik'}\right) \frac{dW^*}{d\tau_n} + \left(\sum_{k'=1}^{N_i} \frac{\partial \wp_{ik}(\cdot)^*}{\partial q_{ik'}} \bar{\lambda}^M_{ik'}\right) \frac{dP^{M*}_i}{d\tau_n}. \tag{37}$$

(ii) An expression analogous to (35) *can be derived with respect to firms prices* $\{p_{ik'}\}_{k'=1}^{N_i}$*. With a slight abuse of notation, let* $\mathcal{P}_i(\mathbf{p}_i)$ *be a function such that* $P_i = \mathcal{P}_i(\mathbf{p}_i)$*. Then, a version of* (35) *is given by*

$$\frac{dP^*_i}{d\tau_n} = \sum_{k'=1}^{N_i} \frac{\partial \mathcal{P}_i(\cdot)^*}{\partial p_{ik'}} \frac{dp_{ik'}}{d\tau_n}. \tag{38}$$

Upon substituting (32) into (35), it holds that

$$\begin{aligned}
\frac{dP^*_i}{d\tau_n} &= \sum_{k'=1}^{N_i} \frac{\partial \mathcal{P}_i(\cdot)^*}{\partial q_{ik'}} \left(\bar{\lambda}^L_{ik'} \frac{dW^*}{d\tau_n} + \bar{\lambda}^M_{ik'} \frac{dP^{M*}_i}{d\tau_n}\right) \\
&= \bar{\lambda}^L_{i\cdot} \frac{dW^*}{d\tau_n} + \bar{\lambda}^M_{i\cdot} \frac{dP^{M*}_i}{d\tau_n},
\end{aligned} \tag{39}$$

where $\bar{\lambda}^L_{i\cdot} := \sum_{k'=1}^{N_i} \frac{\partial \mathcal{P}_i(\cdot)^*}{\partial q_{ik'}} \bar{\lambda}^L_{ik'}$ and $\bar{\lambda}^M_{i\cdot} := \sum_{k'=1}^{N_i} \frac{\partial \mathcal{P}_i(\cdot)^*}{\partial q_{ik'}} \bar{\lambda}^M_{ik'}$. These are a weighted sum of the elasticities of sectoral price index with respect to firms' quantities, with the weight assigned to a firm's index of strategic complementarity in that sector. From the expression (39), $\bar{\lambda}^L_{i\cdot}$ and $\bar{\lambda}^M_{i\cdot}$ can be interpreted as representing a pass-through of a change in the wage and material input cost to the sectoral price index, respectively.

Example A.4 (Monopolistic Competition)**.** *Continuing Example A.3 and assuming that* $\bar{\lambda}^L_{i1}$, $\bar{\lambda}^L_{i2}$, $\bar{\lambda}^M_{i1}$ *and* $\bar{\lambda}^M_{i2}$ *have all turned out to be negative, I can proceed to calculate* $\bar{\lambda}^L_{i\cdot}$ *and* $\bar{\lambda}^M_{i\cdot}$*. Due to the law of demand (i.e.,* $\frac{\partial \mathcal{P}_i(\cdot)^*}{\partial q_{ik'}} < 0$ *for all* $k' \in \mathbf{N}_i$*), these are both positive. In light of* (39)*, this in turn implies a higher sectoral price index in response to higher wage and material cost index, which accords with a lower output quantity seen in Example A.3.*

Meanwhile, the equilibrium material cost index P^{M*}_i satisfies the following unit cost condition:

$$P^{M*}_i \quad = \quad \min_{\{m_{ik,j}\}_{j\in\mathbf{N}}} \sum_{j=1}^{N} (1-\tau_i) P^*_j m_{ik,j} \qquad s.t. \qquad \mathcal{G}_i(\{m_{ik,j}\}_{j=1}^N) \geq 1,$$

from which I can write P^{M*}_i as a function of the sectoral price indices and the sector-specific subsidy, i.e.,

$$P^{M*}_i = \mathcal{P}^M_i(\{P^*_j\}_{j=1}^N, \tau_i). \tag{40}$$

Note that the function $\mathcal{P}^M_i(\cdot)$ encodes the information about the production network, carrying over from the aggregator $\mathcal{G}_i(\cdot)$; specifically, it embodies the shares of sectoral goods in material good used by firms in sector i.

Taking total derivatives of (40), it holds that

$$\frac{dP_i^{M*}}{d\tau_n} = \sum_{j=1}^{N} \frac{\partial \mathcal{P}_i^M(\cdot)^*}{\partial P_j} \frac{dP_j^*}{d\tau_n} + \frac{\partial \mathcal{P}_i^M(\cdot)}{\partial \tau_n} \mathbb{1}_{\{n=i\}}, \tag{41}$$

where $\mathbb{1}_{\{n=i\}}$ takes one if $n = i$, and zero otherwise. Substituting (39) for $\{\frac{dP_j^*}{d\tau_n}\}_{j=1}^N$ into (41), I obtain

$$\frac{dP_i^{M*}}{d\tau_n} = \left(\sum_{j=1}^{N} \frac{\partial \mathcal{P}_i^M(\cdot)^*}{\partial P_j} \bar{\lambda}_{j\cdot}^L\right) \frac{dW^*}{d\tau_n} + \sum_{j=1}^{N} \frac{\partial \mathcal{P}_i^M(\cdot)^*}{\partial P_j} \bar{\lambda}_{j\cdot}^M \frac{dP_j^{M*}}{d\tau_n} + \frac{\partial \mathcal{P}_i^M(\cdot)^*}{\partial \tau_n} \mathbb{1}_{\{n=i\}}. \tag{42}$$

The equation 42 holds true for all sectors, constituting a system of equations (simultaneous/structural equations). The next step is to solve these equations for comparative statics, or to derive "reduced-form" equations. Denoting $\Gamma_1 := \left[\frac{\partial \mathcal{P}_i^M(\cdot)^*}{\partial P_j} \bar{\lambda}_{j\cdot}^L\right]_{i,j=1}^N$ and $\Gamma_2 := \left[\frac{\partial \mathcal{P}_i^M(\cdot)^*}{\partial P_j} \bar{\lambda}_{j\cdot}^M\right]_{i,j=1}^N$, and letting $\iota := [1, 1, \ldots, 1]'$ be a $N \times 1$ vector of ones, I stack (42) over sectors to obtain the following system of equations:

$$\begin{aligned}
\begin{bmatrix} \frac{dP_1^{M*}}{d\tau_n} \\ \vdots \\ \frac{dP_N^{M*}}{d\tau_n} \end{bmatrix} &= \Gamma_1 \iota \frac{dW^*}{d\tau_n} + \Gamma_2 \begin{bmatrix} \frac{dP_1^{M*}}{d\tau_n} \\ \vdots \\ \frac{dP_N^{M*}}{d\tau_n} \end{bmatrix} + \begin{bmatrix} \frac{\partial \mathcal{P}_1^M(\cdot)^*}{\partial \tau_n} \mathbb{1}_{\{n=1\}} \\ \vdots \\ \frac{\partial \mathcal{P}_N^M(\cdot)^*}{\partial \tau_n} \mathbb{1}_{\{n=N\}} \end{bmatrix} \\
\therefore (I - \Gamma_2) \begin{bmatrix} \frac{dP_1^{M*}}{d\tau_n} \\ \vdots \\ \frac{dP_N^{M*}}{d\tau_n} \end{bmatrix} &= \Gamma_1 \iota \frac{dW^*}{d\tau_n} + \begin{bmatrix} \frac{\partial \mathcal{P}_1^M(\cdot)^*}{\partial \tau_n} \mathbb{1}_{\{n=1\}} \\ \vdots \\ \frac{\partial \mathcal{P}_N^M(\cdot)^*}{\partial \tau_n} \mathbb{1}_{\{n=N\}} \end{bmatrix}
\end{aligned} \tag{43}$$

where I represents an $N \times N$ identity matrix.

To ensure a unique solution, I impose the following regularity condition.

Assumption A.2 (Regularity Condition 2)**.** *The matrix $(I - \Gamma_2)$ is nonsingular.*

This assumption guarantees that the premultiplying term in 43 is invertible. Under Assumption A.2, it thus follows that

$$\begin{bmatrix} \frac{dP_1^{M*}}{d\tau_n} \\ \vdots \\ \frac{dP_N^{M*}}{d\tau_n} \end{bmatrix} = (I - \Gamma_2)^{-1} \Gamma_1 \iota \frac{dW^*}{d\tau_n} + (I - \Gamma_2)^{-1} \begin{bmatrix} \frac{\partial \mathcal{P}_1^M(\cdot)^*}{\partial \tau_n} \mathbb{1}_{\{n=1\}} \\ \vdots \\ \frac{\partial \mathcal{P}_N^M(\cdot)^*}{\partial \tau_n} \mathbb{1}_{\{n=N\}} \end{bmatrix}. \tag{44}$$

Observe here that Γ_2 is a version of the adjacency matrix capturing the input-output linkages among sectors. Hence, $(I - \Gamma_2)^{-1}$ can be conceived as a type of the Leontief inverse matrix, augmented by measures of strategic competition in the source sectors $\bar{\lambda}_{j\cdot}^M$ (i.e., market distortion).

The (i, n) entry of this strategic-complementarity-adjusted Leontief inverse, denoted by $h_{i,n}^M$, can be written as a geometric sum:[108] if $n \neq i$,

$$\bar{\lambda}_{n\cdot}^M \frac{\partial \mathcal{P}_i^M(\cdot)^*}{\partial P_n} + \sum_{j=1}^N \bar{\lambda}_{n\cdot}^M \frac{\partial \mathcal{P}_i^M(\cdot)^*}{\partial P_j} \bar{\lambda}_{j\cdot}^M \frac{\partial \mathcal{P}_j^M(\cdot)^*}{\partial P_n} + \sum_{j=1}^N \sum_{j'=1}^N \bar{\lambda}_{n\cdot}^M \frac{\partial \mathcal{P}_j^M(\cdot)^*}{\partial P_n} \bar{\lambda}_{j\cdot}^M \frac{\partial \mathcal{P}_{j'}^M(\cdot)^*}{\partial P_j} \bar{\lambda}_{j'\cdot}^M \frac{\partial \mathcal{P}_i^M(\cdot)^*}{\partial P_{j'}} + \dots, \tag{45}$$

and if $n = i$,

$$1 + \bar{\lambda}_{n\cdot}^M \frac{\partial \mathcal{P}_i^M(\cdot)^*}{\partial P_n} + \sum_{j=1}^N \bar{\lambda}_{n\cdot}^M \frac{\partial \mathcal{P}_i^M(\cdot)^*}{\partial P_j} \bar{\lambda}_{j\cdot}^M \frac{\partial \mathcal{P}_j^M(\cdot)^*}{\partial P_n} + \sum_{j=1}^N \sum_{j'=1}^N \bar{\lambda}_{n\cdot}^M \frac{\partial \mathcal{P}_j^M(\cdot)^*}{\partial P_n} \bar{\lambda}_{j\cdot}^M \frac{\partial \mathcal{P}_{j'}^M(\cdot)^*}{\partial P_j} \bar{\lambda}_{j'\cdot}^M \frac{\partial \mathcal{P}_i^M(\cdot)^*}{\partial P_{j'}} + \dots. \tag{46}$$

To gain some intuition for this infinite sum expression, suppose that sector i uses sector n's ($n \neq i$) intermediate good both directly and indirectly along the production network. For the sake of brevity, assume in addition that $\bar{\lambda}_{j\cdot}^M > 0$ for all $j \in \mathbf{N}$. When sector n is subsidized, the reduced input cost stimulates the production in that sector, leading to a lower sectoral output price index of sector n according to (39). The pass-through ratio is given by $\bar{\lambda}_{n\cdot}^M$. This change in sector n's output price index affects the cost index of sector i through multiple channels. The first term of (45) stands for the first-order spillover effect: the lower price index of sector n directly reduces sector i's input cost. The second term captures the second-order spillover effect coming via a third sector j. The output price index of sector j decreases as firms in sector j can produce more of their goods by taking advantage of cheaper input costs. This effect is captured by $\bar{\lambda}_{j\cdot}$. This chain of reductions in input cost takes place along the network. I refer to this comovement of sectoral variables reflected in $h_{i,n}$ as the *macro complementarity*. In general, however, the sign and magnitude of the macro complementarity are ambiguous, because they are mediated by the source sector's overall strategic complementarities, encoded in $\bar{\lambda}_{j\cdot}$, which I call the *micro complementarity*.

Proof of Proposition 2.1. Part (ii) of this proposition is immediate from (38). Parts (i) and (iii) follows, respectively, from (42) and (37) as soon as acknowledging $\frac{dW^*}{d\tau_n} = 0$ by the premise of this proposition.

□

[108] For any square matrix A, the corresponding Leontief inverse matrix, if exists, can be written as $(I - A)^{-1} = \sum_{m=0}^{\infty} A^m$ where I define $A^0 = I$.

A.2 Cost Minimization 1: Input Decision

In equilibrium, firm k's cost minimization problem in sector i satisfies the following constrained cost minimization problem:[109]

$$(\ell_{ik}^*, m_{ik}^*) \quad \in \quad \arg\min_{\ell_{ik}, m_{ik}} W^* \ell_{ik} + P_i^{M*} m_{ik} \qquad s.t. \qquad f_i(\ell_{ik}, m_{ik}; z_{ik}) \geq q_{ik}^*.$$

The associated Lagrange function is

$$\mathcal{L}_i(\ell_{ik}, m_{ik}, \xi_{ik}) := W^* \ell_{ik} + P_i^{M*} m_{ik} - \xi_{ik}\Big(f_i(\ell_{ik}, m_{ik}; z_{ik}) - q_{ik}^*\Big).$$

In equilibrium, the following first order conditions are satisfied at $(\ell_{ik}, m_{ik}) = (\ell_{ik}^*, m_{ik}^*)$:

$$\begin{aligned}
&[\ell_{ik}] : W^* = \xi_{ik}^* \frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}} \\
&[m_{ik}] : P_i^{M*} = \xi_{ik}^* \frac{\partial f_i(\cdot)^*}{\partial m_{ik}} \\
&[\xi_{ik}] : f_i(\ell_{ik}^*, m_{ik}^*; z_{ik}) = q_{ik}^*,
\end{aligned}$$

where ξ_{ik}^* is the marginal cost of production at the given quantity q_{ik}^*. Note that under Assumption 2.4 (i), ξ_{ik}^* equals the average cost, i.e., $\xi_{ik}^* = \frac{TC_{ik}^*}{q_{ik}^*}$ where $TC_{ik}^* := TC_{ik}(W, P_i^M, q_{ik})\big|_{(W, P_i^M, q_{ik}) = (W^*, P_i^{M*}, q_{ik}^*)}$ with $TC_{ik}(\cdot)$ denoting, with a slight abuse of notation, the firm's total cost function (see Fact C.1).

Remark A.5. *Two sets of "optimal" labor and material inputs $(\bar{\ell}_{ik}^*, \bar{m}_{ik}^*)$ and (ℓ_{ik}^*, m_{ik}^*) need to be distinguished. They reside on the same production possibility frontier, but do not necessarily coincide. It is the latter that minimizes the total cost of producing q_{ik}^*.*

Totally differentiating these equations yields

$$\frac{dW^*}{d\tau_n} = \frac{d\xi_{ik}^*}{d\tau_n} \frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}} + \xi_{ik}^* \left(\frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik}^2} \frac{d\ell_{ik}^*}{d\tau_n} + \frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik} \partial m_{ik}} \frac{dm_{ik}^*}{d\tau_n} \right) \tag{47}$$

$$\frac{dP_i^{M*}}{d\tau_n} = \frac{d\xi_{ik}^*}{d\tau_n} \frac{\partial f_i(\cdot)^*}{\partial m_{ik}} + \xi_{ik}^* \left(\frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik} m_{ik}} \frac{d\ell_{ik}^*}{d\tau_n} + \frac{\partial^2 f_i(\cdot)^*}{\partial m_{ik}^2} \frac{dm_{ik}^*}{d\tau_n} \right) \tag{48}$$

$$\frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}} \frac{d\ell_{ik}^*}{d\tau_n} + \frac{\partial f_i(\cdot)^*}{\partial m_{ik}} \frac{dm_{ik}^*}{d\tau_n} = \frac{dq_{ik}^*}{d\tau_n}. \tag{49}$$

Notice that (49) dictates the changes of labor and material input along the new production possibility frontier induced by the change in output quantity.

Observe here that

$$\begin{aligned}
\frac{d\xi_{ik}^*}{d\tau_n} &= \frac{1}{q_{ik}^*} \left(\frac{\partial TC_{ik}(\cdot)^*}{\partial W} \frac{dW^*}{d\tau_n} + \frac{\partial TC_{ik}(\cdot)^*}{\partial P_i^M} \frac{dP_i^{M*}}{d\tau_n} + \frac{\partial TC_{ik}(\cdot)^*}{\partial q_{ik}} \frac{dq_{ik}^*}{d\tau_n} \right) - \frac{1}{q_{ik}^*} \frac{TC_{ik}^*}{q_{ik}^*} \frac{dq_{ik}^*}{d\tau_n} \\
&= \frac{1}{q_{ik}^*} \left(\ell_{ik}^* \frac{dW^*}{d\tau_n} + m_{ik}^* \frac{dP_i^{M*}}{d\tau_n} + \xi_{ik}^* \frac{dq_{ik}^*}{d\tau_n} \right) - \frac{1}{q_{ik}^*} \xi_{ik}^* \frac{dq_{ik}^*}{d\tau_n}
\end{aligned}$$

[109] See Remark A.1.

$$= \frac{\ell_{ik}^*}{q_{ik}^*}\frac{dW^*}{d\tau_n} + \frac{m_{ik}^*}{q_{ik}^*}\frac{dP_i^{M*}}{d\tau_n}. \tag{50}$$

where the second equality is a consequence of the Shephard lemma and the fact that the marginal cost equals average cost under Assumption 2.4 (i). It then follows from from (47) and (50),

$$\xi_{ik}^*\frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik}^2}\frac{d\ell_{ik}^*}{d\tau_n} + \xi_{ik}^*\frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik}\partial m_{ik}}\frac{dm_{ik}^*}{d\tau_n} = \left(1 - \frac{\ell_{ik}^*}{q_{ik}^*}\frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}}\right)\frac{dW^*}{d\tau_n} - \frac{m_{ik}^*}{q_{ik}^*}\frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}}\frac{dP_i^{M*}}{d\tau_n}. \tag{51}$$

Likewise, from (48) and (50),

$$\xi_{ik}^*\frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik}\partial m_{ik}}\frac{d\ell_{ik}^*}{d\tau_n} + \xi_{ik}^*\frac{\partial^2 f_i(\cdot)^*}{\partial m_{ik}^2}\frac{dm_{ik}^*}{d\tau_n} = -\frac{\ell_{ik}^*}{q_{ik}^*}\frac{\partial f_i(\cdot)^*}{\partial m_{ik}}\frac{dW^*}{d\tau_n} + \left(1 - \frac{m_{ik}^*}{q_{ik}^*}\frac{\partial f_i(\cdot)^*}{\partial m_{ik}}\right)\frac{dP_i^{M*}}{d\tau_n}. \tag{52}$$

Notice that under Assumption 2.4 (i), (51) and (52) are essentially identical. Hence, the first order conditions (47) – (49) can be summarized by (49) and (51) (or equivalently (49) and (52)), and thus can be compactly expressed as the following single equation:

$$\begin{bmatrix} \xi_{ik}^*\frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik}^2} & \xi_{ik}^*\frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik}\partial m_{ik}} \\ \frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}} & \frac{\partial f_i(\cdot)^*}{\partial m_{ik}} \end{bmatrix} \begin{bmatrix} \frac{d\ell_{ik}^*}{d\tau_n} \\ \frac{dm_{ik}^*}{d\tau_n} \end{bmatrix} = \begin{bmatrix} 1 - \frac{\ell_{ik}^*}{q_{ik}^*}\frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}} & -\frac{m_{ik}^*}{q_{ik}^*}\frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}} \\ \bar{\lambda}_{ik}^L & \bar{\lambda}_{ik}^M \end{bmatrix} \begin{bmatrix} \frac{dW^*}{d\tau_n} \\ \frac{dP_i^{M*}}{d\tau_n} \end{bmatrix}. \tag{53}$$

It is immediate to show that (53) can be inverted for $\frac{d\ell_{ik}^*}{d\tau_n}$ and $\frac{dm_{ik}^*}{d\tau_n}$ as soon as acknowledging the following fact.

Fact A.1. *Suppose that Assumption 2.4 holds. Then, the matrix*

$$\begin{bmatrix} \xi_{ik}^*\frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik}^2} & \xi_{ik}^*\frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik}\partial m_{ik}} \\ \frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}} & \frac{\partial f_i(\cdot)^*}{\partial m_{ik}} \end{bmatrix}$$

is nonsingular, i.e., invertible.

Proof. By Assumption 2.4 (i), it holds by Euler's theorem for homogeneous functions that for each firm k,

$$\frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}}\ell_{ik}^* + \frac{\partial f_i(\cdot)^*}{\partial m_{ik}}m_{ik}^* = q_{ik}^*$$

and

$$\frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik}^2}\ell_{ik}^* + \frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik}\partial m_{ik}}m_{ik}^* = 0. \tag{54}$$

Then the determinant of the matrix in question is given by

$$\begin{vmatrix} \xi_{ik}^*\frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik}^2} & -\xi_{ik}^*\frac{\partial f_i^2(\cdot)^*}{\partial \ell_{ik}\partial m_{ik}} \\ \frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}} & \frac{\partial f_i(\cdot)^*}{\partial m_{ik}} \end{vmatrix} = \begin{vmatrix} -\xi_{ik}^*\frac{m_{ik}^*}{\ell_{ik}^*}\frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik}\partial \ell_{ik}} & \xi_{ik}^*\frac{\partial f_i^2(\cdot)^*}{\partial \ell_{ik}\partial m_{ik}} \\ \frac{q_{ik}^*}{\ell_{ik}^*} - \frac{m_{ik}^*}{\ell_{ik}^*}\frac{\partial f_i(\cdot)^*}{\partial m_{ik}} & \frac{\partial f_i(\cdot)^*}{\partial m_{ik}} \end{vmatrix}$$

$$= -\xi_{ik}^* \frac{q_{ik}^*}{\ell_{ik}^*} \frac{\partial f_i^2(\cdot)^*}{\partial \ell_{ik} \partial m_{ik}}$$

$$< 0,$$

where the last strict inequality is due to Assumptions 2.4 (v). This means that the matrix is nonsingular, as claimed. □

In light of Fact A.1, the system of equations (53) can be uniquely solved for $\frac{d\ell_{ik}^*}{d\tau_n}$ and $\frac{dm_{ik}^*}{d\tau_n}$:

$$\begin{bmatrix} \frac{d\ell_{ik}^*}{d\tau_n} \\ \frac{dm_{ik}^*}{d\tau_n} \end{bmatrix} = \underbrace{-\left(\xi_{ik}^* \frac{q_{ik}^*}{\ell_{ik}^*} \frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik} \partial m_{ik}}\right)^{-1} \begin{bmatrix} \frac{\partial f_i(\cdot)^*}{\partial m_{ik}} & -\xi_{ik}^* \frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik} \partial m_{ik}} \\ -\frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}} & \xi_{ik}^* \frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik}^2} \end{bmatrix} \begin{bmatrix} 1 - \frac{\ell_{ik}^*}{q_{ik}^*} \frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}} & -\frac{m_{ik}^*}{q_{ik}^*} \frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}} \\ \bar{\lambda}_{ik}^L & \bar{\lambda}_{ik}^M \end{bmatrix}}_{\text{firm } k\text{'s input elasticities}} \underbrace{\begin{bmatrix} \frac{dW^*}{d\tau_n} \\ \frac{dP_i^{M*}}{d\tau_n} \end{bmatrix}}_{\text{policy shocks}}. \tag{55}$$

The leading three terms of the right hand side of (55) jointly account for the responsiveness of the firm's labor and material input decisions to the changes in wage and the cost index due to a policy shift, which are given by the last term.[110]

Now, notice from (32), (36), (39) and (55) that $\frac{dq_{ik}^*}{d\tau_n}$, $\frac{dp_{ik}^*}{d\tau_n}$, $\frac{d\ell_{ik}^*}{d\tau_n}$, $\frac{dm_{ik}^*}{d\tau_n}$ and $\frac{dP_i^*}{d\tau_n}$ are expressed in terms of $\frac{dW^*}{d\tau_n}$ and $\frac{dP_i^{M*}}{d\tau_n}$. But I also know from (44) that $\frac{dP_i^{M*}}{d\tau_n}$ can be written by $\frac{dW^*}{d\tau_n}$. Hence, it remains to "solve" for $\frac{dW^*}{d\tau_n}$. This is accomplished by making use of the labor market clearing condition (11).

First, let

$$D_{ik} = \begin{bmatrix} d_{ik,11} & d_{ik,12} \\ d_{ik,21} & d_{ik,22} \end{bmatrix}$$

be the 2×2 matrix expressing the firm's input elasticities' part of (55), i.e.,

$$D_{ik} := -\left(\xi_{ik}^* \frac{q_{ik}^*}{\ell_{ik}^*} \frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik} \partial m_{ik}}\right)^{-1} \begin{bmatrix} \frac{\partial f_i(\cdot)^*}{\partial m_{ik}} & -\xi_{ik}^* \frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik} \partial m_{ik}} \\ -\frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}} & \xi_{ik}^* \frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik}^2} \end{bmatrix} \begin{bmatrix} 1 - \frac{\ell_{ik}^*}{q_{ik}^*} \frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}} & -\frac{m_{ik}^*}{q_{ik}^*} \frac{\partial f_i(\cdot)^*}{\partial \ell_{ik}} \\ \bar{\lambda}_{ik}^L & \bar{\lambda}_{ik}^M \end{bmatrix}. \tag{56}$$

Then, (55) can be written as

$$\frac{d\ell_{ik}^*}{d\tau_n} = d_{ik,11} \frac{dW^*}{d\tau_n} + d_{ik,12} \frac{dP_i^{M*}}{d\tau_n}, \tag{57}$$

$$\frac{dm_{ik}^*}{d\tau_n} = d_{ik,21} \frac{dW^*}{d\tau_n} + d_{ik,22} \frac{dP_i^{M*}}{d\tau_n}. \tag{58}$$

[110] The former is determined independently of the latter. Because of this, once the former is obtained, (55) can be viewed as a "reduced-form" relationship between the changes of labor and material inputs and those of wage and material cost index.

Next, letting $\vartheta_{1,i}$ and $\vartheta_{2,i}$ be the i-th elements of $(I-\Gamma_2)^{-1}\Gamma_1\iota$ and $(I-\Gamma_2)^{-1}\left[\frac{\partial\mathcal{P}_1^M(\cdot)}{\partial\tau_n}\mathbb{1}_{\{n=1\}},\ldots,\frac{\partial\mathcal{P}_N^M(\cdot)}{\partial\tau_n}\mathbb{1}_{\{n=N\}}\right]'$, respectively, the ith element of (44) can be written as

$$\frac{dP_i^{M*}}{d\tau_n} = \vartheta_{1,i}\frac{dW^*}{d\tau_n} + \vartheta_{2,i}. \tag{59}$$

Therefore, upon substituting (59) into (57), I have

$$\begin{aligned}\frac{d\ell_{ik}^*}{d\tau_n} &= d_{ik,12}\left(\vartheta_{1,i}\frac{dW^*}{d\tau_n} + \vartheta_{2,i}\right) + d_{ik,11}\frac{dW^*}{d\tau_n}\\ &= (d_{ik,11} + \vartheta_{1,i}d_{ik,12})\frac{dW^*}{d\tau_n} + \vartheta_{2,i}d_{ik,12}.\end{aligned} \tag{60}$$

To ensure the unique solution, I maintain the following regularity condition.

Assumption A.3 (Regularity Condition 3)**.** $\sum_{i=1}^N\sum_{k=1}^{N_i}(d_{ik,11} + \vartheta_{1,i}d_{ik,12}) \neq 0.$

Totally differentiating the labor market clearing condition (11) delivers

$$\frac{dL}{d\tau_n} = \sum_{i=1}^N\sum_{k=1}^{N_i}\frac{d\ell_{ik}^*}{d\tau_n}.$$

Since here labor supply is inelastic, it then must be $\frac{dL}{d\tau_n} = 0$, so that

$$0 = \sum_{i=1}^N\sum_{k=1}^{N_i}\frac{d\ell_{ik}^*}{d\tau_n}. \tag{61}$$

Substituting (60) for $\frac{d\ell_{ik}^*}{d\tau_n}$ into (61) leads to

$$0 = \sum_{i=1}^N\sum_{k=1}^{N_i}\left\{(d_{ik,11} + \vartheta_{1,i}d_{ik,12})\frac{dW^*}{d\tau_n} + \vartheta_{2,i}d_{ik,12}\right\}, \tag{62}$$

which, under Assumption A.3, can be rearranged to

$$\frac{dW^*}{d\tau_n} = -\frac{\sum_{i=1}^N\sum_{k=1}^{N_i}\vartheta_{2,i}d_{ik,12}}{\sum_{i=1}^N\sum_{k=1}^{N_i}(d_{ik,11} + \vartheta_{1,i}d_{ik,12})}. \tag{63}$$

Combining (63) with (32), (36), (39), (44) and (55), I can "solve" for $\frac{dq_{ik}^*}{d\tau_n}$, $\frac{dp_{ik}^*}{d\tau_n}$, $\frac{d\ell_{ik}^*}{d\tau_n}$, $\frac{dm_{ik}^*}{d\tau_n}$, $\frac{dP_i^*}{d\tau_n}$, $\frac{dP_i^{M*}}{d\tau_n}$ and $\frac{dW^*}{d\tau_n}$ in terms of the endogenous variables in the current equilibrium, exogenous variables and the policy-invariant functions.

Now, it remains to study the responsiveness of the derived demand for sectoral goods with respect to a marginal change in the subsidy $\frac{dm_{ik,j}^*}{d\tau_n}$.

A.3 Cost Minimization 2: Derived Demand for Sectoral Goods

In equilibrium, firm k in sector i purchases sectoral intermediate goods according to the following cost minimization problem:

$$\{m_{ik,j}^*\}_{j=1}^N \quad \in \quad \underset{\{m_{ik,j}\}_{j\in\mathbf{N}}}{\arg\min} \sum_{j=1}^N (1-\tau_i) P_j^* m_{ik,j} \qquad s.t. \qquad \mathcal{G}_i(\{m_{ik,j}\}_{j=1}^N) \geq m_{ik}^*.$$

leading to the derived demand for sectoral goods:

$$m_{ik,j}^* = m_{ik,j}(\{P_j^*\}_{j=1}^N, \tau_i, m_{ik}^*), \tag{64}$$

where $m_{ik,j}(\cdot)$ is a mapping from a combination $(\{P_j\}_{j=1}^N,\ \tau_i,\ m_{ik})$ to a real value representing the demand for sector j's intermediate good $m_{ik,j}$.

Totally differentiating (64) (and evaluating at the equilibrium values of its arguments) delivers

$$\frac{dm_{ik,j}^*}{d\tau_n} = \sum_{j'=1}^N \frac{\partial m_{ik,j}(\cdot)^*}{\partial P_{j'}} \frac{dP_{j'}^*}{d\tau_n} + \frac{\partial m_{ik,j}(\cdot)^*}{\partial \tau_n} \mathbb{1}_{\{n=i\}} + \frac{\partial m_{ik,j}(\cdot)^*}{\partial m_{ik}} \frac{dm_{ik}^*}{d\tau_n}, \tag{65}$$

where $\mathbb{1}_{\{n=i\}}$ is an indicator function that takes one if $n=i$, and zero otherwise. Since both $\frac{dP_{j'}^*}{d\tau_n}$ and $\frac{dm_{ik}^*}{d\tau_n}$ are already solved above, (65) in turn gives $\frac{dm_{ik,j}^*}{d\tau_n}$.

A.4 An Illustrative Example

To gain a clear view of how macro and micro complementarities work, this subsection considers a special case of the general model of Section 2. The model of this subsection posits a constant elasticity of substitution (CES) production function for sectoral aggregators, and a Cobb-Douglas production function for individual firms and the economy-wide aggregator. A version of this parametric setup is widely used in the macroeconomics and international trade literature (e.g., Atkeson and Burstein 2008; Gaubert and Itskhoki 2020; Gaubert et al. 2021; Bigio and La'O 2020; La'O and Tahbaz-Salehi 2022).

A.4.1 Setup

The economy-wide aggregator $\mathcal{F}(\cdot)$ in (2) is given by a Cobb-Douglas production function:

$$\mathcal{F}(\{X_j\}_{j=1}^N) \coloneqq \prod_{j=1}^N X_j^{\beta_j},$$

where β_j is the elasticity parameter with respect to the sector j's good. The sectoral aggregator $F_i(\cdot)$ in (3) takes the form of a constant elasticity of substitution (CES) production function:

$$F_i(\{q_{ik}\}_{k=1}^{N_i}) := \left(\sum_{k=1}^{N_i} \delta_{ik} q_{ik}^{\frac{\sigma_i - 1}{\sigma_i}} \right)^{\frac{\sigma_i}{\sigma_i - 1}},$$

where δ_{ik} is a firm-specific demand shifter and $\sigma_i > 0$ represents elasticity of substitution. The associated sectoral price index is

$$P_i = \left(\sum_{k=1}^{N_i} \delta_{ik}^{\sigma_i} p_{ik}^{1-\sigma_i} \right)^{\frac{1}{1-\sigma_i}}. \tag{66}$$

The firm-level production function $f_i(\cdot)$ in (4) is a Cobb-Douglas aggregator with productivity being Hicks-neutral:

$$f_i(\ell_{ik}, m_{ik}; z_{ik}) := z_{ik} \ell_{ik}^{\alpha_i} m_{ik}^{1-\alpha_i},$$

where α_i is a sector-specific parameter indicating the output-labor ratio. The material aggregator $\mathcal{G}_i(\cdot)$ in (5) is again given by a Cobb-Douglas production:

$$\mathcal{G}(\{m_{ik,j}\}_{j=1}^{N}) := \prod_{j=1}^{N} m_{ik,j}^{\gamma_{i,j}},$$

where $\gamma_{i,j}$ corresponds to the input share of sector j's intermediate good, reflecting the production network Ω. The associated unit cost condition yields the material cost index:

$$P_i^M = \prod_{j=1}^{N} \frac{1}{\gamma_{i,j}^{\gamma_{i,j}}} \Big\{ (1 - \tau_i) P_j \Big\}^{\gamma_{i,j}}. \tag{67}$$

The firm's profit maximization problem (7) can be formulated as

$$q_{ik}^* \in \arg\max_{q_{ik}} \left\{ \frac{\delta_{ik} q_{ik}^{\frac{\sigma_i - 1}{\sigma_i}}}{\sum_{k'=1}^{N_i} \delta_{ik'} q_{ik'}^{\frac{\sigma_i - 1}{\sigma_i}}} R_i - mc_{ik} q_{ik} \right\},$$

where R_i is the total income of the sectoral aggregator. The equilibrium prices and quantities are given by the following system of firms' pricing equations:

$$\begin{aligned} p_{ik}^* &= \frac{\sigma_i}{(1 - \sigma_i)(1 - s_{ik})} mc_{ik} \\ s_{ik}^* &= \delta_{ik}^{\sigma_i} \left(\frac{p_{ik}}{P_i^*} \right)^{1-\sigma_i}, \end{aligned}$$

where s_{ik} is firm k's market share. Note that the firm k's marginal revenue function $mr_{ik}(\cdot)$ is given by

$$mr_{ik}(\{q_{ik'}\}_{k'=}^{N}) = \frac{\sigma_i - 1}{\sigma_i} p_{ik}(1 - s_{ik}).$$

Moreover, it is immediate to verify that

$$\frac{\partial p_{ik}(\cdot)}{\partial q_{ik}} = \begin{cases} \frac{p_{ik}}{q_{ik}} \left\{ \frac{\sigma_i - 1}{\sigma_i}(1 - s_{ik}) - 1 \right\} & \text{if } k' = k \\ -\frac{\sigma_i - 1}{\sigma_i} \frac{p_{ik}}{q_{ik'}} s_{ik'} & \text{if } k' \neq k, \end{cases}$$

and

$$\frac{\partial(1 - s_{ik}(\cdot))}{\partial q_{ik}} = \begin{cases} -\frac{\sigma_i - 1}{\sigma_i} \frac{1}{q_{ik}} s_{ik}(1 - s_{ik}) & \text{if } k' = k \\ -\frac{\sigma_i - 1}{\sigma_i} \frac{1}{q_{ik'}} s_{ik} s_{ik'} & \text{if } k' \neq k. \end{cases}$$

In equilibrium, it follows from (66) that

$$\frac{\partial \mathcal{P}_i(\cdot)}{\partial q_{ik}^*} = -\frac{s_{ik}^*}{q_{ik}^*} P_i^* \qquad \forall k \in \mathbf{N}_i,$$

and from (67) that

$$\begin{aligned} \frac{\partial \mathcal{P}_i^M(\cdot)}{\partial P_j^*} &= \gamma_{i,j} \frac{P_i^{M*}}{P_j^*} \qquad \forall j \in \mathbf{N} \\ \frac{\partial \mathcal{P}_i^M(\cdot)}{\partial \tau_n} &= -\frac{P_i^{M*}}{1 - \tau_i} \mathbb{1}_{\{n=i\}}. \end{aligned}$$

Proposition A.1. *Consider the economy defined in Appendix A.4.1. For each sector $i \in \mathbf{N}$, the following statements hold:*

(i) If $\sigma_i > 1$, then (a) for each $k \in \mathbf{N}_i$, $\frac{\partial mr_{ik}(\cdot)^*}{\partial q_{ik}} < 0$; and (b) for each $k \in \mathbf{N}_i$ and $k' \in \mathbf{N}_i \backslash \{k\}$, $\frac{\partial mr_{ik}(\cdot)^*}{\partial q_{ik'}} < 0$ if $s_{ik} < \frac{1}{2}$, $\frac{\partial mr_{ik}(\cdot)^*}{\partial q_{ik'}} = 0$ if $s_{ik} = \frac{1}{2}$ and $\frac{\partial mr_{ik}(\cdot)^*}{\partial q_{ik'}} > 0$ otherwise.

(ii) If $\sigma_i < 1$, then (a) for each $k \in \mathbf{N}_i$, $\frac{\partial mr_{ik}(\cdot)^*}{\partial q_{ik}} < 0$ if $s_{ik} > -\frac{1}{2(\sigma_1 - 1)}$, $\frac{\partial mr_{ik}(\cdot)^*}{\partial q_{ik}} = 0$ if $s_{ik} = -\frac{1}{2(\sigma_1 - 1)}$ and $\frac{\partial mr_{ik}(\cdot)^*}{\partial q_{ik}} < 0$ otherwise; and (b) for each $k \in \mathbf{N}_i$ and $k' \in \mathbf{N}_i \backslash \{k\}$, $\frac{\partial mr_{ik}(\cdot)^*}{\partial q_{ik'}} < 0$ if $s_{ik} < \frac{1}{2}$, $\frac{\partial mr_{ik}(\cdot)^*}{\partial q_{ik'}} = 0$ if $s_{ik} = \frac{1}{2}$ and $\frac{\partial mr_{ik}(\cdot)^*}{\partial q_{ik'}} > 0$ otherwise.

Proof. (i) Suppose $\sigma_i > 1$.

(a) Observe that

$$\frac{\partial mr_{ik}(\cdot)}{\partial q_{ik}} \gtreqless 0 \Longleftrightarrow -\frac{1}{2(\sigma_i - 1)} \gtreqless s_{ik}. \tag{68}$$

Given the hypothesis (i.e., $\sigma_i > 1$), the left hand side of (68) is negative, while s_{ik} is by definition positive. Hence, it is always true that $\frac{\partial mr_{ik}(\cdot)}{\partial q_{ik}} < s_{ik}$, from which it follows that $\frac{\partial mr_{ik}(\cdot)}{\partial q_{ik}} < 0$.

(b) Observe that

$$\frac{\partial mr_{ik}(\cdot)}{\partial q_{ik'}} \gtreqless 0 \Longleftrightarrow \frac{1}{2} \lesseqgtr s_{ik}.$$

This proves the statement.

(ii) Suppose $\sigma_i < 1$.

(a) Observe that

$$\frac{\partial mr_{ik}(\cdot)}{\partial q_{ik}} \gtreqless 0 \Longleftrightarrow -\frac{1}{2(\sigma_i - 1)} \gtreqless s_{ik}. \tag{69}$$

According to the hypothesis (i.e., $\sigma_i < 1$), the left hand side of (69) is positive. Then there can be three configurations depending on the value of s_{ik}. This observation directly leads to the statement.

(b) Observe that

$$\frac{\partial mr_{ik}(\cdot)}{\partial q_{ik'}} \gtreqless 0 \Longleftrightarrow \frac{1}{2} \lesseqgtr s_{ik}.$$

This proves the statement. □

Notice that in Proposition A.1, the part (b) of (i) is identical to that of (ii), i.e., they do not depend on the value of σ_i. This observation immediately leads to the following corollaries.

Corollary A.1. *Consider the economy defined in Appendix A.4.1.*

(i) If there exists a firm $\bar{k} \in \mathbf{N}_i$ such that $s_{i\bar{k}} > \frac{1}{2}$, then $\frac{\partial mr_{i\bar{k}}(\cdot)^*}{\partial q_{ik'}} > 0$ for all $k' \in \mathbf{N}_i \backslash \{\bar{k}\}$; and $\frac{\partial mr_{ik}(\cdot)^*}{\partial q_{ik'}} < 0$ for all $k, k' \in \mathbf{N}_i \backslash \{\bar{k}\}$ such that $k \neq k'$, regardless of the value of σ_i.

(ii) If $s_{ik} < \frac{1}{2}$ for all $k \in \mathbf{N}_i$, then for each $k \in \mathbf{N}_i$, $\frac{\partial mr_{ik}(\cdot)^*}{\partial q_{ik'}} < 0$ for all $k' \in \mathbf{N}_i \backslash \{k\}$, regardless of the value of σ_i.

These corollaries can yield further implications in the case of duopoly.

A.4.2 Duopoly

Consider the same setup as above. But suppose that each sector is populated by two firms, i.e., $N_i = \{1, 2\}$ for all $i \in \mathbf{N}$. Here, observe that in this case, one firm accounts for more than half of the market share, while the other explains less than a half.[111] Thus, with out loss of generality, I let $s_{i1} > \frac{1}{2}$, which in turn means that $s_{i2} < \frac{1}{2}$, i.e., firm 1 has a larger market share.

Corollary A.2. *In duopoly, wherein $s_{i1} > \frac{1}{2}$, it holds that $\frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i2}} > 0$ and $\frac{\partial mr_{i2}(\cdot)^*}{\partial q_{i1}} < 0$.*

Corollary A.3. *In duopoly, wherein $s_{i1} > \frac{1}{2}$ and $\sigma_i > 1$, it holds that (i) $\frac{\partial mr_{ik}(\cdot)^*}{\partial q_{ik}} < 0$ for all $k \in \{1, 2\}$; (ii) $\frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i2}} > 0$; and (iii) $\frac{\partial mr_{i2}(\cdot)^*}{\partial q_{i1}} < 0$, so that $det(\Lambda_{i,1}) > 0$.*

[111] That is, there always exists such firms $\bar{k} \in \mathbf{N}_i$ and $\bar{k}' \in \mathbf{N}_i \backslash \{\bar{k}\}$ that $s_{i\bar{k}} > \frac{1}{2}$ and $s_{i\bar{k}'} < \frac{1}{2}$.

Noticing that the firm's marginal costs are constant in the firm's profit maximization problem, the following corollary is almost trivial.

Corollary A.4. *(i) Firm 1's quantity decision is a strategic complement to firm 2's quantity decision. (ii) Firm 2's quantity decision is a strategic substitute to firm 1's quantity decision.*

Proof. It is immediate to see that

$$0 < \frac{\partial mr_{i1}(\cdot)}{\partial q_{i2}} = \frac{\partial (mr_{i1}(\cdot) - mc_{i1})}{\partial q_{i2}} = \frac{\partial \frac{\partial \pi_{i1}(\cdot)}{\partial q_{i1}}}{\partial q_{i2}}.$$

An analogous argument applies to firm 2, completing the proof. $\square$

Turning to micro complementarities, I focus on $\bar{\lambda}_{i\cdot}^{M}$ in the subsequent analysis. A parallel argument holds for $\bar{\lambda}_{i\cdot}^{L}$ as well. In what follows, I assume that $\sigma_i > 1$. First,

$$\begin{aligned}
\bar{\lambda}_{i1}^{M} &= \frac{1}{det(\Lambda_{i,1})}\left(\frac{m_{i1}^*}{q_{i1}^*}\frac{\partial mr_{i2}(\cdot)^*}{\partial q_{i2}} - \frac{m_{i2}^*}{q_{i2}^*}\frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i2}}\right)\\
\bar{\lambda}_{i2}^{M} &= \frac{1}{det(\Lambda_{i,1})}\left(-\frac{m_{i1}^*}{q_{i1}^*}\frac{\partial mr_{i2}(\cdot)^*}{\partial q_{i1}} + \frac{m_{i2}^*}{q_{i2}^*}\frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i1}}\right),
\end{aligned}$$

where $det(\Lambda_{i,1}) = \frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i1}}\frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i1}} - \frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i2}}\frac{\partial mr_{i2}(\cdot)^*}{\partial q_{i1}}$. From Corollary A.3, it follows that $\bar{\lambda}_{i1}^{M} < 0$ as well as $det(\Lambda_{i,1}) > 0$.

The following lemma characterize the sign of $\bar{\lambda}_{i2}^{M}$ in terms of partial derivatives of marginal revenue functions and firms' productivities.

Lemma A.1. $\bar{\lambda}_{i2}^{M} \lesseqgtr 0 \iff \frac{z_{i1}}{z_{i2}}\frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i1}} \lesseqgtr \frac{\partial mr_{i2}(\cdot)^*}{\partial q_{i1}}$.

Proof. First, observe that

$$\bar{\lambda}_{i2}^{M} \lesseqgtr 0 \iff \frac{\frac{m_{i2}}{q_{i2}}}{\frac{m_{i1}}{q_{i1}}}\frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i1}} \lesseqgtr \frac{\partial mr_{i2}(\cdot)^*}{\partial q_{i2}}.$$

Here, under the Cobb-Douglas production function, the material productivity is proportional to the inverse of the firm's productivity:

$$\frac{m_{ik}^*}{q_{ik}^*} = z_{ik}^{-1}\left(\frac{\alpha_i}{1-\alpha_i}\right)^{-\alpha_i}\left(\frac{P_i^{M*}}{W^*}\right)^{\alpha_i}.$$

Substituting this into the above equivalence proves the claim. $\square$

Remark A.6. *Due to the presumption (i.e., $s_{i1} > s_{i2}$), it holds that $\frac{z_{i1}}{z_2} > 1$.*

The following proposition gives a sufficient condition for $\bar{\lambda}_{i\cdot}^{M}$ to be positive, and states that if firm 2 is a "relatively strong" strategic substitute, then the sectoral measure of strategic complementarity is positive.

Proposition A.2. *Suppose* $\frac{z_{i1}}{z_{i2}} \frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i1}} < \frac{\partial mr_{i2}(\cdot)^*}{\partial q_{i1}}$*. Then,* $\bar{\lambda}^M_{i\cdot} > 0$.

Proof. First, by construction, $P_i Q_i = R_i$. Differentiation with respect to q_{ik} leads to

$$\frac{\partial \mathcal{P}_i(\cdot)}{\partial q_{ik}} = -\frac{s_{ik}}{q_{ik}} P_i.$$

Next, by definition,

$$\bar{\lambda}^M_{i\cdot} = \frac{\partial \mathcal{P}_i(\cdot)}{\partial q^*_{i1}} \bar{\lambda}^M_{i1} + \frac{\partial \mathcal{P}_i(\cdot)}{\partial q^*_{i2}} \bar{\lambda}^M_{i2} = -\left(\frac{s^*_{i1}}{q^*_{i1}} \bar{\lambda}^M_{i1} + \frac{s^*_{i2}}{q^*_{i2}} \bar{\lambda}^M_{i2} \right) P^*_i.$$

Acknowledging that $\bar{\lambda}^M_{i1} < 0$ due to Corollary A.3, and $\bar{\lambda}^M_{i2} < 0$ because of Lemma A.1, it follows that $\bar{\lambda}^M_{i\cdot} > 0$. □

Notice that the hypothesis of this proposition reads

$$\frac{\partial \frac{\partial \pi_{i2}(\cdot)^*}{\partial q_{i2}}}{\partial q_{i1}} \Bigg/ \frac{\partial \frac{\partial \pi_{i1}(\cdot)^*}{\partial q_{i1}}}{\partial q_{i1}} \in \left(\frac{z_{i1}}{z_{i2}}, \infty \right).$$

This requires that firm 2's output is a "relatively strong" strategic substitute in the sense that the proportion of the sensitivity of firm 2's marginal profit to firm 1's quantity adjustment relative to that of firm 1's marginal profit to its own quantity change is at least as large as the productivity ratio between the two firms.[112] Note that the converse of Proposition A.2 is not true.[113] Nevertheless, a positive micro complementarity can be viewed as an indication that firm 2 might possibly be a "relatively strong" strategic substitute. Moreover, the contrapositive suggests that a negative micro complementarity is evidence of firm 2's being a "relatively modest" strategic substitute.

Remark A.7. *The converse is not true. A necessary and sufficient condition for the sign of* $\bar{\lambda}^M_{i\cdot}$ *reads*

$$\bar{\lambda}^M_{i\cdot} \gtreqless 0 \Longleftrightarrow \bar{\lambda}^M_{i2} \lesseqgtr -\frac{p^*_{i1}}{p^*_{i2}} \bar{\lambda}^M_{i1}.$$

While it is possible to further rewrite this in terms of partial derivatives of marginal revenue functions, its economic content is not easy to interpret.

[112] By setup, $\frac{z_{i1}}{z_{i2}} > 1$.

[113] Although it is possible to characterize the necessary and sufficient condition in terms of firms' strategic complementarities, its economic content is not clear. See Remark A.7.

B Detail of Data

This section provides a detailed account of the data source used in my paper, and explains how I construct the empirical counterparts of the variables set out in Section 2.

B.1 Aggregate Data

Data on the wage-related concepts are obtained from the U.S. Bureau of Labor Statistics (BLS) through the Federal Reserve Bank of St. Louis (FRED) at annual frequency. In my model, labor is assumed to be frictionlessly mobile across sectors so that wage is common for all sectors. Thus, I use *Average hourly earnings of all employees, total private* as the empirical analogue of the wage W in my model. In addition, I also obtain the measure of the average hours worked per employee per year (*Average weekly hours of all employees, total private*). It should be remarked that these data exclude agricultural workers mainly due to the peculiarities of the structure of the agricultural industry and characteristics of its workers — e.g., various definitions of agriculture, farms, famers and farmworkers; and considerable seasonal fluctuation in the employment (Daberkow and Whitener 1986). Note also that these data do not include information about government employees, either. Sectoral price index data is available at the Bureau of Economic Analysis (BEA). I use *U.Chain-Type Price Indexes for Gross Output by Industry — Detail Level (A)* as the empirical counterparts of $\{P_i^*\}_{i=1}^N$.

These are summarized in the following fact.

Fact B.1 (Wage & Sectional Price Index). *The wage W^* and sectoral price indices $\{P_i^*\}_{i=1}^N$ are directly observed in the data.*

B.2 Sector-Level Data: Industry Economic Accounts (IEA)

My analysis involves two types of sector-level data: namely, the input-output table and sector-specific tax/subsidy, both of which come from the input-output accounts data of the Bureau of Economic Analysis (BEA). In line with the global economic accounting standards, such as the System of National Accounts 2008 (UN 2008), the BEA input-output table consists of two tables, namely, the use and supply table.

The use table shows the uses of commodities (goods and services) by industries as intermediate inputs and by final users, with the columns indicating the industries and final users and the rows representing commodities. This table reports three pieces of information, namely, intermediate inputs, final demand and value added. Each cell in the intermediate input section records the amount of a commodity purchased by each industry as an intermediate input, valued at producer' or purchasers' prices.[114] The final demand section accounts for expenditure-side components of

[114] Typically, the IEA is valued at either of the producers', basic, or purchasers' prices. The producers' prices are the total amount of monetary units received from the purchasers for a unit of a good and service that is sold. The basic prices mean the total amount retained by the producer for a unit of a good and service. This price plays a pivotal role in the producer's decision making about production and sales. The purchasers' prices refer to the total amount

GDP. The value-added part bridges the difference between an industry's total output and its total cost for intermediate inputs. This part will further be expanded in the upcoming section (Appendix B.2.2).

The supply table shows total supply of commodities by industries, with the columns indicating the industries and the rows representing commodities. This table comprises domestic output and imports. Each cell of the domestic output section presents the total amount of each commodity supplied domestically by each industry, valued at the basic prices. The import section records the total amount of each commodity imported from foreign countries, valued at the importers' customs frontier price (i.e., the c.i.f. valuation).[115]

Segmentation. My analysis is based on the BEA's industry classification at the summary level, which is roughly equivalent to the three-digit NAICS (North American Industry Classification System). I make four major modifications in accordance with other aggregate and firm-level data as well as my model (Section 2). First, I omit several industries and products from my analysis. Following Bigio and La'O (2020), I exclude the finance, insurance, real estate, rental and leasing (FIRE) sectors from my analysis. In the BEA's input-output table, these sectors are indexed by 521CI, 523, 524, 525, HS, ORE, and 532RL. I also follow Baqaee and Farhi (2020) in dropping the scrap, used and secondhand goods industry/commodity and the noncomparable imports and rest-of-the-world adjustment industry/commodity. The former is indexed by Used and the latter by Others. I again follow Baqaee and Farhi (2020) in removing the government sectors, which are reported with the indices 81, GFGD, GFGN, GFE, GSLG, and GSLE. This is in line with both my model (Section 2) and aggregate data (Appendix B.1). Second, drawing on Gutiérrez and Philippon (2017), I merge several BEA's industries. This manipulation ensures that each industry has a good coverage of the Compustat firms (Gutiérrez and Philippon 2017).[116] Third, I eliminate the farms industry (BEA code 111CA), and the forestry, fishing, and related activities (BEA code 113FF), in view of the construction of the aggregate employment data (Appendix B.1). Consistent with this, I also omit the food services and drinking places (BEA code 722) from my analysis. Lastly, I drop the health care industries (BEA code 621, 622, 623 and 624) because my model may not capture several key aspects of the industry's competition nature.[117] After all, I am left with 30 industries listed in Table B.2.

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paid by the purchasers for a unit of a good and service that they purchase. This is the key for the purchasers to make their purchasing decisions. By definition, the basic prices are equal to the producers' prices minus taxes payable for a unit of a good and service plus any subsidy receivable for a unit of a good and service; and the purchasers' prices are equivalent to the sum of the producers' prices and any wholesale, retail or transportation markups charged by intermediaries between producers and purchasers. See BEA (2009) and Young et al. (2015) for the detail.

[115] The importers' customs frontier price is calculated as the cost of the product at foreign port value plus insurance and freight charges to move the product to the domestic port. See Young et al. (2015) for the detail.

[116] For example, the nonparametric estimation of the share regression using the polynomials of degree 2 requires at least 6 observations in the same sector. See Appendix E.2.

[117] Recent works model the health care industry as a mix-oligopoly, in which public and private providers compete to maximize, respectively, the consumer surplus and profits (e.g., Jofre-Bonet 2000; Bisceglia et al. 2023).

Table 3: Mapping of BEA Industry Codes to Segments

BEA code	Industry	Mapped segment
111CA	Farms	Omitted
113FF	Forestry, fishing, and related activities	Omitted
211	Oil and gas extraction	Oil and gas extraction, and mining
212	Mining, except oil and gas	Oil and gas extraction, and mining
213	Support activities for mining	Support activities for mining
22	Utilities	Omitted
23	Construction	Construction
321	Wood products	Wood and nonmetallic mineral products
327	Nonmetallic mineral products	Wood and nonmetallic mineral products
331	Primary metals	Primary metals
332	Fabricated metal products	Fabricated metal products
333	Machinery	Machinery
334	Computer and electronic products	Computer and electronic products
335	Electrical equipment, appliances, and components	Electrical equipment, appliances, and components
3361MV	Motor vehicles, bodies and trailers, and parts	Motor vehicles and other transportation equipment
3364OT	Other transportation equipment	Motor vehicles and other transportation equipment
337	Furniture and related products	Furniture and related products
339	Miscellaneous manufacturing	Miscellaneous manufacturing
311FT	Food and beverage and tobacco products	Food and beverage and tobacco products
313TT	Textile mills and textile product mills	Textile-related mills and apparel products
315AL	Apparel and leather and allied products	Textile-related mills and apparel products
322	Paper products	Paper products and printing-related services
323	Printing and related support activities	Paper products and printing-related services
324	Petroleum and coal products	Petroleum and coal products
325	Chemical products	Chemical products
326	Plastics and rubber products	Plastics and rubber products
42	Wholesale trade	Wholesale trade
441	Motor vehicle and parts dealers	Retail trade
445	Food and beverage stores	Retail trade
452	General merchandise stores	Retail trade
4A0	Other retail	Retail trade
481	Air transportation	Air and ground transportation
482	Rail transportation	Air and ground transportation
483	Water transportation	Water transportation
484	Truck transportation	Air and ground transportation
485	Transit and ground passenger transportation	Air and ground transportation
486	Pipeline transportation	Other transportation and support activities
487OS	Other transportation and support activities	Other transportation and support activities
493	Warehousing and storage	Other transportation and support activities
511	Publishing industries, except internet (includes software)	Publishing industries and information services
512	Motion picture and sound recording industries	Mass media and telecommunications
513	Broadcasting and telecommunications	Mass media and telecommunications
514	Data processing, internet publishing, and other information services	Publishing industries and information services
521CI	Federal Reserve banks, credit intermediation, and related activities	Omitted
523	Securities, commodity contracts, and investments	Omitted
524	Insurance carriers and related activities	Omitted
525	Funds, trusts, and other financial vehicles	Omitted
HS	Housing	Omitted
ORE	Other real estate	Omitted
532RL	Rental and leasing services and lessors of intangible assets	Omitted
5411	Legal services	Legal, scientific, and technical services
5412OP	Miscellaneous professional, scientific, and technical services	Legal, scientific, and technical services

BEA code	Industry	Mapped segment
5415	Computer systems design and related services	Legal, scientific, and technical services
55	Management of companies and enterprises	Omitted
561	Administrative and support services	Administrative and waste services
562	Waste management and remediation services	Administrative and waste services
61	Educational services	Educational services
621	Ambulatory health care services	Omitted
622	Hospitals	Omitted
623	Nursing and residential care facilities	Omitted
624	Social assistance	Omitted
711AS	Performing arts, spectator sports, museums, and related activities	Arts, Entertainment, and Recreation
713	Amusements, gambling, and recreation industries	Arts, Entertainment, and Recreation
721	Accommodation	Accommodation
722	Food services and drinking places	Omitted
81	Other services, except government	Omitted
GFGD	Federal general government (defense)	Omitted
GFGN	Federal general government (nondefense)	Omitted
GFE	Federal government enterprises	Omitted
GSLG	State and local general government	Omitted
GSLE	State and local government enterprises	Omitted
Used	Scrap, used and secondhand goods	Omitted
Other	Noncomparable imports and rest-of-the-world adjustment	Omitted
999	Nonclassifiable Establishments	Omitted

Note: This table shows the correspondence between the BEA's industry classification (at summary level) and my segmentation, which draws heavily on Gutiérrez and Philippon (2017). The first two columns ("BEA code" and "Industry") list the BEA codes and the corresponding industries as used in the BEA's input-output table. The third column ("Mapped segment") indicates the names of the segments I define.

B.2.1 Transformation to Symmetric Input-Output Tables

Although the use table comes very close to representing an empirical counterpart of the production network of my model, it cannot be directly adopted in my empirical analysis as it only shows the uses of each commodity by each industry, not the uses of each industrial product by each industry. This is because the BEA's accounting system allows for each industry to produce multiple commodities (e.g., secondary production), being incompatible with my conceptualization. Hence, I first need to convert the use table to a symmetric industry-by-industry input-output table by transferring inputs and outputs over the rows in the use and supply tables, respectively.[118] To this end, I impose an assumption about how each commodity is used.

[118] For example, if there is a non-zero entry in the cell of the supply table whose column is agriculture and whose row is manufacturing products, it is recorded in the use table as the supply of manufacturing products, the largest component of which should be accounted for by the supply from manufacturing industry. Now my goal is to modify this attribution in a way that the supply of manufacturing products by agriculture industry is treated as agricultural products. To this end, I need to subtract the contributions of agriculture industry from the use of manufacturing products, and transfer them to the agricultural commodities, thereby changing the classification of the row from commodity to industry.

Assumption B.1 (Fixed Product Sales Structures, (Eurostat 2008))**.** *Each product has its own specific sales structure, irrespective of the industry where it is produced.*

The term 'sales structure' here refers to the shares of the respective intermediate and final users in the sales of a commodity. Under Assumption B.1, each commodity is used at constant rates regardless of in which industry it is produced. For example, a unit of a manufacturing product supplied by the agriculture industry will be transferred from the use of manufacturing products to that of agricultural products in the use table in the same proportion to the use of manufacturing products.[119] Note that the value-added part remains intact throughout this manipulation. Recorded in each cell of the intermediate inputs section of the resulting industry-by-industry table is the empirical counterpart of my $(1-\tau_i)\sum_{k=1}^{N_i} P_i^* m_{ik,j}^*$, and each cell of the compensation of employee corresponds to $\sum_{k=1}^{N_i} W^* \ell_{ik}^*$. These are the data that are used to construct the production network in my empirical analysis, as shown in the following fact.

Fact B.2. *Under Assumption B.1, the input-output linkages $\boldsymbol{\omega}_L$ and Ω are recovered from the observables.*

Proof. By Shephard lemma,[120] it holds that for each $i, j \in \mathbf{N}$, the cost-based intermediate expenditure shares $\omega_{i,j}$ satisfies

$$\omega_{i,j} = \frac{(1-\tau_i)\sum_{k=1}^{N_i} P_j m_{ik,j}}{\sum_{j'=1}^{N}(1-\tau_i)\sum_{k=1}^{N_i} P_{j'} m_{ik,j'} + \sum_{k=1}^{N_i} W \ell_{ik}}. \tag{70}$$

Also, for each $i \in \mathbf{N}$, cost-based equilibrium factor expenditure shares $\omega_{i,L}$ satisfies:

$$\omega_{i,L} = \frac{\sum_{k=1}^{N_i} W \ell_{ik}}{\sum_{j'=1}^{N}(1-\tau_i)\sum_{k=1}^{N_i} P_{j'} m_{ik,j'} + \sum_{k=1}^{N_i} W \ell_{ik}}.$$

Since $\left\{(1-\tau_i)\sum_{k=1}^{N_i} P_j m_{ik,j}\right\}_{i,j=1}^{N}$ and $\{\sum_{k=1}^{N_i} W \ell_{ik}\}_{i=1}^{N}$ are directly observed in the transformed industry-by-industry input-output table, I can immediately recover $\boldsymbol{\omega}_L$ and Ω, as desired. $\square$

Figure 3 compares the input-output table based on the use table and transformed industry-by-industry input-output table.

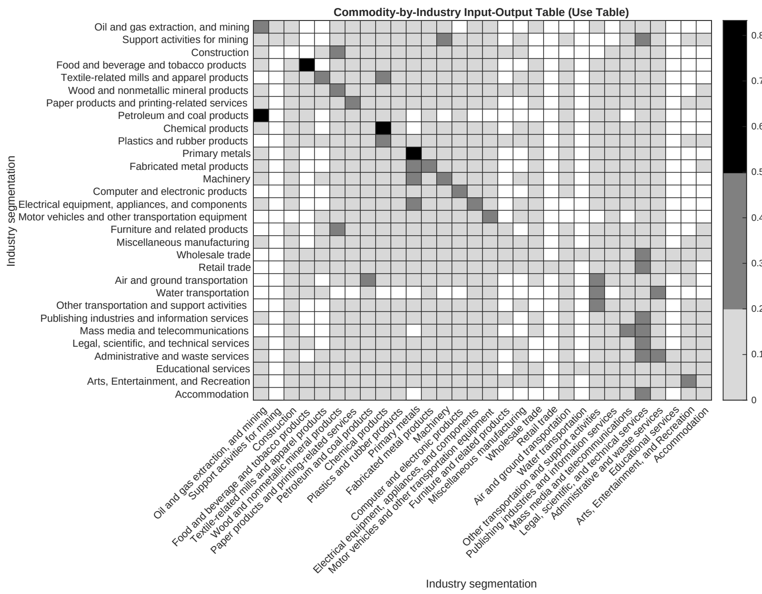
B.2.2 Sectoral Tax/Subsidy

Given that the use table has been transformed into a symmetric industry-by-industry input-output table, I can proceed to back out the tax/subsidy from the transformed table. In this step, I exploit the feature of the use table that reports value added at basic and purchasers' prices. The value added measured at basic prices is composed of (i) compensation of employees (V001), (ii) gross operating surplus (V003), and (iii) other taxes on production (T00OTOP) less subsidies (T00OSUB). The

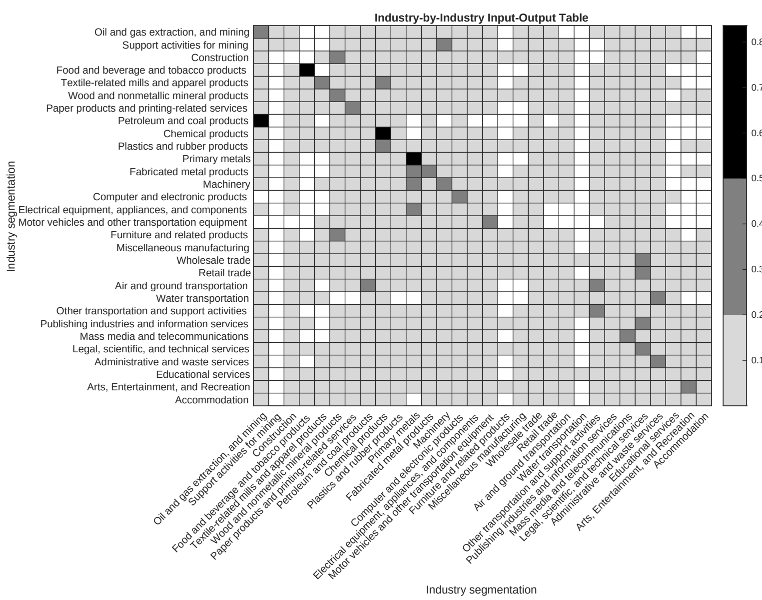
[119] Related to this assumption is the fixed industry sales structure assumption, in which . However, it is Assumption B.1 that is widely used by statistical offices for various reasons. See Eurostat (2008) for the detail.

[120] See Liu (2019), Baqaee and Farhi (2020) and Bigio and La'O (2020) for application and reference.

Figure 3: Comparison of Input-Output Tables



(a) Use table



(b) Transformed industry-by-industry table

Note: This figure illustrates the input-output table in terms of the cost share of sectoral goods. Panel (a) shows the use table that is provided by BEA, while panel (b) reports the transformed industry-by-industry table. White cells indicate zero, while light, medium and dark grey cells represent the low (0 $\sim$ 0.2), medium (0.2 $\sim$ 0.5) and high (0.5 $\sim$ 1.0) cost shares, respectively.

value added at producers' prices further entails (*iv*) taxes on products (T00TOP) and imports less subsidies (T00SUB).[121] According to BEA (2009), the tax-related components of (*iii*) and (*iv*) jointly include, among many others, sales and excise taxes, customs duties, property taxes, motor vehicle licenses, severance taxes, other taxes and special assessments as well as commodity taxes, while the subsidy-related components refer to monetary grants paid by government agencies to private businesses and to government enterprises at another level of government.

I consider the sum of (*iii*) and (*iv*) to be the empirical counterpart of the policy expenditure in my model. This choice is motivated by the mapping between the BEA's data construction and my conceptualization. To see this, observe that the construction of data states

$$\begin{aligned} Profit_i &= (Revenue_i + TaxSubsidy1_i) - (LaborCost_i + MaterialCost_i + TaxSubsidy2_i) \\ \therefore \underbrace{Revenue - MaterialCost_i}_{\text{Value-added}} &= \underbrace{Profit_i}_{\text{Gross operating surplus}} + \underbrace{LaborCost_i}_{\text{Compensation of employees}} \\ &\quad - \underbrace{(TaxSubsidy1_i - TaxSubsidy2_i)}_{\text{Value-added taxes less subsidies}}, \end{aligned} \tag{71}$$

where $TaxSubsidy1_i$ is taxes less subsidies on revenues, and $TaxSubsidy2_i$ those on input costs. Notice that the value-added taxes less subsidies ($TaxSubsidy1_i - TaxSubsidy2_i$) are available in the data. The theoretical counterpart of the data construction (71) stems from the definition of the sector-level profit:

$$\begin{aligned} \sum_{k=1}^{N_i} \pi_{ik}^* &= \sum_{k=1}^{N_i} p_{ik}^* q_{ik}^* - \left\{ W^* \ell_{ik}^* + (1-\tau_i) \sum_{j=1}^{N} P_i^{M^*} m_{ik,j}^* \right\} \\ \therefore \underbrace{\sum_{k=1}^{N_i} p_{ik}^* q_{ik}^* - \sum_{j=1}^{N} P_i^{M^*} m_{ik,j}^*}_{\text{Value-added}} &= \underbrace{\sum_{k=1}^{N_i} \pi_{ik}^*}_{\text{Gross operating surplus}} + \underbrace{W^* \ell_{ik}^*}_{\text{Compensation of employees}} \\ &\quad - \underbrace{\tau_i \sum_{j=1}^{N} P_i^{M^*} m_{ik,j}^*}_{\text{Value-added taxes less subsidies}} . \end{aligned} \tag{72}$$

Comparing (71) and (72), the data on ad-valorem taxes/subsidy can be backed out from the constructed input-output table, as summarized in the following fact.

Fact B.3. *Under Assumption B.1, sector-specific subsidies $\boldsymbol{\tau} \coloneqq \{\tau_i\}_{i=1}^N$ are recovered from the observables.*

Proof. For each sector (industry) $i \in \mathbf{N}$, I have

$$(1-\tau_i) \sum_{j=1}^{N} \sum_{k=1}^{N_i} P_j^* m_{ik,j}^* = \sum_{j=1}^{N} IntermExpend_{i,j}, \tag{73}$$

[121] By construction, the sum of the latter across all industries has to coincide with GDP for the economy.

where $InterExpend_{i,j}$ means the sector i's total expenditure on sector j, which is observed in the (i, j) entry of the industry-by-industry input-output table constructed in Appendix B.2.1. Meanwhile, comparing (71) to (72), I obtain

$$\tau_i \sum_{j=1}^{N} \sum_{k=1}^{N_i} P_j^* m_{ik,j}^* = VAT_i, \tag{74}$$

where VAT_i stands for the sector i's value-added taxes less subsidies, reported in the BEA use table.

Rearranging (73) and (74), I can recover the data for sector-specific taxes/subsidies, i.e.,

$$\tau_i = \frac{VAT_i}{VAT_i + \sum_{j=1}^{N} InterExpend_{i,j}}.$$

□

Remark B.1. *Operationalizing the ad-valorem taxes/subsidies in this way, its empirical content should be understood as an overall extent of wedges that promotes or demotes the purchase of input goods.*

Moreover, comparing (71) with (72) in terms of gross operating surplus delivers a constant linking the scale of sectoral variables and that of firm-level variables.

Definition B.1 (Scale Constant)**.** *For each sector $i \in \mathbf{N}$, the scaling constant $\backepsilon_i$ is defined as*

$$\backepsilon_i := \frac{Profit_i}{\sum_{k=1}^{N_i} \pi_{ik}^*}, \tag{75}$$

where $Profit_i$ represents gross operating surplus reported in the use table, and π_{ik}^* is the firm-level profit available from the firm-level data.

This scaling constant is used in compiling the firm-level data, as illustrated below.

B.3 Firm-Level Data: Compustat Data

The data source for firm-level data is the Compustat data provided by the Wharton Research Data Services (WRDS). This database provides detailed information about a firm's fundamentals, based on financial accounts. For the analysis of this paper, I use the following items: Sales (SALE), Costs of Goods Sold (COGS), Selling, General & Administrative Expense (XSGA), and Number of Employees (EMP). Though the coverage is limited to publicly traded firms, they tend to be much larger than private firms and thus account for the dominant part of the industry dynamics (Grullon et al. 2019). The construction of the empirical counterparts of the variables in my model follows the existing literature in dropping outliers, as summarized in Appendix B.3.3.

In line with De Loecker et al. (2020, 2021), I consider SALE corresponding to the firm's revenue, COGS to the firm's variable costs, and XSGA to the firm's fixed costs. Although my model abstracts away from fixed entry costs, I need to apportion labor and material inputs between the variable and fixed costs to recover labor and material inputs. To this end, De Loecker et al. (2020) rely on a parametric assumption, while my framework avoid imposing a specific functional form restriction on the firm-level production. Thus, I instead use the direct measurement of the number of employees (EMP) and assume that the cost shares of labor and material are the same across fixed and variable costs.

Assumption B.2 (Constant Cost Share)**.** *For each sector* $i \in \mathbf{N}$ *and each firm* $k \in \mathbf{N}_i$*,* $VariableLaborCost_{ik} : VariableMaterialCost_{ik} = FixedLaborCost_{ik} : FixedMaterialCost_{ik} = \delta_{ik} : 1 - \delta_{ik}$*, where* $\delta_{ik} \in [0, 1]$ *is a constant specific to firm* k*.*

This assumption states that $COGS_{ik}$ and $XSGA_{ik}$ are made up of the same proportion of labor and material inputs.

B.3.1 Labor & Material Inputs

As in De Loecker et al. (2021), my construction starts from combining $COGS_{ik}$ and $XSGA_{ik}$ to compute the total costs. The firm k's total costs are given by

$$\begin{aligned}
TotalCosts_{ik} &= TotalLaborCost_{ik} + TotalMaterialCost_{ik} \\
&= \underbrace{VariableLaborCost_{ik} + VariableMaterialCost_{ik}}_{COGS_{ik}} \qquad (76) \\
&\quad + \underbrace{FixedLaborCost_{ik} + FixedMaterialCost_{ik}}_{XSGA_{ik}} \\
&= COGS_{ik} + XSGA_{ik}. \qquad (77)
\end{aligned}$$

Since both $COGS_{ik}$ and $XSGA_{ik}$ are observed in the data, I can compute the firm k's total expense ($TotalCost_{ik}$).

Next, the total expenditure on labor input is

$$\begin{aligned}
TotalLaborCosts_{ik} &= VariableLaborCosts_{ik} + FixedLaborCosts_{ik} \\
&= W \times AverageHoursWorked \times \underbrace{Employees_{ik}}_{EMP_{ik}} \\
&= W \times \frac{TotalHours}{TotalEmployees} \times EMP_{ik}. \qquad (78)
\end{aligned}$$

From Appendix B.1, both W and $TotalHours/TotalEmployees$ are directly observed in the data. Moreover, the Compustat data provide information about the number of employees (EMP_{ik}). Hence, I can calculate the firm k's total labor expense ($TotalLaborCosts_{ik}$). Then, the total expenditure on material input is, in turn, obtained as

$$TotalMaterialCosts_{ik} = TotalCosts_{ik} - TotalLaborCosts_{ik}. \qquad (79)$$

Now, I invoke Assumption B.2 to derive,

$$\delta_{ik} = \frac{TotalMaterialCost_{ik}}{TotalLaborCost_{ik} + TotalMaterialCost_{ik}}, \tag{80}$$

where both $TotalLaborCost_{ik}$ and $TotalMaterialCost_{ik}$ can be calculated according to (78) and (79), respectively. Since δ_{ik} is given by (80), I can recover $VariableLaborCost_{ik}$ (the empirical counterpart of $W^*\ell^*_{ik}$) and $VariableMaterialCost_{ik}$ (the empirical counterpart of $P_i^{M*}m^*_{ik}$) according to

$$\begin{aligned} VariableLaborCost_{ik} &= \delta_{ik}COGS_{ik} \\ VariableMaterialCost_{ik} &= (1-\delta_{ik})COGS_{ik}. \end{aligned}$$

In view of Fact B.1, once outlier eliminations are done (explained in Appendix B.3.3), I can divide the former by the wage W^*, and the latter by the sectoral cost index P_i^{M*} to obtain the firm's labor ℓ^*_{ik} and material input m^*_{ik}. These are summarized in the following fact.

Fact B.4 (Labor & Material Inputs)**.** *Under Assumption B.2, the firm-level labor input ℓ^*_{ik} and material input m^*_{ik} are recovered from the data.*

Remark B.2. *In deriving the firm-level input variables ℓ^*_{ik} and m^*_{ik}, firm's revenue and total costs are scaled up/down by $\backepsilon_i$ (see Definition B.1), so that the sectoral profits computed from the firm-level data become equal to those reported directly in the input-output table.*

B.3.2 Derived Demand for Sectoral Intermediate Goods

Since I lack separate data on the firm-level input demand for sectoral intermediate goods, I have to divid the firm's expenditure on material input in a way that is consistent with the configuration of the input-output linkage. To this end, I make additional assumptions on the form of the aggregator function $\mathcal{G}_i(\cdot)$ in (4). Specifically, I assume that the material input m_{ik} aggregates sectoral intermediate goods according to the Cobb-Douglas production function.[122]

Assumption B.3. *The material input m_{ik} comprises sectoral intermediate goods according to the Cobb-Douglas production function:*

$$m_{ik} = \prod_{j=1}^{N} m_{ik,j}^{\gamma_{i,j}},$$

where $m_{ik,j}$ is sector j's intermediate good demanded by firm k in sector i and $\gamma_{i,j}$ denotes the elasticity of output with respect to sector j's intermediate good, with $\sum_{j=1}^{N}\gamma_{i,j} = 1$.

[122] In principle, this assumption is necessitated in order to compensate for the limitation of the dataset at hand. This assumption could be relaxed to the extent which allows the researcher to recover the material input and demand for sectoral intermediate goods. Also, this assumption could even be omitted if detailed data on firm-to-firm trade are available, as studied for the Belgium data (Dhyne et al. 2021), the Chilean data (Huneeus 2020) and the Japanese data (Bernard et al. 2019).

In view of the structure of the input markets, it is implicit that the input share is the same within sector i. The producer price index for material input P_i^M is defined through the cost minimization problem, formulated as

$$P_i^M := \min_{\{m^\circ_{ik,j}\}_{j=1}^N} \sum_{j=1}^N (1-\tau_i) P_j m^\circ_{ik,j} \qquad s.t. \quad \prod_{j=1}^N (m^\circ_{ik,j})^{\gamma_{i,j}} \geq 1. \tag{81}$$

Under Assumption B.3, together with (81), I can recover both the cost index of material input and the input demand for sectoral intermediate goods from the observables.

Fact B.5 (Identification of $\gamma_{i,j}$, P_i^{M*} & $m^*_{ik,j}$)**.** *Suppose that Assumption B.3 holds. Then, (i) for each sector $i \in \mathbf{N}$, the input shares $\{\gamma_{i,j}\}_{j=1}^N$, and the cost index for material input P_i^{M*} are identified from the observables; and iii) for each sector $i \in \mathbf{N}$ and for each firm $k \in \mathbf{N}_i$, the input demand for composite intermediate goods $\{m^*_{ik,j}\}_{j=1}^N$ are identified from the observables.*

Proof. (i) From the first order conditions for the cost minimization, I have

$$(1-\tau_i) P^*_{j'} m^*_{ik,j'} = \frac{\gamma_{i,j'}}{\gamma_{i,j}} (1-\tau_i) P^*_j m^*_{ik,j},$$

Substituting this into (70) leads to

$$\omega_{i,j} = \frac{\sum_{k=1}^{N_i} (1-\tau_i) P^*_j m^*_{ik,j}}{\frac{1}{\gamma_{i,j}} \sum_{k=1}^{N_i} (1-\tau_i) P^*_j m^*_{ik,j} + \sum_{k=1}^{N_i} W^* \ell^*_{ik}},$$

where I note $\sum_{j'=1}^N \gamma_{i,j'} = 1$ by assumption. Rearranging this yields

$$\gamma_{i,j} = \frac{\sum_{k=1}^{N_i} (1-\tau_i) P^*_j m^*_{ik,j}}{\frac{1}{\omega_{i,j}} \sum_{k=1}^{N_i} (1-\tau_i) P^*_j m^*_{ik,j} - \sum_{k=1}^{N_i} W^* \ell^*_{ik}} = \frac{\omega_{i,j}}{\sum_{j'=1}^N \omega_{i,j'}}.$$

Since the terms in the rightmost expression $\{\omega_{i,j'}\}_{j'=1}^N$ are available in the data (see Appendix B.2.1), the parameter $\gamma_{i,j}$ can thus be identified for all $i \in \mathbf{N}$.

From (81), the equilibrium value of the cost index for material input P_i^{M*} is given by

$$P_i^{M*} = \prod_{j=1}^N \frac{1}{\gamma_{i,j}^{\gamma_{i,j}}} \{(1-\tau_i) P^*_j\}^{\gamma_{i,j}}. \tag{82}$$

Given that $\{\gamma_{i,j}\}_{j=1}^N$ are identified above, P_i^{M*} is also identified.

(ii) Now, using again the first order condition for the cost minimization problem, I have

$$(1-\tau_i) P^*_j = \nu_{ik} \gamma_{i,j} \frac{m^*_{ik}}{m^*_{ik,j}},$$

where ν_{ik} is the marginal cost of constructing additional unit of material input (De Loecker and

Warzynski 2012; De Loecker et al. 2016, 2020), which equals P_i^M. Hence, if follows

$$m_{ik,j}^* = \gamma_{i,j} \frac{P_i^{M*}}{(1-\tau_i)P_j^*} m_{ik}^*, \tag{83}$$

from which $m_{ik,j}^*$, the input demand for sector j's composite intermediate good from sector i, is identified. This completes the poof. $\square$

B.3.3 Data Construction

I follow the existing literature (e.g., Baqaee and Farhi 2020; De Loecker et al. 2021) in eliminating entries with missing data or zeros, and in dropping firms in the top and bottom 1% percentiles. The procedure for constructing firm-level dataset is summarized as follows:

Step 1: Eliminate entries with missing data or zeros in either SALE, COGS, XSGA or EMP.

Step 2: Drop firms with negative profits, which is calculated as SALE minus COGS minus XSGA.

Step 3: Omit firms with SALE-to-COGS and SALE-to-XSGA ratios in the top and bottom 1%.

Step 4: Scale up/down the firm's revenue and total costs according to $\backepsilon_i$ (see Definition B.1).

Step 5: Apply the results developed in Appendices B.3.1 and B.3.2 to construct the dataset for firm-level variables.

B.3.4 Market Concentration

To study the degree of oligopoly in each industry, I calculate the four-firm concentration ratio (CR4), a measure defined as the sum of market shares of the four largest firms in an industry. Table 4 displays the CR4, along with the number of firms, of each industry. These statistics before the outlier elimination (Step 3 of Appendix B.3.3) are compared to the ones after the outlier elimination. As shown in this table, the market concentration is overall unaltered by the data cleaning. According to the classification proposed in Shepherd (2018), it is fair to say that all the industries in my dataset fall into the categories of either loose oligopoly or tight oligopoly.[123]

B.3.5 Treatment of Capital

My model is static and abstracts away from capital accumulation over periods of time, whereby the object of interest being the value added. In reality, however, capital plays a great important role in a firm's production and input decisions. Moreover, policymakers may be interested in the gross output, rather than the value added. One way to accommodate capital in my model is to

[123] It should be remarked that the industry definition of my analysis is closer to the three-digit NAICS classification, a broader categorization than the four- or five-digit classifications, which are commonly used in the analysis of market concentration, such as antitrust and meager analyses. Due to this coarser definition, the CR4 in this paper may well appear to be lower compared to other studies based on much finer industry codes.

Table 4: Number of Firms and Concentration Ratio

Industry	Before Outlier Elimination		After Outlier Elimination	
	Number of firms	CR4	Number of firms	CR4
Oil and gas extraction, and mining	153.00	0.34	145.00	0.36
Support activities for mining	26.00	0.75	22.00	0.75
Construction	52.00	0.36	49.00	0.39
Food and beverage and tobacco products	83.00	0.33	81.00	0.34
Textile-related mills and apparel products	43.00	0.59	39.00	0.48
Wood and nonmetallic mineral products	28.00	0.57	24.00	0.64
Paper products and printing-related services	26.00	0.56	23.00	0.58
Petroleum and coal products	23.00	0.58	19.00	0.61
Chemical products	199.00	0.19	192.00	0.19
Plastics and rubber products	18.00	0.79	15.00	0.81
Primary metals	37.00	0.52	33.00	0.56
Fabricated metal products	54.00	0.44	52.00	0.44
Machinery	103.00	0.29	97.00	0.29
Computer and electronic products	256.00	0.36	246.00	0.37
Electrical equipment, appliances, and components	33.00	0.57	30.00	0.58
Motor vehicles and other transportation equipment	87.00	0.43	83.00	0.43
Furniture and related products	18.00	0.59	15.00	0.66
Miscellaneous manufacturing	60.00	0.43	57.00	0.43
Wholesale trade	104.00	0.53	98.00	0.55
Retail trade	139.00	0.46	133.00	0.45
Air and ground transportation	25.00	0.49	22.00	0.55
Water transportation	30.00	0.66	26.00	0.58
Other transportation and support activities	19.00	0.89	15.00	0.93
Publishing industries and information services	237.00	0.56	226.00	0.57
Mass media and telecommunications	85.00	0.45	82.00	0.48
Legal, scientific, and technical services	100.00	0.28	98.00	0.28
Administrative and waste services	60.00	0.36	56.00	0.37
Educational services	25.00	0.58	23.00	0.59
Arts, Entertainment, and Recreation	20.00	0.60	16.00	0.64
Accommodation	23.00	0.45	19.00	0.50

Note: This table display the number of firms and CR4 of each industry. These statistics before the outlier elimination (Step 3 of Appendix B.3.3) are compared to the ones after the outlier elimination. The definition of industry is based on the segmentation shown in Table B.2.

introduce the law of capital accumulation, which is left for future work. Alternatively, the model of this paper can be viewed as a stationary representation of a dynamic model. In this case, capital is effectively treated as an "endowment" of the agents in the analysis of the gross output, while it is considered to be a part of firm's productivity in the analysis of the value added.[124]

[124] In the latter scenario, the firm's productivity should be understood as the overall capability of production — a composite of the production efficiency and capital.

C Identification

The goal of this section is to prove Theorem 4.1. The proof requires recovering firm-level quantities and prices, and comparative statics of both sector- and firm-level variables. Moreover, these in turn require the identification of derivatives of firm-level production and inverse demand functions. To this end, I exploit the identification assumptions detailed in Section 4 in conjunction with the model defined in Section 2 and the data described in Section 3. In what follows, I first derive (84), before proceeding to the identification of firm-level price and quantity, and the identification of derivatives of the production and inverse demand functions.

C.1 Derivation of (84)

The derivation of (84) builds on the characterization result concerning exchangeable functions, which has recently been developed in the literature on computer science. For the sake of exposition, the main result is summarized as a lemma below.

Lemma C.1 (Subdecomposition (Zaheer et al. 2018; Wagstaff et al. 2019))**.** *Let $J \in \mathbb{N}$, and let $h : [0,1]^J \to \mathbb{R}$ be a continuous function. Then, $h(x_1, \ldots, x_J)$ is exchangeable in $(x_1, \ldots, x_J)$ if and only if it can be expressed as $h(x_1, \ldots, x_J) = \upsilon(\sum_{j=1}^J \rho(x_j))$ for some outer function $\upsilon : \mathbb{R}^{J+1} \to \mathbb{R}$ and some inner function $\rho : \mathbb{R} \to \mathbb{R}^{J+1}$.*

Proof. See Zaheer et al. (2018) and Wagstaff et al. (2019). □

Now, the expression (84) can be proved by the multiple application of this lemma.

Proposition C.1. *Suppose that Assumption 4.4 holds. Then, for each $i \in \mathbf{N}$, there exists a constant $M_i \in \mathbb{N}$ such that there exist some continuous functions $\mathcal{H}_{i,1}, \ldots, \mathcal{H}_{i,M_i} : \mathscr{Z}_i^{\mathbf{N}_i} \to \mathbb{R}$ and $\chi_i : \mathscr{Z}_i \times \mathbb{R}^{M_i} \to \mathbb{R}_+$ such that*

$$q^*_{ik} = \chi_i(z_{ik}; \mathcal{H}_{i,1}(\mathbf{z}_i), \ldots, \mathcal{H}_{i,M_i}(\mathbf{z}_i)), \tag{84}$$

where $\mathcal{H}_{i,m}(\mathbf{z}_i)$ is exchangeable in $(z_{i1}, \ldots, z_{iN_i})$ for all $m \in \{1, \ldots, M_i\}$.

Proof. First of all, it follows from Assumption 4.4 (ii) and Lemma C.1 that there exist continuous functions $\upsilon_0 : \mathbb{R}^{N_i+1} \to \mathbb{R}$ and $\rho_0 : \mathbb{R} \to \mathbb{R}^{N_i+1}$ such that

$$A_i(\{q_{ik'}\}_{k'=1}^{N_i}) = \upsilon_0(\sum_{k'=1}^{N_i} \rho_0(q_{ik'})).$$

In consequence, the partial derivative of $A_i(\cdot)$ with respect to q_{ik} is given by

$$\frac{\partial A_i(\cdot)}{\partial q_{ik}} = (\upsilon_0'(\sum_{k'=1}^{N_i} \rho_0(q_{ik'})))^T \rho_0'(q_{ik}),$$

where $\upsilon_0'(\cdot)$ and $\rho_0'(\cdot)$ are both $(N_i+1)\times 1$ vectors whose kth entry indicates the derivatives of $\upsilon_i(\cdot)$ and $\rho_0(\cdot)$ with respect to the kth argument, respectively; and T denotes the transpose of a vector.

Next, let $mc_{ik} = mc_i(z_{ik})$ be the firm k's marginal cost. Due to Assumption 2.4 *(i)*, mc_{ik} is independent of the firm's output quantity q_{ik}. Under Assumption 4.4, the Cournot-Nash equilibrium quantities satisfy the following system of first-order conditions:

$$\Phi_i \Psi_i'\left(\frac{q_{ik}}{A_i(\{q_{ik'}\}_{k'=1}^{N_i})}\right)\frac{A_i(\{q_{ik'}\}_{k'=1}^{N_i}) - \frac{\partial A_i(\cdot)}{\partial q_{ik}}}{A_i(\{q_{ik'}\}_{k'=1}^{N_i})^2} = mc_{ik},$$

for all $k \in \mathbf{N}_i$. Note here that firm k's identity can also be traced via the marginal costs mc_{ik} instead of the index k. Thus, it holds by symmetry that there exists a constant $M_i \in \mathbb{N}$ such that $H_{i,1},\ldots,H_{i,M_i} : \mathbb{R}_+^{N_i} \to \mathbb{R}$ and $\chi_i^a : \mathscr{Z} \times \mathbb{R}^{M_i} \to \mathbb{R}$ such that

$$q_{ik}^* = \chi_i^a\big(mc_{ik}; H_{i,1}(\{mc_{ik'}\}_{k'\neq k}),\ldots,H_{i,M_i}(\{mc_{ik'}\}_{k'\neq k})\big),$$

where each of $H_{i,1}(\cdot),\ldots,H_{i,M_i}(\cdot)$ is exchangeable in $(mc_{i1},\ldots,mc_{i(k-1)},mc_{i(k+1)},\ldots,mc_{iN_i})$. Again by Lemma C.1, this can further be rewritten as

$$\begin{aligned}
q_{ik}^* &= \chi_i^a\Bigg(mc_{ik}; \upsilon_1^a\Big(\sum_{k'\neq k}\rho_1(mc_{ik'})\Big),\ldots,\upsilon_{M_i}^a\Big(\sum_{k'\neq k}\rho_{M_i}(mc_{ik'})\Big)\Bigg)\\
&= \chi_i^b\Bigg(mc_{ik}; \sum_{k'\neq k}\rho_1(mc_{ik'}),\ldots,\sum_{k'\neq k}\rho_{M_i}(mc_{ik'})\Bigg)\\
&= \chi_i^b\Bigg(mc_{ik}; \sum_{k'=1}^{N_i}\rho_1(mc_{ik'}) - \rho_1(mc_{ik}),\ldots,\sum_{k'=1}^{N_i}\rho_{M_i}(mc_{ik'}) - \rho_{M_i}(mc_{ik})\Bigg)\\
&= \chi_i^c\Bigg(mc_{ik}; \sum_{k'=1}^{N_i}\rho_1(mc_{ik'}),\ldots,\sum_{k'=1}^{N_i}\rho_{M_i}(mc_{ik'})\Bigg)\\
&= \chi_i^d\Bigg(mc_{ik}; \upsilon_1^b\Big(\sum_{k'=1}^{N_i}\rho_1(mc_{ik'})\Big),\ldots,\upsilon_{M_i}^b\Big(\sum_{k'=1}^{N_i}\rho_{M_i}(mc_{ik'})\Big)\Bigg),
\end{aligned}$$

for some functions $\{\rho_m(\cdot)\}_{m=1}^{M_i}$, $\{\upsilon_m^a(\cdot)\}_{m=1}^{M_i}$, $\{\upsilon_m^b(\cdot)\}_{m=1}^{M_i}$, $\chi_i^b(\cdot)$, $\chi_i^c(\cdot)$ and $\chi_i^d(\cdot)$, each of which is appropriately defined. Applying once again Lemma C.1, it follows that for each $m = 1,\ldots,M_i$,

$$\check{H}_{i,m}(\{mc_{ik'}\}_{k'=1}^{N_i}) := \upsilon_m^b\Big(\sum_{k'=1}^{N_i}\rho_m(mc_{ik'})\Big)$$

is exchangeable in $(mc_{i1},\ldots,mc_{iN_i})$. Hence, the equilibrium quantity can be written as

$$q_{ik}^* = \chi_i^d\big(mc_{ik}; \check{H}_{i,1}(\{mc_{ik'}\}_{k'=1}^{N_i}),\ldots,\check{H}_{i,M_i}(\{mc_{ik'}\}_{k'=1}^{N_i})\big).$$

Since $mc_{ik} = mc_i(z_{ik})$, this can in turn be rearranged so that there exist some functions $\mathcal{H}_{i,1},\ldots,\mathcal{H}_{i,M_i}$:

$\mathscr{Z}^{N_i} \to \mathbb{R}$ and $\chi_i : \mathscr{Z} \times \mathbb{R}^{M_i} \to \mathbb{R}$ such that

$$q^*_{ik} = \chi_i\big(z_{ik}; \mathcal{H}_{i,1}(\{z_{ik'}\}_{k'=1}^{N_i}), \ldots, \mathcal{H}_{i,M_i}(\{z_{ik'}\}_{k'=1}^{N_i})\big),$$

where each of $\mathcal{H}_{i,1}(\cdot), \ldots, \mathcal{H}_{i,M_i}(\cdot)$ is, by construction, exchangeable in $(z_{i1}, \ldots, z_{iN_i})$. This proves the proposition. □

C.1.1 Detail of Example 4.2

As in Example 4.1, suppose that the sectoral aggregator takes the form of a CES function: $F_i(\{q_{ik}\}_{k\in\mathbf{N}_i}) := \big(\sum_{k=1}^{N_i} \delta_i q_{ik}^{\frac{\sigma-1}{\sigma}}\big)^{\frac{\sigma}{\sigma-1}}$. As shown in Example 4.1, the associated inverse demand function is given by $p_{ik} = \frac{\Phi_i}{q_{ik}} \frac{\delta_i q_{ik}^{\frac{\sigma-1}{\sigma}}}{\sum_{k'=1}^{N_i} \delta_i q_{ik'}^{\frac{\sigma-1}{\sigma}}}$, and the quantity index can be expressed as $A_i(\mathbf{q}_i) = \frac{1}{B_0}\sum_{k'=1}^{N_i} \delta_i q_{ik'}^{\frac{\sigma-1}{\sigma}}$. In the interest of clarity of exposition, assume that there are only three firms in each sector, i.e., $\mathbf{N}_i = \{1, 2, 3\}$, and consider the case of $\sigma = 2$, $\delta_i = 1$ and $B_0 = 1$. Assume in addition that firm's production technology is given by a Cobb-Douglas function: $q_{ik} = z_{ik}\ell_{ik}^{\alpha} m_{ik}^{1-\alpha}$ (the material aggregator $\mathcal{G}_i(\cdot)$ can be arbitrary).

Under this setup, the Cournot-Nash equilibrium quantities $\{q^*_{ik}\}_{k=1}^{3}$ satisfy the following system of equations:

$$\begin{aligned}
\frac{\frac{\sigma-1}{\sigma} {q^*_{i1}}^{-\frac{1}{\sigma}} (A^*_i - {q^*_{i1}}^{\frac{\sigma-1}{\sigma}})}{{A^*_i}^2}\Phi_i &= mc_{i1} \\
\frac{\frac{\sigma-1}{\sigma} {q^*_{i2}}^{-\frac{1}{\sigma}} (A^*_i - {q^*_{i2}}^{\frac{\sigma-1}{\sigma}})}{{A^*_i}^2}\Phi_i &= mc_{i2} \\
\frac{\frac{\sigma-1}{\sigma} {q^*_{i3}}^{-\frac{1}{\sigma}} (A^*_i - {q^*_{i3}}^{\frac{\sigma-1}{\sigma}})}{{A^*_i}^2}\Phi_i &= mc_{i3},
\end{aligned}$$

where A^*_i represents the equilibrium value of the quantity index, and $mc_{ik} := z_{ik}^{-1} mc_i$ is the firm k's marginal cost.[125] In particular, when $\sigma = 2$, this system can be solved for the equilibrium quantities, yielding

$$q^*_{ik} = \Big(\frac{A^*_i\Phi_i}{2{A^*_i}^2 mc_{ik} + \Phi_i}\Big)^2 \tag{85}$$

for each $k \in \{1, 2, 3\}$. By the construction, the equilibrium quantity index A^*_i satisfies

$$\begin{aligned}
A^*_i &= {q^*_{i1}}^{\frac{1}{2}} + {q^*_{i2}}^{\frac{1}{2}} + {q^*_{i3}}^{\frac{1}{2}} \\
&= \frac{A^*_i\Phi_i}{2{A^*_i}^2 mc_{i1} + \Phi_i} + \frac{A^*_i\Phi_i}{2{A^*_i}^2 mc_{i2} + \Phi_i} + \frac{A^*_i\Phi_i}{2{A^*_i}^2 mc_{i3} + \Phi_i}.
\end{aligned}$$

[125] Precisely, mc_i represents the component of the marginal cost common across all firms, and it is given by $mc_i = \alpha^{-\alpha}(1-\alpha)^{1-\alpha} W^{\alpha} (P_i^M)^{1-\alpha}$.

Rearranging this leads to

$$8mc_{i1}mc_{i2}mc_{i3}A_i^{*6} - 2(mc_{i1} + mc_{i2} + mc_{i3})\Phi_i^2 A_i^{*2} - 2\Phi_i^3 = 0.$$

Noticing that A_i^* has to be a real number, it follows from the general cubic formula (or the Cardano formula) that

$$A_i^{*2} = -\sqrt[3]{B} - \sqrt[3]{C}, \tag{86}$$

where $B = \frac{3\sqrt{3}t+\sqrt{27t^2+s^3}}{6\sqrt{3}}$ and $C = \frac{3\sqrt{3}t-\sqrt{27t^2+s^3}}{6\sqrt{3}}$ with $s = -\frac{mc_{i1}+mc_{i2}+mc_{i3}}{4mc_{i1}mc_{i2}mc_{i3}}\Phi_i = -\frac{z_{i1}^{-1}+z_{i2}^{-1}+z_{i3}^{-1}}{4(z_{i1}z_{i2}z_{i3})^{-1}mc_i^2}$ and $t = -\frac{\Phi_i^3}{4mc_{i1}mc_{i2}mc_{i3}} = -\frac{\Phi_i^3}{4(z_{i1}z_{i2}z_{i3})^{-1}mc_i^3}$.

Combining (85) and (86), one obtains

$$\begin{aligned} q_{ik}^* &= \frac{\Phi_i^2 A_i^{*2}}{(2mc_{ik}A_i^{*2} + \Phi_i)^2} \\ &= \chi_i(z_{ik}; \mathcal{H}_{i,1}(\{z_{ik'}\}_{k'=1}^3), \mathcal{H}_{i,2}(\{z_{ik'}\}_{k'=1}^3)), \end{aligned}$$

for some continuous function $\chi_i(\cdot)$, where $\mathcal{H}_{i,1}(\{z_{ik'}\}_{k'=1}^3) \coloneqq z_{i1}^{-1} + z_{i2}^{-1} + z_{i3}^{-1}$ and $\mathcal{H}_{i,2}(\{z_{ik'}\}_{k'=1}^3) \coloneqq z_{i1}z_{i2}z_{i3}$. Note here that both $\mathcal{H}_{i,1}(\cdot)$ and $\mathcal{H}_{i,2}(\cdot)$ are clearly exchangeable in (z_{i1}, z_{i2}, z_{i3}).

Next, the subsequent input choice — specifically, the inner optimization of (6) — is constrained by the production possibility frontier

$$\chi_i(z_{ik}; \mathcal{H}_{i,1}(\{z_{ik'}\}_{k'=1}^3), \mathcal{H}_{i,2}(\{z_{ik'}\}_{k'=1}^3)) = q_{ik}^* = z_{ik}\ell_{ik}^{\alpha} m_{ik}^{1-\alpha}.$$

Since $\chi_i(\cdot)$ obviously satisfies Assumption 4.5, this equation can be solved for z_{ik}. By the quadratic formula, it holds in equilibrium that

$$\begin{aligned} z_{ik} &= \frac{-(4mc_i {\ell_{ik}^*}^{\alpha} {m_{ik}^*}^{1-\alpha} A_i^{*2}\Phi_i - A_i^{*2}\Phi_i^2) \pm \sqrt{(4mc_i {\ell_{ik}^*}^{\alpha} {m_{ik}^*}^{1-\alpha} A_i^{*2}\Phi_i - A_i^{*2}\Phi_i^2)^2 - 16mc_i^2({\ell_{ik}^*}^{\alpha} {m_{ik}^*}^{1-\alpha})^2 A_i^{*2}\Phi_i}}{2{\ell_{ik}^*}^{\alpha} {m_{ik}^*}^{1-\alpha}\Phi_i} \\ &\eqqcolon \mathcal{M}_i(\ell_{ik}^*, m_{ik}^*; \mathcal{H}_{i,1}(\{z_{ik'}\}_{k'=1}^3), \mathcal{H}_{i,2}(\{z_{ik'}\}_{k'=1}^3)). \end{aligned}$$

This shows the existence of a function $\mathcal{M}_i(\cdot)$ by giving it an analytical expression. $\square$

C.2 Recovering the Values of Firm-Level Quantity and Price

In this subsection, I first derive the identification of the firm-level markups, and then turn to the identification of firm-level prices and quantities, followed by that of the firm-level inverse demand function.

C.2.1 Identification of the Values of Markup

It can be shown that the firm-level markups are recovered from the observables under the assumptions imposed in the main text (these assumptions are presented in Section 2.3 and summarized below for ease of reference).[126]

Assumption C.1 (Input Markets)**.** *(i) The input markets are perfectly competitive. (ii) All inputs are variable.*

Fact C.1. *Suppose that Assumptions 2.4 and C.1 hold. For each sector $i \in \mathbf{N}$ and each firm $k \in \mathbf{N}_i$, the value of the firm-level markup μ_{ik}^* can be recovered from the data.*

Proof. Under Assumption C.1, the equilibrium value of the firm's markup μ_{ik}^* can be expressed as:

$$\mu_{ik}^* \coloneqq \frac{p_{ik}^*}{MC_{ik}^*} = \frac{Revenue_{ik}^*}{TC_{ik}^*} \frac{AC_{ik}^*}{MC_{ik}^*},$$

where MC_{ik}^*, AC_{ik}^*, and TC_{ik}^* represent the equilibrium values of the marginal, average, and total costs, respectively. Note here that $\frac{AC_{ik}^*}{MC_{ik}^*}$ is the elasticity of cost with respect to quantity (Syverson 2019), which equals one due to Assumption 2.4 (i). Hence, I have

$$\mu_{ik}^* = \frac{Revenue_{ik}^*}{TC_{ik}^*},$$

i.e., the value of the firm's markup equals the ratio of revenue to total costs, both of which are observed in the data. Thus, the value of the firm-level markup μ_{ik}^* is identified from the observables, as desired. □

C.2.2 Identification of the Values of Quantity and Price

Let $\mathscr{R}_i$, $\mathscr{L}_i$ and $\mathscr{M}_i$ be the observed supports of revenue r_{ik}, labor input ℓ_{ik} and material input m_{ik}, respectively. To facilitate exposition, I introduce a tilde notation to denote the logarithm of each variable. For instance, I write the logarithms of the firm's revenue, labor and material inputs, and productivity as $\tilde{r}_{ik}$, $\tilde{\ell}_{ik}$, $\tilde{m}_{ik}$ and $\tilde{z}_{ik}$, respectively. Correspondingly, the observed supports for r_{ik}, ℓ_{ik} and m_{ik} are denoted by $\tilde{\mathscr{R}}_i$, $\tilde{\mathscr{L}}_i$ and $\tilde{\mathscr{M}}_i$, respectively. Also, the logarithms of a firm's output quantity and price are expressed as

$$\tilde{q}_{ik} \coloneqq \ln q_{ik} = \tilde{f}_i(\tilde{\ell}_{ik}, \tilde{m}_{ik}; \tilde{z}_{ik}), \tag{87}$$

and

$$\tilde{p}_{ik} \coloneqq \ln p_{ik} = \tilde{\wp}_i(\tilde{q}_{ik}, \tilde{A}_i(\tilde{\mathbf{q}}_i); \mathcal{I}_i), \tag{88}$$

[126]See Syverson (2019), De Loecker et al. (2020) and Kasahara and Sugita (2020) for discussion.

where $\tilde{f}_i(\cdot) \coloneqq (\ln \circ f_i \circ \exp)(\cdot)$, $\tilde{\wp}_i(\cdot) \coloneqq (\ln \circ \wp_i \circ \exp)(\cdot)$, and $\tilde{A}_i(\cdot) \coloneqq (\ln \circ A_i \circ \exp)(\cdot)$. In what follows, I let the quantity index $\tilde{A}_i(\cdot)$ and the information set $\mathcal{I}_i$ be absorbed in the sector index i for the sake of brevity.

Let $\frac{\partial \tilde{f}_i(\cdot)^*}{\partial \tilde{\ell}_{ik}}$ and $\frac{\partial \tilde{f}_i(\cdot)^*}{\partial \tilde{m}_{ik}}$, respectively, denote the equilibrium values of the first-order derivatives of the log-production function with respect to log-labor and log-material, i.e.,

$$\frac{\partial \tilde{f}_i(\cdot)^*}{\partial \tilde{\ell}_{ik}} \coloneqq \left.\frac{\partial \tilde{f}_i(\cdot)}{\partial \tilde{\ell}_{ik}}\right|_{(\tilde{\ell}_{ik}, \tilde{m}_{ik}) = (\tilde{\ell}^*_{ik}, \tilde{m}^*_{ik})},$$

and $\frac{\partial \tilde{f}_i(\cdot)^*}{\partial \tilde{m}_{ik}}$ is analogously defined.

It can easily be shown that $\frac{\partial \tilde{f}_i(\cdot)^*}{\partial \tilde{\ell}_{ik}}$ and $\frac{\partial \tilde{f}_i(\cdot)^*}{\partial \tilde{m}_{ik}}$ are identified from the data.

Proposition C.2. *Suppose that Assumptions 2.4 and C.1 hold. Then, the equilibrium values of the derivative of the log-production function with respect to log labor and log material can be recovered from the observables.*

Proof. Under Assumptions 2.4 and C.1, the firm's input cost minimization problem is well-defined and has interior solutions. For a given level of output $\tilde{q}^*_{ik}$, the associated Lagrange function[127] in terms of the logarithm variables reads

$$\tilde{\mathcal{L}}(\tilde{\ell}_{ik}, \tilde{m}_{ik}, \xi_{ik}) \coloneqq \exp\{\tilde{W} + \tilde{\ell}_{ik}\} + \exp\{\tilde{P}^M_i + \tilde{m}_{ik}\} - \xi_{ik}\bigg(\exp\{\tilde{f}_i(\tilde{\ell}_{ik}, \tilde{m}_{ik}; \tilde{z}_{ik})\} - \exp\{\tilde{q}^*_{ik}\}\bigg),$$

where ξ_{ik} represents the Lagrange multiplier indicating the marginal cost of producing an additional unit of output at the given level $\tilde{q}^*_{ik}$ (De Loecker and Warzynski 2012; De Loecker et al. 2016, 2020). In equilibrium, the first order conditions at $\tilde{q}^*_{ik}$ look like

$$[\tilde{\ell}_{ik}] : \exp\{\tilde{W}^* + \tilde{\ell}^*_{ik}\} - \xi^*_{ik}\frac{\partial \tilde{f}_i(\cdot)^*}{\partial \tilde{\ell}_{ik}} \exp\{\tilde{f}_i(\tilde{\ell}^*_{ik}, \tilde{m}^*_{ik}; \tilde{z}_{ik})\} = 0 \tag{89}$$

$$[\tilde{m}_{ik}] : \exp\{\tilde{P^M_i}^* + \tilde{m}^*_{ik}\} - \xi^*_{ik}\frac{\partial \tilde{f}_i(\cdot)^*}{\partial \tilde{m}_{ik}} \exp\{\tilde{f}_i(\tilde{\ell}^*_{ik}, \tilde{m}^*_{ik}; \tilde{z}_{ik})\} = 0, \tag{90}$$

where $\tilde{\ell}^*_{ik}$ and $\tilde{m}^*_{ik}$, respectively, are the equilibrium quantities of labor and material inputs corresponding to the given output level q^*_{ik}. Taking the ratio between (89) and (90), I have

$$\frac{\frac{\partial \tilde{f}_i(\cdot)^*}{\partial \tilde{\ell}_{ik}}}{\frac{\partial \tilde{f}_i(\cdot)^*}{\partial \tilde{m}_{ik}}} = \frac{\exp\{\tilde{W}^* + \tilde{\ell}^*_{ik}\}}{\exp\{\tilde{P^M_i}^* + \tilde{m}^*_{ik}\}}. \tag{91}$$

[127] To simplify the exposition, I leverage the equivalence explained in Remark A.1, and consider the simultaneous decision of labor and material inputs, instead of the sequential one.

Here, due to Assumption 2.4 (i),

$$\frac{\partial \tilde{f}_i(\cdot)^*}{\partial \tilde{\ell}_{ik}} + \frac{\partial \tilde{f}_i(\cdot)^*}{\partial \tilde{m}_{ik}} = 1,$$

so that (91) gives

$$\begin{aligned}
\frac{\partial \tilde{f}_i(\cdot)^*}{\partial \tilde{\ell}_{ik}} &= \frac{\exp\{\tilde{W}^* + \tilde{\ell}_{ik}^*\}}{\exp\{\tilde{W}^* + \tilde{\ell}_{ik}^*\} + \exp\{\tilde{P_i^M}^* + \tilde{m}_{ik}^*\}} \\
\frac{\partial \tilde{f}_i(\cdot)^*}{\partial \tilde{m}_{ik}} &= \frac{\exp\{\tilde{P_i^M}^* + \tilde{m}_{ik}^*\}}{\exp\{\tilde{W}^* + \tilde{\ell}_{ik}^*\} + \exp\{\tilde{P_i^M}^* + \tilde{m}_{ik}^*\}}.
\end{aligned}$$

Since both $\exp\{\tilde{W}^* + \tilde{\ell}_{ik}^*\}$ and $\exp\{\tilde{P_i^M}^* + \tilde{m}_{ik}^*\}$ are available in the data (Appendix B), I thus can identify $\frac{\partial \tilde{f}_i(\cdot)^*}{\partial \tilde{\ell}_{ik}}$ and $\frac{\partial \tilde{f}_i(\cdot)^*}{\partial \tilde{m}_{ik}}$ from the observables, as claimed. $\square$

Next, I closely follow Kasahara and Sugita (2020) in identifying the equilibrium values of firm-level output quantity and price. Because of this, the notations are intentionally set closed to theirs.

To begin with, I admit a measurement error $\tilde{\eta}_{ik}$ in the observed log-revenue:[128]

$$\begin{aligned}
\tilde{r}_{ik} &= \tilde{\wp}_i(\tilde{q}_{ik}) + \tilde{q}_{ik} + \tilde{\eta}_{ik} \\
&= \tilde{\varphi}_i(\tilde{q}_{ik}) + \tilde{\eta}_{ik} \\
&= \tilde{\varphi}_i(\tilde{f}_i(\tilde{\ell}_{ik}, \tilde{m}_{ik}, \tilde{\mathcal{M}}_i(\tilde{\ell}_{ik}, \tilde{m}_{ik})) + \tilde{\eta}_{ik} \\
&= \tilde{\phi}_i(\tilde{\ell}_{ik}, \tilde{m}_{ik}) + \tilde{\eta}_{ik},
\end{aligned}$$

where $\tilde{\varphi}_i(\tilde{q}_{ik}) \coloneqq \tilde{\wp}_i(\tilde{q}_{ik}) + \tilde{q}_{ik}$, and $\tilde{\phi}_i(\cdot)$ is the nonparametric component of the revenue function in terms of labor and material inputs satisfying $\tilde{\phi}_i(\tilde{\ell}_{ik}, \tilde{m}_{ik}) = \tilde{\varphi}_i(\tilde{f}_i(\tilde{\ell}_{ik}, \tilde{m}_{ik}, \tilde{\mathcal{M}}_i(\tilde{\ell}_{ik}, \tilde{m}_{ik}))$. The additive separability of the log measurement error $\tilde{\eta}_{ik}$ is chosen to conform to the bulk of the literature on identification and estimation of production functions.[129]

Towards identification, it is posited that the econometrician has knowledge about the following regularity conditions.

Assumption C.2 (Regularity Conditions)**.** *(i) (Strict Exogeneity)* $E[\tilde{\eta}_{ik}|\tilde{\ell}_{ik}, \tilde{m}_{ik}] = 0$. *(ii) (Continuous Differentiability)* $\phi_i(\cdot)$ *is at least first differentiable in each of its argument. (iii) (Normalization) For each* $i \in \mathbf{N}$ *and each* $k \in \mathbf{N}_i$*, there exists a pair of labor and material inputs* $(\tilde{\ell}_{ik}^\circ, \tilde{m}_{ik}^\circ) \in \tilde{\mathscr{L}}_i \times \tilde{\mathscr{M}}_i$ *such that* $\tilde{f}_i(\tilde{\ell}_{ik}^\circ, \tilde{m}_{ik}^\circ; \tilde{z}_{ik}) = 0$.

[128] The measurement error is supposed to capture the variation in revenue that cannot be explained by firm-level input variables nor aggregate variables. This can be conceived as i) a shock to the firm's production that is unanticipated to the firm and hits after the firm's decision has been made, ii) the coding error in the measurement used by the econometrician.

[129] This specification is equivalent to assume that the error terms enter in a multiplicative way the system of structural equations in terms of the original variables. The additive separability of the measurement errors in terms of the logarithm variables are canonically employed in the literature (Olley and Pakes 1996; Levinsohn and Petrin 2003; Ackerberg et al. 2015; Gandhi et al. 2019).

Lemma C.2. *Suppose that Assumptions 2.4, C.1, and C.2 hold. Then, the logarithms of the firm-level output quantity $\tilde{q}^*_{ik}$ and price $\tilde{p}^*_{ik}$ can be identified up to scale from the observables.*

Proof. The proof proceeds in three steps.

Step 1:

The first step identifies the firm's revenue free of the measurement errors $\bar{\tilde{r}}_{ik}$ in terms of $(\tilde{\ell}_{ik}, \tilde{m}_{ik})$, eliminating the measurement error $\tilde{\eta}_{ik}$. From Assumption C.2, I can identify $\tilde{\phi}_i(\cdot)$, $\bar{\tilde{r}}_{ik}$ and $\tilde{\varepsilon}_{ik}$ according to

$$\begin{aligned}
&\tilde{\phi}_i(\tilde{\ell}_{ik}, \tilde{m}_{ik}) = E[\tilde{r}_{ik}|\tilde{\ell}_{ik}, \tilde{m}_{ik}];\\
&\bar{\tilde{r}}_{ik} = \tilde{\phi}_i(\tilde{\ell}_{ik}, \tilde{m}_{ik}); \text{and}\\
&\tilde{\eta}_{ik} = \tilde{r}_{ik} - \bar{\tilde{r}}_{ik}.
\end{aligned}$$

Step 2:

Next, I aim to identify the derivative of the inverse of the revenue function $\tilde{\varphi}_i$. By definition, it is true that

$$\tilde{f}_i(\tilde{\ell}_{ik}, \tilde{m}_{ik}, \tilde{\mathcal{M}}_i(\tilde{\ell}_{ik}, \tilde{m}_{ik})) = \tilde{\varphi}_i^{-1}(\bar{\tilde{r}}_{ik}). \tag{92}$$

Noticing that $\bar{\tilde{r}}_{ik} = \tilde{\phi}_i(\tilde{\ell}_{ik}, \tilde{m}_{ik})$ is identified above, one can take derivatives of (92) with respect to $\tilde{\ell}_{ik}$ and $\tilde{m}_{ik}$ to obtain

$$\frac{\partial \tilde{f}_i(\cdot)}{\partial \tilde{\ell}_{ik}} + \frac{\partial \tilde{f}_i(\cdot)}{\partial \tilde{z}_{ik}} \frac{\partial \tilde{\mathcal{M}}_i(\cdot)}{\partial \tilde{\ell}_{ik}} = \frac{\partial \tilde{\varphi}_i^{-1}(\cdot)}{\partial \bar{\tilde{r}}_{ik}} \frac{\partial \tilde{\phi}_i(\cdot)}{\partial \tilde{\ell}_{ik}} \tag{93}$$

$$\frac{\partial \tilde{f}_i(\cdot)}{\partial \tilde{m}_{ik}} + \frac{\partial \tilde{f}_i(\cdot)}{\partial \tilde{z}_{ik}} \frac{\partial \tilde{\mathcal{M}}_i(\cdot)}{\partial \tilde{m}_{ik}} = \frac{\partial \tilde{\varphi}_i^{-1}(\cdot)}{\partial \bar{\tilde{r}}_{ik}} \frac{\partial \tilde{\phi}_i(\cdot)}{\partial \tilde{m}_{ik}} \tag{94}$$

for all $(\tilde{\ell}_{ik}, \tilde{m}_{ik}) \in \tilde{\mathscr{L}}_i \times \tilde{\mathscr{M}}_i$. Here notice that $\frac{d\tilde{\varphi}_i^{-1}(\cdot)^*}{d\tilde{r}^*_{ik}} = \left(\frac{d\tilde{r}^*_{ik}}{d\tilde{q}^*_{ik}}\right)^{-1}$, with the right-hand side being the firm's markup (Kasahara and Sugita 2020). Owing to Fact C.1, the equilibrium firm's markup (in log) $\tilde{\mu}^*_{ik}$ is obtained by $\tilde{\mu}^*_{ik} = \bar{\tilde{r}}_{ik} - \tilde{TC}_{ik}(\tilde{\ell}^*_{ik}, \tilde{m}^*_{ik})$, where $\tilde{TC}_{ik}(\tilde{\ell}_{ik}, \tilde{m}_{ik}) := \ln[\exp\{\tilde{W}^* + \tilde{\ell}_{ik}\} + \exp\{\tilde{P_i^{M*}} + \tilde{m}_{ik}\}]$. Thus, $\frac{d\tilde{\varphi}_i^{-1}(\cdot)^*}{d\bar{\tilde{r}}_{ik}}$ is identified as

$$\frac{\partial \tilde{\varphi}_i^{-1}(\cdot)}{\partial \bar{\tilde{r}}_{ik}} = \tilde{\phi}_i(\tilde{\ell}_{ik}, \tilde{m}_{ik}) - \ln[\exp\{\tilde{W} + \tilde{\ell}_{ik}\} + \exp\{\tilde{P}_i^M + \tilde{m}_{ik}\}].$$

Since the equilibrium values of $\frac{\partial \tilde{f}_i(\cdot)^*}{\partial \tilde{\ell}_{ik}}$ and $\frac{\partial \tilde{f}_i(\cdot)^*}{\partial \tilde{m}_{ik}}$ are identified in Proposition C.2, (93) and (94) can be rearranged to identify, respectively, $\frac{\partial \tilde{f}_i(\cdot)^*}{\partial \tilde{z}_{ik}} \frac{\partial \tilde{\mathcal{M}}_i(\cdot)^*}{\partial \tilde{\ell}_{ik}}$ and $\frac{\partial \tilde{f}_i(\cdot)^*}{\partial \tilde{z}_{ik}} \frac{\partial \tilde{\mathcal{M}}_i(\cdot)^*}{\partial \tilde{m}_{ik}}$, i.e.,

$$\frac{\partial \tilde{f}_i(\cdot)^*}{\partial \tilde{z}_{ik}} \frac{\partial \tilde{\mathcal{M}}_i(\cdot)^*}{\partial \tilde{\ell}_{ik}} = \frac{\partial \tilde{\varphi}_i^{-1}(\cdot)^*}{\partial \bar{\tilde{r}}_{ik}} \frac{\partial \tilde{\phi}_i(\cdot)^*}{\partial \tilde{\ell}_{ik}} - \frac{\partial \tilde{f}_i(\cdot)^*}{\partial \tilde{\ell}_{ik}}, \tag{95}$$

and

$$\frac{\partial \tilde{f}_i(\cdot)^*}{\partial \tilde{z}_{ik}} \frac{\partial \tilde{\mathcal{M}}_i(\cdot)^*}{\partial \tilde{m}_{ik}} = \frac{\partial \tilde{\varphi}_i^{-1}(\cdot)^*}{\partial \bar{\tilde{r}}_{ik}} \frac{\partial \tilde{\phi}_i(\cdot)^*}{\partial \tilde{m}_{ik}} - \frac{\partial \tilde{f}_i(\cdot)^*}{\partial \tilde{m}_{ik}}. \tag{96}$$

Step 3:

The final step recovers the realized value of firm-level output quantity by means of integration:

$$\begin{aligned}\tilde{q}_{ik}^* &= \tilde{f}_i(\tilde{\ell}_{ik}^*, \tilde{m}_{ik}^*, \tilde{z}_{ik}) \\ &= \int_{\tilde{\ell}_{ik}^\circ}^{\tilde{\ell}_{ik}^*} \left(\frac{\partial \tilde{f}_i}{\partial \tilde{\ell}_{ik}} + \frac{\partial \tilde{f}_i}{\partial \tilde{z}_{ik}} \frac{\partial \tilde{\mathcal{M}}_i}{\partial \tilde{\ell}_{ik}} \right)(s, \tilde{m}_{ik}^*) ds + \int_{\tilde{m}_{ik}^\circ}^{\tilde{m}_{ik}^*} \left(\frac{\partial \tilde{f}_i}{\partial \tilde{m}_{ik}} + \frac{\partial \tilde{f}_i}{\partial \tilde{z}_{ik}} \frac{\partial \tilde{\mathcal{M}}_i}{\partial \tilde{m}_{ik}} \right)(\tilde{\ell}_{ik}^\circ, s) ds,\end{aligned}$$

where the value of $\tilde{f}_i(\tilde{\ell}_{ik}^\circ, \tilde{m}_{ik}^\circ, \tilde{z}_{ik})$ is assumed to be known to the econometrician (Assumption C.2 (iii)).

Lastly, I can also recover the realized value of the firm-level output price $\tilde{p}_{ik}^*$ through

$$\tilde{p}_{ik}^* = \bar{\tilde{r}}_{ik} - \tilde{q}_{ik}^*.$$

This completes the proof. □

Remark C.1. *(i) Lemma C.2 rests on the identifiability of the value of the firm-level markup μ_{ik} (Fact C.1). Kasahara and Sugita (2020) instead exploit the panel structure of their dataset to first identify the firm's productivity from the observables. My framework, by contrast, is static in nature, which prohibits the use of panel data. In light of this, the use of Fact C.1 can be considered a compromise between the data availability and the model assumptions. (ii) The proof of Lemma C.2 does not require the identification of the firm's productivity per se, and thus it does not invoke the feature of the Hicks-neutral productivity in the firm-level production function (Assumption 4.3). Thus, this lemma also applies to the case of non-Hicks-neutral productivity as studied in Demirer (2022) and Pan (2022). Under Hicks-neutrality, it holds $\frac{\partial \tilde{f}_i(\cdot)}{\partial \tilde{z}_{ik}} = 1$. (iii) As discussed in Kasahara and Sugita (2020, 2023), Lemma C.2 identifies the firm-level quantity and price only up to a scale constant. Nevertheless, it is straightforward to verify that this is innocuous for the purpose of this paper, as the scale constants end up canceling out with each other. Hence, the presence of the scale constants is made implicit throughout the exposition.*

Having Lemma C.2 established, the firm-level quantity and price can immediately be recovered by reverting (87) and (88).

Proposition C.3. *Suppose that the assumptions required in Lemma C.2 hold. Then the equilibrium values of the firm-level quantity q_{ik}^* and price p_{ik}^* are identified up to scale from the observables.*

C.3 Recovering Demand Function (Sectoral Aggregator)

C.3.1 HSA Demand System

With the notation defined so far, the HSA demand system in Assumption 4.4 can be expressed as follows. First, by definition

$$\Phi_i \coloneqq \sum_{k=1}^{N_i} p_{ik}^* q_{ik}^*,$$

where p_{ik}^* and q_{ik}^* are the equilibrium (realized) values of firm-level price and quantity. Then, I can take

$$\Phi_i = \sum_{k=1}^{N_i} \varphi_i(q_{ik}^*), \tag{97}$$

where $r_{ik} = \varphi_i(q_{ik})$ with $\varphi_i(\cdot) \coloneqq (\exp \circ \tilde{\varphi}_i \circ \ln)(\cdot)$.

Next, let $\wp_i(q_{ik}, \mathbf{q}_{i,-k}) = \wp_{ik}(\mathbf{q}_i)$ be the residual inverse demand function faced by firm k in sector i. Under Assumption 4.4, it takes the form of

$$p_{ik} = \frac{\Phi_i}{q_{ik}} \Psi_i \left(\frac{q_{ik}}{A_i(\mathbf{q}_i)} \right) \eqqcolon \wp_i(q_{ik}; \mathbf{q}_{i,-k}), \tag{98}$$

where

$$\Psi_i(q_{ik}) = \frac{\varphi_i(q_{ik})}{\Phi_i}, \tag{99}$$

with

$$\sum_{k=1}^{N_i} \Psi_i \left(\frac{q_{ik}}{A_i(\mathbf{q}_i)} \right) = 1. \tag{100}$$

In the remainder of this subsection, I study the identification of the quantity index, residual inverse demand functions, marginal revenue functions and their first- and second-order derivatives, as well as sectoral price index. To derive analytical expressions incurs notational overhead. Taking derivatives of the both hand sides of (100), one obtains

$$\frac{q_{ik}}{A_i(\mathbf{q}_i)} \frac{\partial A_i(\cdot)}{\partial q_{ik}} = \frac{\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}} \exp\{\tilde{\varphi}_i(\tilde{x}_{ik})\}}{\sum_{k'=1}^{N_i} \frac{d\tilde{r}_{ik'}}{d\tilde{x}_{ik'}} \exp\{\tilde{\varphi}_i(\tilde{x}_{ik'})\}}, \tag{101}$$

where $x_{ik} = \frac{q_{ik}}{A_i(\mathbf{q}_i)}$, and $\tilde{x}_{ik} = \ln x_{ik}$. Notice here that the right hand side of (101) represents a weighted revenue-based market share with the weight attached to the derivatives of log revenue with respect to $\tilde{x}_{ik}$. Given this observation, denote $\tilde{u}_{ik} = \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}} \exp\{\tilde{\varphi}_i(\tilde{x}_{ik})\}$ and $\varpi_{ik} = \frac{\tilde{u}_{ik}}{\sum_{k'=1}^{N_i} \tilde{u}_{ik'}}$. Define moreover $\varrho_{ik} \coloneqq \left(\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\right)^{-1} \frac{d^2\tilde{r}_{ik}}{d\tilde{x}_{ik}^2} + \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}$ and $t_{ik} \coloneqq \frac{\varpi_{ik}}{q_{ik}} \left(\varrho_{ik} - \frac{\sum_{k'=1}^{N_i} \varrho_{ik'} \tilde{u}_{ik'}}{\sum_{k'=1}^{N_i} \tilde{u}_{ik'}}\right)$. Note that with Proposition C.3 established, all of these values (i.e., $\tilde{u}_{ik}$, ϖ_{ik}, ϱ_{ik} and t_{ik}) are identified from the observables.

C.3.2 Proofs

Quantity index. I first identify the quantity index $A_i(\cdot)$ over the entire support $\mathscr{S}_i^{N_i}$. This is shown in Kasahara and Sugita (2020).

Lemma C.3 (Identification of A_i; Kasahara and Sugita (2020))**.** *Suppose that the same assumptions in Lemma C.2 are satisfied. Assume moreover that Assumption 4.4 holds with* (97) – (100)*. Then, the quantity index $A_i(\mathbf{q}_i)$ is identified.*

Proof. See Kasahara and Sugita (2020). □

In Lemma C.3, the quantity index $A_i(\cdot)$ is nonparametrically identified as a function of $\mathbf{q}_i$, so that its derivatives can also be nonparametrically identified. The analytical expressions are summarized in the following corollary.

Corollary C.1 (Identification of $\frac{\partial A_i(\cdot)}{\partial q_{ik}}$ and $\frac{\partial^2 A_i(\cdot)}{\partial q_{ik} q_{ik'}}$)**.** *Suppose that the same assumptions required in Lemma C.3 hold. Then, for each $i \in \mathbf{N}$, (i) $\frac{\partial A_i(\cdot)}{\partial q_{ik'}}$ and (ii) $\frac{\partial^2 A_i(\cdot)}{\partial q_{ik} \partial q_{ik'}}$ are identified for all $k, k' \in \mathbf{N}_i$.*

Proof. (i) Rearranging (101) yields $\frac{\partial A_i(\cdot)}{\partial q_{ik}} = \frac{A_i(\mathbf{q}_i)}{q_{ik}} \varpi_{ik}$, according to which the partial derivative of the quantity index with respect to individual firm's output is identified.

(ii) One can apply another differentiation to the result obtained in part (i). The analytical expression for the second-order partial derivative of $A_i(\cdot)$ with respect to q_{ik} is given by

$$\frac{\partial^2 A_i(\cdot)}{\partial q_{ik}^2} = -(1 - \varrho_{ik}) \frac{A_i(\mathbf{q}_i)}{q_{ik}^2} (1 - \varpi_{ik}) \varpi_{ik} - \frac{A_i(\mathbf{q}_i)}{q_{ik}} \varpi_{ik} t_{ik}.$$

The mixed partial derivatives of $A_i(\cdot)$ with respect to q_{ik} and $q_{ik'}$, with $k' \neq k$, are given by

$$\frac{\partial^2 A_i(\cdot)}{\partial q_{ik} \partial q_{ik'}} = (1 - \varrho_{ik}) \frac{A_i(\mathbf{q}_i)}{q_{ik} q_{ik'}} \varpi_{ik} \varpi_{ik'} - \frac{A_i(\mathbf{q}_i)}{q_{ik}} \varpi_{ik} t_{ik'},$$

completing the proof. □

Responsivenesses of demand functions. With the firm-level output quantity and price identified in Proposition C.3, I can further recover the residual inverse demand functions faced by firms and their elasticities with respect to firm's quantity. I write $\frac{\partial \wp_{ik}(\cdot)}{\partial q_{ik'}} = \frac{\partial \wp_i(q_{ik}, \wp_{i,-k})}{\partial q_{ik'}}$.

Lemma C.4. *Suppose that the same assumptions required in Lemma C.3 hold. Then, the first- and second-order derivatives of the residual inverse demand functions $\wp_i(\cdot)$ can be identified from the observables.*

Proof. For each $i \in \mathbf{N}$ and $k \in \mathbf{N}_i$, taking the partial derivatives of (98) with respect to q_{ik} and $q_{ik'}$ ($k' \neq k$), respectively, yield

$$\frac{\partial \wp_{ik}(\cdot)}{\partial q_{ik}} = -\frac{p_{ik}}{q_{ik}}\Big\{1 - \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}(1 - \varpi_{ik})\Big\}, \tag{102}$$

and

$$\frac{\partial \wp_{ik}(\cdot)}{\partial q_{ik'}} = -\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}} p_{ik} \frac{\varpi_{ik'}}{q_{ik'}}.$$

Taking further the partial derivatives of (102), it is immediate to obtain

$$\begin{aligned}\frac{\partial^2 \wp_{ik}(\cdot)}{\partial q_{ik}^2} = \frac{p_{ik}}{q_{ik}}\Bigg[&\Big\{1 - \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}(1 - \varpi_{ik})\Big\}\Big\{2 - \frac{d\tilde{r}_{ik}}{\tilde{x}_{ik}}(1 - \varpi_{ik})\Big\} + (1 - \varpi_{ik})\Big\{\frac{d^2\tilde{r}_{ik}}{d\tilde{x}_{ik}^2}(1 - \varpi_{ik}) - \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\varrho_{ik}\varpi_{ik}\Big\}\Bigg] \\ &+ \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\frac{p_{ik}}{q_{ik}}\varpi_{ik} t_{ik},\end{aligned}$$

and

$$\frac{\partial^2 \wp_{ik}(\cdot)}{\partial q_{ik} q_{ik'}} = -\frac{p_{ik}}{q_{ik} q_{ik'}}\varpi_{ik'}\Bigg[\frac{d^2\tilde{r}_{ik}}{d\tilde{x}_{ik}^2}(1 - \varpi_{ik}) - \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\Big\{\Big(1 - \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\Big) + \Big(\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}} + \varrho_{ik}\Big)\varpi_{ik}\Big\}\Bigg] + \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\frac{p_{ik}}{q_{ik}}\varpi_{ik} t_{ik'}.$$

for all $k' \neq k$, which completes the proof. $\square$

Remark C.2. *Analogous results can be derived for monopolistic competition:* $\frac{\partial \wp_{ik}(\cdot)}{\partial q_{ik}} = -\frac{p_{ik}}{q_{ik}}\big(1 - \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\big)$, *and* $\frac{\partial \wp_{ik}(\cdot)}{\partial q_{ik'}} = 0$ *for all* $k' \neq k$; *and* $\frac{\partial^2 \wp_{ik}(\cdot)}{\partial q_{ik}^2} = \frac{p_{ik}}{q_{ik}}\Big\{\big(1 - \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\big)\big(2 - \frac{d\tilde{r}_{ik}}{\tilde{x}_{ik}}\big) + \frac{d^2\tilde{r}_{ik}}{d\tilde{x}_{ik}^2}\Big\}$, *and* $\frac{\partial^2 \wp_{ik}(\cdot)}{\partial q_{ik}\partial q_{ik'}} = 0$ *for all* $k' \neq k$.

Responsivenesses of marginal revenue functions. For each sector $i \in \mathbf{N}$ and for each firm $k \in \mathbf{N}_i$, let $mr_{ik} : \mathscr{S}_i \times \mathscr{S}_i^{N_i - 1} \to \mathbb{R}$ be the marginal revenue function; that is, $mr_{ik}(q_{ik}, \mathbf{q}_{i,-k}; \mathcal{I}_i) := \frac{\partial \wp_{ik}(\cdot)}{\partial q_{ik}} q_{ik} + p_{ik}$. Given Lemma C.3, it is immediate to show that for each $i \in \mathbf{N}$ and $k \in \mathbf{N}_i$, $mr_{ik}(\cdot)$ and its partial derivatives $\frac{\partial mr_{ik}(\cdot)}{\partial q_{ik'}}$ for all $k' \in \mathbf{N}_i$ are identified.

Lemma C.5 (Identification of Marginal Revenue Function)**.** *Suppose that the assumptions required in Lemma C.3 are satisfied. Then, (i) the firm-level marginal revenue function* $mr_{ik}(\cdot)$ *and (ii) its partial derivatives* $\frac{\partial mr_{ik}(\cdot)}{\partial q_{ik'}}$ *for all* $k' \in \mathbf{N}_i$ *are identified.*

Proof. (i) By the setup, $r_{ik} = \exp\{\tilde{\varphi}_i(\tilde{x}_{ik})\}$, wehre $\tilde{x}_{ik} = \ln x_{ik}$ with $x_{ik} = x_{ik}(q_{ik}, \mathbf{q}_{i,-k})$. By the definition of the marginal revenue, it follows

$$mr_{ik} = \frac{dr_{ik}}{dq_{ik}} = \exp\{\tilde{\varphi}_i(\tilde{x}_{ik})\}\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\frac{d\tilde{x}_{ik}}{dx_{ik}}\frac{\partial x_{ik}(\cdot)}{\partial q_{ik}} = \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}} p_{ik}(1 - \varpi_{ik}) =: mr_i(q_{ik}, \mathbf{q}_{i,-k}).$$

(ii) By taking the derivative of the part (i) with respect to q_{ik} yields

$$\frac{\partial mr_{ik}(\cdot)}{\partial q_{ik}} = \frac{p_{ik}}{q_{ik}}(1 - \varpi_{ik})\Bigg[\frac{d^2\tilde{r}_{ik}}{d\tilde{x}_{ik}^2}(1 - \varpi_{ik}) - \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\Big\{\Big(1 - \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\Big) + \Big(\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}} + \varrho_{ik}\Big)\varpi_{ik}\Big\}\Bigg] + \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}} p_{ik}\varpi_{ik} t_{ik}.$$

Analogously, the derivative with respect to $q_{ik'}$ ($k' \neq k$) leads to

$$\frac{\partial mr_{ik}(\cdot)}{\partial q_{ik'}} = -p_{ik}\frac{\varpi_{ik'}}{q_{ik'}}\left[\frac{d^2\tilde{r}_{ik}}{d\tilde{x}_{ik}^2}(1-\varpi_{ik}) + \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\left\{\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}} - \left(\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}} + \varrho_{ik}\right)\varpi_{ik}\right\}\right] + \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}p_{ik}\varpi_{ik}t_{ik'}$$

$\square$

Remark C.3. *Notice that there are general relationships between the derivatives of the demand function and marginal revenue function, namely,* $\frac{\partial \wp_{ik}(\cdot)}{\partial q_{ik}}q_{ik} + p_{ik} = mr_i(q_{ik}, \mathbf{q}_{i,-k})$*,* $\frac{\partial^2 \wp_{ik}(\cdot)}{\partial q_{ik}^2}q_{ik} + 2\frac{\partial \wp_{ik}(\cdot)}{\partial q_{ik}} = \frac{\partial mr_{ik}(\cdot)}{\partial q_{ik}}$ *and* $\frac{\partial \wp_{ik}(\cdot)}{\partial q_{ik}\partial q_{ik'}}q_{ik} + \frac{\partial \wp_{ik}(\cdot)}{\partial q_{ik'}} = \frac{\partial mr_{ik}(\cdot)}{\partial q_{ik'}}$*. These equalities offer an alternative route from Lemma C.4 to Lemma C.5, or the other way around.*

Remark C.4. *Analogous results are true for the case of monopolistic competition:* $mr_{ik}(\cdot) = \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}p_{ik}$*,* $\frac{\partial mr_{ik}(\cdot)}{\partial q_{ik}} = \frac{p_{ik}}{q_{ik}}\left\{\frac{d^2\tilde{r}_{ik}}{d\tilde{x}_{ik}^2} - \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\left(1 - \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\right)\right\}$ *and* $\frac{\partial mr_{ik}(\cdot)}{\partial q_{ik'}} = 0$ *for all* $k' \neq k$*.*

Example C.1 (CES Sectoral Aggregator)**.** *Consider that the sectoral aggregator in sector* i *takes the form of a CES function with the elasticity of substitution being* σ_i*. In this case, it is straightforward to see that* $\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}} = \frac{\sigma_i - 1}{\sigma_i}$*,* $\frac{d^2\tilde{r}_{ik}}{d\tilde{x}_{ik}^2} = 0$*, and thus* $\varpi_{ik} = \frac{\sigma_i-1}{\sigma_i}$ *and* $t_{ik} = 0$ *for all* $k \in \mathbf{N}_i$*.*

Aggregate quantity and price. I can further recover the sectoral aggregator $F_i(\cdot)$ and its partial derivatives with respect to q_{ik} (denoted by $\frac{\partial F_i(\cdot)}{\partial q_{ik}}$) as well as the partial derivatives of $\mathcal{P}_i(\cdot)$ with respect to q_{ik} (denoted by $\frac{\partial \mathcal{P}_i(\cdot)}{\partial q_{ik}}$) for all $k \in \mathbf{N}_i$ under an additional normalization condition.

Assumption C.3 (Normalization of HSA Demand System)**.** *There exists a collection of constants* $\{c_{ik}\}_{k=1}^{N_i}$ *such that* $F_i(\{c_{ik}\}_{k=1}^{N_i}) = 1$*.*

Lemma C.6 (Identification of Sectoral Aggregators)**.** *Suppose that the assumptions required in Lemma C.3 are satisfied. Assume moreover that Assumption C.3 holds. Then, (i) the sectoral aggregator* $F_i(\cdot)$*, and (ii) the derivatives* $\frac{\partial F_i(\cdot)}{\partial q_{ik}}$ *and* $\frac{\partial \mathcal{P}_i(\cdot)}{\partial q_{ik}}$ *for each* $k \in \mathbf{N}_i$*, are identified as a function of* $\mathbf{q}_i$*. (iii) In particular, evaluated at the realized values, it holds that* $\frac{\partial F_i(\cdot)^*}{\partial q_{ik}} = \frac{p_{ik}^*}{P_i^*}$ *and* $\frac{\partial \mathcal{P}_i(\cdot)^*}{\partial q_{ik}} = -\frac{p_{ik}^*}{Q_i^*}$*.*

Proof. (i) By Proposition 1 (i) and Remark 3 (self-duality) of Matsuyama and Ushchev (2017), there exists a unique monotone, convex, continuous and homothetic rational preference over the support of $\mathbf{q}_i$ associated to the HSA inverse demand system (98) – (100). Clearly, this preference corresponds to the sectoral aggregator $F_i(\cdot)$. Moreover, a variant of Proposition 1 (ii) of Matsuyama and Ushchev (2017) implies that $F_i(\cdot)$ can be expressed as[130]

$$\ln F_i(\mathbf{q}_i) = \ln A_i(\mathbf{q}_i) + \sum_{k=1}^{N_i}\int_{c_{ik}}^{q_{ik}/A_i(\mathbf{q}_i)} \frac{\Psi_i(\zeta)}{\zeta}d\zeta, \tag{103}$$

where $\{c_{ik}\}_{k=1}^{N_i}$ are the constants satisfying Assumption C.3.

[130] See also Kasahara and Sugita (2020).

Since, by Lemma C.3, $A_i(\cdot)$ is identified, it remains to prove that for each $k \in \mathbf{N}$, the integrand $\frac{\Psi_i(\zeta)}{\zeta}$ is identified for all $\zeta \in [c_{ik}, \frac{q_{ik}}{A_i(\mathbf{q}_i)}]$. Observe that $\varphi_i(\cdot)$ in (99) is obtained by taking the continuous transformation and inverse of $\tilde{\varphi}_i^{-1}(\cdot)$, which is identified in the proof of Lemma C.2. Notice moreover that for the realized values $\{q_{ik}^*\}_{k=1}^{N_i}$, I can recover Φ_i using (97):

$$\Phi_i = \sum_{k=1}^{N_i} \varphi_i(q_{ik}^*),$$

where Φ_i is a constant that firms take as given. Then, the identification of $\frac{\Psi_i(\zeta)}{\zeta}$, for $\zeta \in [c_{ik}, \frac{q_{ik}}{A_i(\mathbf{q}_i)}]$, comes directly from its construction (99). Tracing (103) therefore restores the identification of $F_i(\cdot)$ as a function of $\mathbf{q}_i$.

(ii) Taking partial derivatives of (103) with respect to q_{ik}: for all $\mathbf{q}_i \in \mathscr{S}_i^{N_i}$,

$$\frac{\frac{\partial F_i(\cdot)}{\partial q_{ik}}}{F_i(\mathbf{q}_i)} = \frac{\frac{\partial A_i(\cdot)}{\partial q_{ik}}}{A_i(\mathbf{q}_i)} + \frac{1}{q_{ik}}\Psi_i\Big(\frac{q_{ik}}{A_i}\Big) - \Big(\sum_{k'=1}^{N_i} \Psi_i\Big(\frac{q_{ik'}}{A_i}\Big)\Big)\frac{1}{A_i(\mathbf{q}_i)}\frac{\partial A_i(\cdot)}{\partial q_{ik}},$$

so that by construction

$$\frac{\partial F_i(\cdot)}{\partial q_{ik}} = \frac{F_i(\mathbf{q}_i)}{\Phi_i}\frac{1}{q_{ik}}\varphi\Big(\frac{q_{ik}}{A_i(\mathbf{q}_i)}\Big).$$

This expression recovers $\frac{\partial F_i(\cdot)}{\partial q_{ik}}$ as a function of $\mathbf{q}_i$.

Moreover, it hods by (97) that $\mathcal{P}_i(\mathbf{q}_i)F_i(\mathbf{q}_i) = \Phi_i$. Then, taking the partial derivatives of the both hand sides with respect to q_{ik}, I obtain

$$\frac{\partial \mathcal{P}_i(\cdot)}{\partial q_{ik}}F_i(\mathbf{q}_i) + \mathcal{P}_i(\mathbf{q}_i)\frac{\partial F_i(\cdot)}{\partial q_{ik}} = 0.$$

Rearranging this identifies $\frac{\partial \mathcal{P}_i(\cdot)}{\partial q_{ik}}$ as a function of $\mathbf{q}_i$.

(iii) For the realized values $\mathbf{q}_i^*$, part (ii) of this lemma further simplifies to

$$\frac{\partial F_i(\cdot)^*}{\partial q_{ik}} = \frac{F_i(\mathbf{q}_i^*)}{\Phi_i}\frac{1}{q_{ik}^*}\varphi\Big(\frac{q_{ik}^*}{A_i(\mathbf{q}_i^*)}\Big) = \frac{p_{ik}^*}{P_i^*},$$

and

$$\frac{\partial \mathcal{P}_i(\cdot)^*}{\partial q_{ik}} = -\frac{\mathcal{P}_i(\mathbf{q}_i^*)}{F_i(\mathbf{q}_i^*)}\frac{p_{ik}^*}{P_i^*} = -\frac{p_{ik}^*}{F_i(\mathbf{q}_i^*)}.$$

This completes the proof. □

Remark C.5. *As discussed in Kasahara and Sugita (2020, 2023), the HSA demand is identified only up to a scale constant. Nevertheless, it is straightforward to verify that this is innocuous for the purpose of this paper, as the scale constants end up canceling out with each other. Hence, the* *presence of the scale constant is made implicit throughout the exposition.*

C.4 Recovering Λ and Γ

Once the partial derivatives of the sector- and firm-level functions are identified, I can also recover the matrices $\Lambda_{i,1}$ and $\Lambda_{i,2}$ in (32), and the matrices Γ_1 and Γ_2 in (43), all of which jointly act as a "bridge" between the partial derivatives and the comparative statics. The identification is constructive in the sense that these are recovered just following their construction derived in Appendix A.

C.4.1 Identification of Λ

Fact C.2 (Identification of $\Lambda_{i,1}$ and $\Lambda_{i,2}$)**.** *Suppose that Proposition C.3 and Lemma C.5 hold. Then, for each sector $i \in \mathbf{N}$, both matrices $\Lambda_{i,1}$ and $\Lambda_{i,2}$ in* (32) *are identified.*

Proof. First, it immediately follows from Lemma C.5 that $\Lambda_{i,1} := \big[\frac{\partial mr_{ik}(\cdot)^*}{\partial q_{ik'}}\big]_{k,k' \in \mathbf{N}_i}$ are identified. Next, $\{q^*_{ik}\}_{k=1}^{N_i}$ are identified by Proposition C.3. Since moreover labor and material inputs are available in the data (Fact B.4), the matrix $\Lambda_{i,2}$ in (32) is identified, as desired. $\square$

Remark C.6. *In view of Fact C.2, each entry of the matrix $\Lambda^{-1}_{i,1}\Lambda_{i,2}$, i.e., $\lambda^{-1}_{ik,k'}$, is also identified.*

Fact C.3 (Identification of $\bar{\lambda}^L_{ik}$ and $\bar{\lambda}^M_{ik}$)**.** *Suppose that the assumptions required in Fact C.2 are satisfied. Then, for each sector $i \in \mathbf{N}$ and each $k \in \mathbf{N}_i$, $\bar{\lambda}^L_{ik}$ and $\bar{\lambda}^M_{ik}$ are identified from the observables.*

Proof. For each sector $i \in \mathbf{N}$, q^*_{ik} is identified for all $k \in \mathbf{N}_i$ (Proposition C.3). Since $\lambda^{-1}_{ik,k'}$ is identified for all $k, k' \in \mathbf{N}_i$ (Fact C.2), $\bar{\lambda}^L_{ik}$ and $\bar{\lambda}^M_{ik}$ are identified by tracing their construction, i.e., $\bar{\lambda}^L_{ik} = \sum_{k'=1}^{N_i} \lambda^{-1}_{ik,k'} \frac{\ell^*_{ik'}}{q^*_{ik'}}$ and $\bar{\lambda}^M_{ik} = \sum_{k'=1}^{N_i} \lambda^{-1}_{ik,k'} \frac{m^*_{ik'}}{q^*_{ik'}}$, where ℓ^*_{ik} and m^*_{ik} are observed in the data (Fact B.4). $\square$

Fact C.4 (Identification of $\bar{\lambda}^L_{i\cdot}$ and $\bar{\lambda}^M_{i\cdot}$)**.** *Suppose that the assumptions required in Fact C.2 are satisfied. Assume moreover that Lemma C.6 holds. Then, for each sector $i \in \mathbf{N}$, $\bar{\lambda}^L_{i\cdot}$ and $\bar{\lambda}^M_{i\cdot}$ are identified.*

Proof. First, $\mathbf{q}^*_i$ and $\mathbf{p}^*_i$ identified by Proposition C.3. Second, $\bar{\lambda}^L_{ik}$ and $\bar{\lambda}^M_{ik}$ are identified by Fact C.3. Moreover, in view of Lemma C.6, $\frac{\partial \mathcal{P}_i(\cdot)^*}{\partial q_{ik}}$ can be expressed in terms of $\mathbf{p}^*_i$ and Q^*_i. Hence, $\bar{\lambda}^L_{i\cdot}$ and $\bar{\lambda}^M_{i\cdot}$ in (39) are identified. $\square$

C.4.2 Identification of Γ

Given that material input is composed according to a Cobb-Douglas aggregator (19), the equilibrium material cost index corresponding to (40) is given by

$$P^{M*}_i = \prod_{j=1}^{N} \frac{1}{\gamma_{i,j}^{\gamma_{i,j}}} \Big\{(1-\tau_i)P^*_j\Big\}^{\gamma_{i,j}}.$$

Fact C.5. *Under the specification* (19), $\frac{\partial \mathcal{P}_i^M(\cdot)^*}{\partial P_j}$ *and* $\frac{\partial \mathcal{P}_i^M(\cdot)^*}{\partial \tau_n}$ *in* (41) *are identified from the observables.*

Proof. Under the specification (19), it holds that $\frac{\partial \mathcal{P}_i^M(\cdot)^*}{\partial P_j} = \gamma_{i,j} \frac{P_i^{M*}}{P_j^*}$ and $\frac{\partial \mathcal{P}_i^M(\cdot)}{\partial \tau_n} = -\frac{P_i^{M*}}{1-\tau_i} \mathbb{1}_{\{n=i\}}$. The right hand sides of these two expressions are directly observed in the data (Appendix B). Hence, $\frac{\partial \mathcal{P}_i^M(\cdot)^*}{\partial P_j}$ and $\frac{\partial \mathcal{P}_i^M(\cdot)^*}{\partial \tau_n}$ are identified. □

Fact C.6. *Suppose that the assumptions required in Fact C.4 are satisfied. Then, the matrices* Γ_1 *and* Γ_2 *in* (43) *are identified.*

Proof. In view of Fact C.5, $\{\frac{\partial \mathcal{P}_i^M(\cdot)^*}{\partial P_j}\}_{i,j \in \mathbf{N}}$ are identified. Moreover, $\{\bar{\lambda}_{j\cdot}^L\}_{j=1}^N$ and $\{\bar{\lambda}_{j\cdot}^M\}_{j=1}^N$ are identified due to Fact C.4. Thus, both Γ_1 and Γ_2 in (43) can be recovered by following their definitions. □

C.5 Recovering Comparative Statics

With the results obtained above (Appendices C.2, C.3 and C.4), I now turn to the identification of comparative statics of firm-level and sector-level variables. As a preliminary, this requires the identification of the first- and second-order derivatives of firm-level production functions. This is accomplished by following the share regression approach of Gandhi et al. (2019), and is deferred to Appendix C.6. Hence, this section takes these as identified.

The identification of the comparative statics is constructive, so that I can follow the theoretical results established in Appendix A.

Fact C.7 (Identification of D_{ik})**.** *Suppose that the assumptions required in Fact C.4 are satisfied. Then, for each sector* $i \in \mathbf{N}$ *and each* $k \in \mathbf{N}_i$*, the matrix* D_{ik} *in* (56) *is identified.*

Proof. First, it holds by Assumption 2.4 (i) that marginal costs equal the average costs, so that $\xi_{ik}^* = \frac{TC_{ik}^*}{q_{ik}^*}$. This expression recovers ξ_{ik}^* because the total cost is directly observed in the data (Appendix B) and the firm-level quantity is recovered by Proposition C.3. Next, both $\bar{\lambda}_{ik}^L$ and $\bar{\lambda}_{ik}^M$ are identified by Fact C.3, and moreover the first- and second-order derivatives of the firm-level production functions are identified (Appendix C.6). Then, I can identify the matrix D_{ik} by tracing its definition (56). □

Proposition C.4 (Identification of $\frac{dW^*}{d\tau_n}$)**.** *Suppose that the assumptions required in Fact C.4 are satisfied. Then,* $\frac{dW^*}{d\tau_n}$ *is identified.*

Proof. From Fact C.5, it is known that $\frac{\partial \mathcal{P}_i^M(\cdot)^*}{\partial \tau_n} = -\frac{P_i^{M*}}{1-\tau_i} \mathbb{1}_{\{n=i\}}$. In addition, it holds from Fact C.6 that Γ_1 and Γ_2 are identified. Thus, $\vartheta_{1,i}$ and $\vartheta_{2,i}$ in (59) are identified. Since moreover each entry of the matrix D_{ik} is identified (Fact C.7), the identification of $\frac{dW^*}{d\tau_n}$ obtains through (63). □

Proposition C.5 (Identification of $\frac{dP_i^{M*}}{d\tau_n}$)**.** *Suppose that the assumptions required in Fact C.4 are satisfied. Then, for each* $i \in \mathbf{N}$, $\frac{dP_i^{M*}}{d\tau_n}$ *is identified.*

Proof. In light of Fact C.5, $\frac{\partial \mathcal{P}_i^M(\cdot)^*}{\partial \tau_n}$ is identified. Both Γ_1 and Γ_2 are recovered in Fact C.6. Given the identification of $\frac{dW^*}{d\tau_n}$ (Proposition C.4), I can thus identify $\frac{dP_i^{M*}}{d\tau_n}$ according to (44). $\square$

Proposition C.6 (Identification of $\frac{dP_i^*}{d\tau_n}$)**.** *Suppose that the assumptions required in Fact C.4 are satisfied. Then, for each $i \in \mathbf{N}$, $\frac{dP_i^*}{d\tau_n}$ is identified.*

Proof. Due to Fact C.4, both $\bar{\lambda}_{i\cdot}^L$ and $\bar{\lambda}_{i\cdot}^M$ are identified. Given the identifications of $\frac{dW^*}{d\tau_n}$ (Proposition C.4) and $\frac{dP_i^{M*}}{d\tau_n}$ (Proposition C.5), I can identify $\frac{dP_i^*}{d\tau_n}$ according to (39). $\square$

Proposition C.7 (Identification of $\frac{dq_{ik}^*}{d\tau_n}$ and $\frac{dp_{ik}^*}{d\tau_n}$)**.** *Suppose that the assumptions required in Fact C.4 are satisfied. Then, for each $i \in \mathbf{N}$ and each $k \in \mathbf{N}_i$, $\frac{dq_{ik}^*}{d\tau_n}$ and $\frac{dp_{ik}^*}{d\tau_n}$ are identified.*

Proof. First, observe that $\bar{\lambda}_{ik}^L$ and $\bar{\lambda}_{ik}^M$ are identified for each $i \in \mathbf{N}$ and each $k \in \mathbf{N}_i$ (Fact C.3). Given the identification of $\frac{dW^*}{d\tau_n}$ (Proposition C.4) and $\frac{dP_i^{M*}}{d\tau_n}$ (Proposition C.5), I can thus identify $\frac{dq_{ik}^*}{d\tau_n}$ according to (32).

Next, $\frac{dp_{ik}^*}{d\tau_n}$ is in turn recovered through $\frac{dp_{ik}^*}{d\tau_n} = \sum_{k'=1}^{N_i} \frac{\partial \wp_{ik}(\cdot)^*}{\partial q_{ik'}} \frac{dq_{ik'}^*}{d\tau_n}$, where the identification of $\frac{\partial \wp_{ik}(\cdot)^*}{\partial q_{ik'}}$ (for all $k' \in \mathbf{N}_i$) is given in Lemma C.4. $\square$

Proposition C.8 (Identification of $\frac{d\ell_{ik}^*}{d\tau_n}$ and $\frac{dm_{ik}^*}{d\tau_n}$)**.** *Suppose that the assumptions required in Fact C.4 are satisfied. Then, for each $i \in \mathbf{N}$ and each $k \in \mathbf{N}_i$, $\frac{d\ell_{ik}^*}{d\tau_n}$ and $\frac{dm_{ik}^*}{d\tau_n}$ are identified.*

Proof. It follows from Fact C.7 that the matrix D_{ik} is identified for each $i \in \mathbf{N}$ and each $k \in \mathbf{N}_i$. Given the identifications of $\frac{dW^*}{d\tau_n}$ (Proposition C.4) and $\frac{dP_i^{M*}}{d\tau_n}$ (Proposition C.5), I can thus identify $\frac{d\ell_{ik}^*}{d\tau_n}$ and $\frac{dm_{ik}^*}{d\tau_n}$ according to (55). $\square$

Notice that if material input is composed according to a Cobb-Douglas aggregator (19), the equilibrium derived demand for sectoral intermediate good corresponding to (64) is given by (20):

$$m_{ik,j}^* = \gamma_{i,j} \frac{P_i^{M*}}{(1-\tau_i) P_j^*} m_{ik}^*.$$

Proposition C.9 (Identification of $\frac{dm_{ik,j}^*}{d\tau_n}$)**.** *Suppose that the assumptions required in Fact C.4 are satisfied. Then, for each $i, j \in \mathbf{N}$ and each $k \in \mathbf{N}_i$, $\frac{dm_{ik,j}^*}{d\tau_n}$ is identified.*

Proof. Under the specification (19), it holds that $\frac{\partial m_{ik,j}(\cdot)^*}{\partial P_{j'}} = -\frac{1}{P_{j'}} m_{ik,j} \mathbb{1}_{\{j'=j\}} + \frac{\gamma_{i,j'}}{P_{j'}^*} m_{ik,j}^*$, $\frac{\partial m_{ik,j}(\cdot)^*}{\partial \tau_n} = -\frac{m_{ik,j}}{1-\tau_i} \mathbb{1}_{\{n=i\}}$ and $\frac{\partial m_{ik,j}(\cdot)^*}{\partial m_{ik}} = \frac{m_{ik,j}^*}{m_{ik}^*}$. Note that these three terms can be directly recovered from the data (Appendix B).

Hence, given the identification of $\{\frac{dP_{j'}^*}{d\tau_n}\}_{j'=1}^N$ (Proposition C.6) and $\frac{dm_{ik}^*}{d\tau_n}$ (Proposition C.8), I can identify $\frac{dm_{ik,j}^*}{d\tau_n}$ according to (65), which proves the claim. $\square$

Remark C.7. *Alternatively, one may directly work on the total differentiation of* (20)*, which is given by*

$$\frac{dm^*_{ik,j}}{d\tau_n} = \left\{ \frac{1}{1-\tau_i} \mathbb{1}_{\{n=i\}} + \frac{1}{P_i^{M*}} \frac{dP_i^{M*}}{d\tau_n} - \frac{1}{P_j^*} \frac{dP_j^*}{d\tau_n} + \frac{1}{m^*_{ik}} \frac{dm^*_{ik}}{d\tau_n} \right\} m^*_{ik,j}.$$

In this case, the identification of $\frac{dm^*_{ik,j}}{d\tau_n}$ *follows from Propositions C.5, C.6 and C.8 as well as Appendix B.*

C.6 Recovering the First- and Second-Order Partial Derivatives of the Firm-Level Production Functions

The goal of this subsection is to identify the equilibrium values of the second-order derivatives of $f_i(\cdot)$ with respect to ℓ_{ik} and m_{ik}.[131] To begin with, observe that under Assumption 4.3, there exits a function $g_i : \mathscr{L}_i \times \mathscr{M}_i \to \mathbb{R}$ such that

$$f_i(\ell_{ik}, m_{ik}; z_{ik}) = z_{ik} g_i(\ell_{ik}, m_{ik}), \tag{104}$$

for all $(\ell_{ik}, m_{ik}, z_{ik}) \in \mathscr{L}_i \times \mathscr{M}_i \times \mathscr{Z}_i$. I define $\tilde{g}_i : \tilde{\mathscr{L}}_i \times \tilde{\mathscr{M}}_i \to \mathbb{R}$ such that

$$\tilde{f}_i(\tilde{\ell}_{ik}, \tilde{m}_{ik}; \tilde{z}_{ik}) = \tilde{z}_{ik} + \tilde{g}_i(\tilde{\ell}_{ik}, \tilde{m}_{ik}). \tag{105}$$

My identification strategy is based on the following relationships between the partial derivatives of $\tilde{g}_i$ and those of f_i.

Fact C.8. *Under Assumption 4.3, it holds that for all* $(\ell_{ik}, m_{ik}, z_{ik}) \in \mathscr{L}_i \times \mathscr{M}_i \times \mathscr{Z}_i$*,*

(i) $\frac{\partial \tilde{f}_i(\cdot)}{\partial \tilde{\ell}_{ik}} = \frac{\partial \tilde{g}_i(\cdot)}{\partial \tilde{\ell}_{ik}}$ *and* $\frac{\partial \tilde{f}_i(\cdot)}{\partial \tilde{m}_{ik}} = \frac{\partial \tilde{g}_i(\cdot)}{\partial \tilde{m}_{ik}}$*;*

(ii) $\frac{\partial f_i(\cdot)}{\partial \ell_{ik}} = \frac{\partial \tilde{g}_i(\cdot)}{\partial \tilde{\ell}_{ik}} \frac{f_i(\cdot)}{\ell_{ik}}$ *and* $\frac{\partial f_i(\cdot)}{\partial m_{ik}} = \frac{\partial \tilde{g}_i(\cdot)}{\partial \tilde{m}_{ik}} \frac{f_i(\cdot)}{m_{ik}}$*;*

(iii) $\frac{\partial^2 f_i(\cdot)}{\partial \ell_{ik}^2} = \frac{f_i(\cdot)}{\ell_{ik}^2} \left\{ \frac{\partial^2 \tilde{g}_i(\cdot)}{\partial \tilde{\ell}_{ik}^2} + \left(\frac{\partial \tilde{g}_i(\cdot)}{\partial \tilde{\ell}_{ik}} \right)^2 - \frac{\partial \tilde{g}_i(\cdot)}{\partial \tilde{\ell}_{ik}} \right\}$, $\frac{\partial^2 f_i(\cdot)}{\partial m_{ik}^2} = \frac{f_i(\cdot)}{m_{ik}^2} \left\{ \frac{\partial^2 \tilde{g}_i(\cdot)}{\partial \tilde{m}_{ik}^2} + \left(\frac{\partial \tilde{g}_i(\cdot)}{\partial \tilde{m}_{ik}} \right)^2 - \frac{\partial \tilde{g}_i(\cdot)}{\partial \tilde{m}_{ik}} \right\}$ *and* $\frac{\partial^2 f_i(\cdot)}{\partial \ell_{ik} \partial m_{ik}} = \frac{f_i(\cdot)}{\ell_{ik} m_{ik}} \left(\frac{\partial^2 \tilde{g}_i(\cdot)}{\partial \tilde{\ell}_{ik} \partial \tilde{m}_{ik}} + \frac{\partial \tilde{g}_i(\cdot)}{\partial \tilde{\ell}_{ik}} \frac{\partial \tilde{g}_i(\cdot)}{\partial \tilde{m}_{ik}} \right)$,

where $f_i(\cdot) \coloneqq f_i(\ell_{ik}, m_{ik}; z_{ik})$ *and* $\tilde{g}_i(\cdot) \coloneqq \tilde{g}_i(\tilde{\ell}_{ik}, \tilde{m}_{ik})$*.*

Proof. The proof is omitted. □

The identification results of Gandhi et al. (2019) rest on Fact C.8 (i) and the timing assumption encoded in (6). I further leverage the insights from Facts C.8 (ii) and (iii). In particular, invoking

[131] Note that the equilibrium values of the first-order derivatives are already identified in Proposition C.2.

(iii) in equilibrium, I have

$$\frac{\partial^2 f_i(\cdot)^*}{\partial m_{ik}^2} = \frac{q_{ik}^*}{(m_{ik}^*)^2}\left\{\frac{\partial^2 \tilde{g}_i(\cdot)^*}{\partial \tilde{m}_{ik}^2} + \left(\frac{\partial \tilde{g}_i(\cdot)^*}{\partial \tilde{m}_{ik}}\right)^2 - \frac{\partial \tilde{g}_i(\cdot)^*}{\partial \tilde{m}_{ik}}\right\} \tag{106}$$

and also in light of Young's theorem,

$$\frac{\partial^2 f_i(\cdot)^*}{\partial m_{ik}\partial \ell_{ik}} = \frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik}\partial m_{ik}} = \frac{q_{ik}^*}{\ell_{ik}^* m_{ik}^*}\left\{\frac{\partial^2 \tilde{g}_i(\cdot)^*}{\partial \tilde{\ell}_{ik}\partial \tilde{m}_{ik}} + \left(\frac{\partial \tilde{g}_i(\cdot)^*}{\partial \tilde{\ell}_{ik}}\right)\left(\frac{\partial \tilde{g}_i(\cdot)^*}{\partial \tilde{m}_{ik}}\right)\right\}. \tag{107}$$

Once these are obtained, I can moreover invoke Euler's theorem for homogeneous functions to derive

$$\frac{\partial^2 f_i(\cdot)^*}{\partial \ell_{ik}^2} = -\frac{m_{ik}^*}{\ell_{ik}^*}\frac{\partial^2 f_i(\cdot)^*}{\partial m_{ik}\partial \ell_{ik}} = \left(\frac{m_{ik}^*}{\ell_{ik}^*}\right)^2\frac{\partial^2 f_i(\cdot)^*}{\partial m_{ik}^2}. \tag{108}$$

Since q_{ik}^* can be identified from Proposition C.3, it remains to identify (the equilibrium values of) the first- and second-order derivatives of $\tilde{g}_i(\cdot)$ with respect to $\tilde{\ell}_{ik}$ and $\tilde{m}_{ik}$. To this end, I follow Gandhi et al. (2019) in nonparametrically identifying the first-oder partial derivatives of $\tilde{g}(\cdot)$ as a function of $\tilde{\ell}_{ik}$ and $\tilde{m}_{ik}$.

The identification equation builds on the one-step profit maximization set out in Appendix A.1. Under Assumption 4.3, multiplying (28) by m_{ik} and dividing by $p_{ik}q_{ik}$ leads to

$$\therefore \frac{1}{\mu_{ik}}\frac{\partial \tilde{g}_i(\cdot)}{\partial \tilde{m}_{ik}} = s_{ik}^m,$$

where $s_{ik}^m := \frac{P_i^M m_{ik}}{p_{ik}q_{ik}}$ is the material cost relative to the revenue. Taking the logarithm of this expression, I obtain

$$\ln s_{ik}^m = \ln \frac{\partial \tilde{g}_i(\cdot)}{\partial \tilde{m}_{ik}} - \ln \mu_{ik}. \tag{109}$$

However, in general this relationship cannot be directly fed into data when the output market is imperfectly competitive, because the firm-level markup μ_{ik} needs to be identified (Kasahara and Sugita 2020). Yet, in my setup, owing to Assumption 2.4 (i), μ_{ik} is recovered in advance of solving (109) for the first-order derivative of $\tilde{g}_i$ with respect to $\tilde{m}_{ik}$ (Fact C.1). Taking stock of this, I adopt the same empirical specification as Gandhi et al. (2019):

$$\tilde{s}_{ik}^{m,\tilde{\mu}} = \ln \mathcal{E}_i^m + \ln \frac{\partial \tilde{g}_i}{\partial \tilde{m}_{ik}}(\tilde{\ell}_{ik}, \tilde{m}_{ik}) - \tilde{\varepsilon}_{ik}^m, \tag{110}$$

where $\tilde{s}_{ik}^{m,\tilde{\mu}} := \ln s_{ik}^m + \ln \mu_{ik}$ can readily be calculated from the data, and $\tilde{\varepsilon}_{ik}^m$ is a measurement error with $E[\tilde{\varepsilon}_{ik}^m \mid \tilde{\ell}_{ik}, \tilde{m}_{ik}] = 0$. The measurement error $\tilde{\varepsilon}_{ik}^m$ captures any unmodeled, non-systematic noise, and is associated with the constant $\mathcal{E}_i^m$ through $\mathcal{E}_i^m = E[\exp\{\tilde{\varepsilon}_{ik}^m\}]$. Inclusion of the mean $\mathcal{E}_i^m$ is based on the suggestion made in Gandhi et al. (2019).

My identification result heavily draws from Gandhi et al. (2019), and is summarized in the following lemma.

Lemma C.7 (Theorem 2 of Gandhi et al. (2019))**.** *Suppose that Assumptions 2.4 and 4.3 hold. Then, the share regression* (110) *identifies the first-order derivatives of* $\tilde{g}_i(\cdot)$ *with respect to log-labor and log-material inputs for all* $(\tilde{\ell}_{ik}, \tilde{m}_{ik}) \in \tilde{\mathscr{L}}_i \times \tilde{\mathscr{M}}_i$.

Proof. First, I start by writing (110) as

$$\tilde{s}_{ik}^{m,\tilde{\mu}} = \ln D_{ik}^m(\tilde{\ell}_{ik}, \tilde{m}_{ik}) - \tilde{\varepsilon}_{ik}^m, \tag{111}$$

where $\ln D_{ik}^m(\tilde{\ell}_{ik}, \tilde{m}_{ik}) \coloneqq \ln \mathcal{E}_i^m + \ln \frac{\partial \tilde{g}_i}{\partial \tilde{m}_{ik}}(\tilde{\ell}_{ik}, \tilde{m}_{ik})$. I can nonparametrically identify $\ln D_{ik}^m(\tilde{\ell}_{ik}, \tilde{m}_{ik})$ according to

$$\ln D_{ik}^m(\tilde{\ell}_{ik}, \tilde{m}_{ik}) = E\big[\tilde{s}_{ik}^{m,\tilde{\mu}} | \tilde{\ell}_{ik}, \tilde{m}_{ik}\big]$$

for all $(\tilde{\ell}_{ik}, \tilde{m}_{ik}) \in \tilde{\mathscr{L}}_i \times \tilde{\mathscr{M}}_i$. The error term $\tilde{\varepsilon}_{ik}^m$ is identified through the specification (111):

$$\tilde{\varepsilon}_{ik}^m = \ln D_{ik}^m(\tilde{\ell}_{ik}, \tilde{m}_{ik}) - \tilde{s}_{ik}^{m,\tilde{\mu}} \tag{112}$$

which in turn identifies the mean $\mathcal{E}_i^m$:

$$\mathcal{E}_i^m = E\big[\exp\{\tilde{\varepsilon}_{ik}^m\}\big] \tag{113}$$

Next, plugging these back into the the definition of $\ln D_{ik}^m$, I identify the log-labor input elasticity of the log-production function:

$$\ln \frac{\partial \tilde{g}_i}{\partial \tilde{m}_{ik}}(\tilde{\ell}_{ik}, \tilde{m}_{ik}) = \ln D_{ik}^m(\tilde{\ell}_{ik}, \tilde{m}_{ik}) - \ln \mathcal{E}_i^m = \ln \frac{D_{ik}^m(\tilde{\ell}_{ik}, \tilde{m}_{ik})}{\mathcal{E}_i^m},$$

yielding

$$\frac{\partial \tilde{g}_i(\tilde{\ell}_{ik}, \tilde{m}_{ik})}{\partial \tilde{m}_{ik}} = \frac{D_{ik}^m(\tilde{\ell}_{ik}, \tilde{m}_{ik})}{\mathcal{E}_i^m} \tag{114}$$

for all $(\tilde{\ell}_{ik}, \tilde{m}_{ik}) \in \tilde{\mathscr{L}}_i \times \tilde{\mathscr{M}}_i$.

Lastly, given the identification of $\frac{\partial \tilde{g}_i(\cdot)}{\partial \tilde{m}_{ik}}$, one can invoke Euler's theorem for homogeneous functions under Assumption 2.4 (i) and Fact C.8 (i) to recover $\frac{\partial \tilde{g}_i(\cdot)}{\partial \tilde{\ell}_{ik}}$ for all $(\tilde{\ell}_{ik}, \tilde{m}_{ik}) \in \tilde{\mathscr{L}}_i \times \tilde{\mathscr{M}}_i$, completing the proof. □

As soon as $\frac{\partial \tilde{g}_i(\cdot)}{\partial \tilde{\ell}_{ik}}$ and $\frac{\partial \tilde{g}_i(\cdot)}{\partial \tilde{m}_{ik}}$ are identified as functions of $\tilde{\ell}_{ik}$ and $\tilde{m}_{ik}$, I can also recover the second-order derivatives of $\tilde{g}_i(\cdot)$.

Corollary C.2. *The second-order derivatives of* $\tilde{g}_{ik}(\cdot)$ *with respect to log-labor and log-material inputs, i.e.,* $\frac{\partial^2 \tilde{g}_i(\cdot)}{\partial \tilde{\ell}_{ik}^2}$, $\frac{\partial^2 \tilde{g}_i(\cdot)}{\partial \tilde{m}_{ik}^2}$, *and* $\frac{\partial^2 \tilde{g}_i(\cdot)}{\partial \tilde{\ell}_{ik} \tilde{m}_{ik}}$, *are nonparametrically identified for all* $(\tilde{\ell}_{ik}, \tilde{m}_{ik}) \in \tilde{\mathscr{L}}_i \times \tilde{\mathscr{M}}_i$.

Now, I prove that it is possible to identify the values of the second-order derivative of the production function corresponding to the equilibrium labor and material inputs.

Lemma C.8. *Suppose that the assumptions required in Proposition C.3 and Lemma C.7 are satisfied. The equilibrium values of the second-order derivatives of the production function are identified from the observables.*

Proof. By Proposition C.3, q^*_{ik} is recovered. Moreover, Lemma C.7 identifies the value of $\frac{\partial \tilde{g}_i(\cdot)}{\partial \tilde{\ell}_{ik}}$ and $\frac{\partial \tilde{g}_i(\cdot)}{\partial \tilde{m}_{ik}}$ at the equilibrium values of inputs $(\tilde{\ell}^*_{ik}, \tilde{m}^*_{ik})$, while Corollary C.2 recovers the equilibrium values of $\frac{\partial^2 \tilde{g}_i(\cdot)}{\partial \tilde{\ell}^2_{ik}}$, $\frac{\partial^2 \tilde{g}_i(\cdot)}{\partial \tilde{m}^2_{ik}}$ and $\frac{\partial^2 \tilde{g}_i(\cdot)}{\partial \tilde{\ell}_{ik}\partial \tilde{m}_{ik}}$. Hence, by tracing (106), (107) and (108), I can recover the equilibrium values of the second-order derivatives of the production function, as claimed. □

Remark C.8. *Lemma C.8 only identifies the values of the second-order derivatives of the firm-level production function at the equilibrium level of labor and material inputs, while being silent about the values at different levels of these inputs. This is because I lack the identification of the production function $f_i(\cdot)$ over the entire support; my approach instead rests on the knowledge about the value of equilibrium quantity, given by Proposition C.3. The punchline is that as far as the identification of* (16) *is concerned, the knowledge about the entire production function is not needed, obviating additional assumptions.*

C.7 Identification of the Object of Interest

Theorem C.1 (Identification of $\frac{dY_i(s)}{ds}$)**.** *Suppose that Assumptions 4.1, 4.3, 4.4 and 4.5 hold. Assume moreover that the regularity conditions (Assumptions C.2 and C.3) are satisfied. Then, the value of $\frac{dY_i(s)}{ds}$ evaluated at any point on $\mathscr{T}$ is identified from the observables.*

Proof. Observe that $\frac{dY_i(s)}{ds}$ evaluated at a point on $s = \boldsymbol{\tau}$ can be decomposed as

$$\left.\frac{dY_i(s)}{ds}\right|_{s=\tau_n} = \sum_{k=1}^{N_i} \frac{dp^*_{ik}}{d\tau_n} q^*_{ik} + \sum_{k=1}^{N_i} p^*_{ik} \frac{dq^*_{ik}}{d\tau_n} - \left(\sum_{k=1}^{N_i} \sum_{j=1}^{N} \frac{dP^*_j}{d\tau_n} m^*_{ik,j} + \sum_{k=1}^{N_i} \sum_{j=1}^{N} P^*_j \frac{dm^*_{ik,j}}{d\tau_n} \right),$$

For all $i, j \in \mathbf{N}$ and $k \in \mathbf{N}_i$, I can recover p^*_{ik} and q^*_{ik} (Proposition C.3), $\frac{dp^*_{ik}}{d\tau_n}$ and $\frac{dq^*_{ik}}{d\tau_n}$ (Proposition C.7), $\frac{dP^*_j}{d\tau_n}$ (Proposition C.6), and $\frac{dm^*_{ik,j}}{d\tau_n}$ (Proposition C.9) over the empirical support. Hence, I can recover the value of $\frac{dY_i(s)}{ds}$ at any point on $\mathscr{T}$. □

Proof of Theorem 4.1. Under Assumption 4.2, Theorem C.1 holds for all values on $[\boldsymbol{\tau}^0, \boldsymbol{\tau}^1]$. Then, the object of interest $\Delta Y(\tau^0_n, \tau^1_n)$ can be recovered according to (15):

$$\Delta Y(\tau^0_n, \tau^1_n) = \sum_{i=1}^{N} \int_{\tau^0_n}^{\tau^1_n} \frac{dY_i(s)}{ds} ds,$$

which proves the theorem. □

A version of Theorem 4.1 remains valid for the case of monopolistic competition with the solution concept appropriately modified.

Corollary C.3. *Suppose that the same assumptions as Theorem 4.1 are satisfied. Assume that firms operate within a structure of monopolistic competition in the output market. Then, the object of interest* (14) *is identified from the observables.*

Proof. The proof is analogous to that of Theorem 4.1 and only requires to modify the responsivenesses of the firm's inverse demand and marginal revenue functions, as explained in Remarks C.2 and C.4. □

Note that Corollary C.3 does not mean that my framework can be agnostic about the nature of the market competition. My framework requires the specification of the market competition prior to analysis.

C.8 Systematic Patterns Induced by Identification Assumptions

The identification assumptions induce several important patterns in the recovered firm's responses. This subsection explores such patterns by classifying them into three categories, namely, i) the patterns induced by the production-side assumptions, ii) those induced by the demand-side assumptions, and iii) those induced by the both types of assumptions.

C.8.1 Systematic Patterns Induced by Production-Side Assumptions

First, I look at the consequences of the assumptions imposed on the firm-level production function. The following lemma tells us that the firm's input choices take a specific form that is proportional to the firm's own output quantity and the inverse of the firm's own productivity.

Lemma C.9. *Suppose that Assumptions 2.4 and 4.3 hold. Then, for each $i \in \mathbf{N}$, there exist $\beta_i^\ell, \beta_i^m \in \mathbb{R}_+$ such that $\ell_{ik}^* = \beta_i^\ell z_{ik}^{-1} q_{ik}^*$ and $m_{ik}^* = \beta_i^m z_{ik}^{-1} q_{ik}^*$.*

Proof. Under Assumption 2.4, the firm's cost-minimization problem implies

$$TC_{ik}(W, P_i^M; q_{ik}^*) = MC_{ik}(W, P_i^M) q_{ik}^*,$$

where $TC_{ik}(\cdot; q_{ik}^*)$ and $MC_{ik}(\cdot)$, respectively, are the firm k's total cost function conditional on output quantity q_{ik}^*, and marginal cost function. Taking derivatives of this equation with respect to W and P_i^M yields

$$\frac{\partial TC_{ik}(\cdot)}{\partial W} = \frac{\partial MC_{ik}(\cdot)}{\partial W} q_{ik}^* \qquad \text{and} \qquad \frac{\partial TC_{ik}(\cdot)}{\partial P_i^M} = \frac{\partial MC_{ik}(\cdot)}{\partial P_i^M} q_{ik}^*.$$

In view of Shephard's lemma, these are equivalently be written as

$$\ell_{ik}^* = \frac{\partial MC_{ik}(\cdot)}{\partial W} q_{ik}^* \qquad \text{and} \qquad m_{ik}^* = \frac{\partial MC_{ik}(\cdot)}{\partial P_i^M} q_{ik}^*.$$

Since $\frac{\partial MC_{ik}(\cdot)}{\partial W}$ and $\frac{\partial MC_{ik}(\cdot)}{\partial P_i^M}$ do not involve the firm's choice variables (i.e., ℓ_{ik} and m_{ik}), they can be treated as constants. I thus define $\beta_{ik}^{\ell} := \frac{\partial MC_{ik}(\cdot)}{\partial W}$ and $\beta_{ik}^{m} := \frac{\partial MC_{ik}(\cdot)}{\partial P_i^m}$, so that

$$\ell_{ik}^* = \beta_{ik}^{\ell} q_{ik}^* \qquad \text{and} \qquad m_{ik}^* = \beta_{ik}^{m} q_{ik}^*. \tag{115}$$

Combined with Hicks-neutrality (Assumption 4.3), (115) suggests

$$z_{ik} g_i(\beta_{ik}^{\ell}, \beta_{ik}^{m}) = 1.$$

Under Assumption 2.4, this is true if and only if there exist $\beta_i^{\ell}, \beta_i^{m} \in \mathbb{R}_+$ such that $\beta_{ik}^{\ell} = \beta_i^{\ell} z_{ik}^{-1}$ and $\beta_{ik}^{m} = \beta_i^{m} z_{ik}^{-1}$ with $g_i(\beta_i^{\ell}, \beta_i^{m}) = 1$. Substituting this back into (115) leads to

$$\ell_{ik}^* = \beta_i^{\ell} z_{ik}^{-1} q_{ik}^* \qquad \text{and} \qquad m_{ik}^* = \beta_i^{m} z_{ik}^{-1} q_{ik}^*,$$

as desired. □

By construction, β_i^{ℓ} and β_i^{m} convey partial information about the marginal cost common to all firms. With this insight in mind, the following corollary is straightforward.

Corollary C.4. *Suppose that Assumptions of Lemma C.9 are satisfied. Then, for each $i \in \mathbf{N}$ and each $k \in \mathbf{N}_i$, $mc_{ik} = mc_i z_{ik}^{-1}$ with $mc_i = \beta_i^{\ell} W + \beta_i^{m} P_i^M$, where β_i^{ℓ} and β_i^{m} are constants appearing in Lemma C.9.*

Proof. Assumption 2.4 implies

$$W \ell_{ik}^* + P_i^M m_{ik}^* = mc_{ik} q_{ik}^*.$$

From Lemma C.9, this further implies

$$(\beta_i^{\ell} W + \beta_i^{m} P_i^M) z_{ik}^{-1} q_{ik}^* = mc_{ik} q_{ik}^*,$$

so that

$$mc_{ik} = (\beta_i^{\ell} W + \beta_i^{m} P_i^M) z_{ik}^{-1}.$$

Upon defining $mc_i = \beta_i^{\ell} W + \beta_i^{m} P_i^M$, the claim is proved. □

Example C.2 (Cobb-Douglas Production Function)**.** *xxx*

C.8.2 Systematic Patterns Induced by Demand-Side Assumptions

Next, I derive several theoretical results that follows from the assumptions imposed on the demand side (i.e., the sectoral aggregator). Here, it is postulated that firms engage in oligopolistic competition in the output market as in the main text, while the case of monopolistic competition is postponed until Section C.8.4.

The following lemma pushes Lemma C.5 forward to derive the system of firms' pricing equations in equilibrium.

Lemma C.10 (Firms' Pricing Equations in Oligopolistic Competition)**.** *Suppose that Assumption 4.4 holds. Then, for each $i \in \mathbf{N}$, $p^*_{ik} = (\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}})^{-1} \frac{1}{1-\varpi_{ik}} mc_{ik}$ for all $k \in \mathbf{N}_i$.*

Proof. Firm's profit-maximization with respect to quantity implies

$$mr_{ik} = mc_{ik} \tag{116}$$

for each firm $k \in \mathbf{N}_i$. Under Assumption 4.4, the left hand side of (116) reads[132]

$$mr_{ik} = \frac{dr_{ik}}{dq_{ik}} = \exp\{\tilde{\varphi}_i(\tilde{x}_{ik})\} \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}} \frac{d\tilde{x}_{ik}}{dx_{ik}} \frac{\partial x_{ik}(\cdot)}{\partial q_{ik}} = p_{ik} \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}} (1 - \varpi_{ik}).$$

Thus, (116) implies

$$p^*_{ik} \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}} (1 - \varpi_{ik}) = mc_{ik},$$

so that

$$p^*_{ik} = \left(\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}} \right)^{-1} (1 - \varpi_{ik})^{-1} mc_{ik},$$

which proves the statement of this lemma. □

The following two facts are concerned with the analytical expression of the derivatives of the firm's marginal revenue function that is derived in the proof of Lemma C.5.

Fact C.9. *Suppose that Assumption 4.4 holds. Then, $\sum_{k'=1}^{N_i} q_{ik'} t_{ik'} = 0$.*

Proof. It immediately follows from the definitions of t_{ik} and ϖ_{ik} that

$$\begin{aligned}
\sum_{k'=1}^{N_i} q_{ik'} t_{ik'} &= \sum_{k'=1}^{N_i} q_{ik'} \frac{1}{q_{ik'}} \varpi_{ik'} \left(\varrho_{ik'} - \frac{\sum_{k''=1}^{N_i} \varrho_{ik''} \tilde{u}_{ik''}}{\sum_{k''=1}^{N_i} \tilde{u}_{ik''}} \right) \\
&= \sum_{k'=1}^{N_i} \varpi_{ik'} \varrho_{ik'} - \frac{\sum_{k''=1}^{N_i} \varrho_{ik''} \tilde{u}_{ik''}}{\sum_{k''=1}^{N_i} \tilde{u}_{ik''}} \sum_{k'=1}^{N_i} \varpi_{ik'} \\
&= \frac{\sum_{k'=1}^{N_i} \varrho_{ik'} \tilde{u}_{ik'}}{\sum_{k''=1}^{N_i} \tilde{u}_{ik''}} - \frac{\sum_{k''=1}^{N_i} \varrho_{ik''} \tilde{u}_{ik''}}{\sum_{k''=1}^{N_i} \tilde{u}_{ik''}} \\
&= 0,
\end{aligned}$$

as desired. □

[132]See also the proof of Lemma C.5.

Now, to simply the exposition, I introduce two additional notations. Denote

$$\hat{B}_{ik} := \frac{d^2\tilde{r}_{ik}}{d\tilde{x}_{ik}^2}(1-\varpi_{ik}) - \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\left\{\left(1-\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\right)+\left(\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}+\varrho_{ik}\right)\varpi_{ik}\right\}$$
$$\check{B}_{ik} := \frac{d^2\tilde{r}_{ik}}{d\tilde{x}_{ik}^2}(1-\varpi_{ik}) - \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\left\{\left(-\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\right)+\left(\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}+\varrho_{ik}\right)\varpi_{ik}\right\},$$

so that

$$\frac{\partial mr_{ik}(\cdot)^*}{\partial q_{ik}} = \frac{p_{ik}^*}{q_{ik}^*}(1-\varpi_{ik})\hat{B}_{ik} + \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}p_{ik}^*\varpi_{ik}t_{ik}$$
$$\frac{\partial mr_{ik}(\cdot)^*}{\partial q_{ik'}} = -p_{ik}^*\frac{\varpi_{ik'}}{q_{ik'}^*}\check{B}_{ik} + \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}p_{ik}^*\varpi_{ik}t_{ik'}.$$

The following fact is immediate.

Fact C.10. *For all $i \in \mathbf{N}$ and $k \in \mathbf{N}_i$, $\hat{B}_{ik} - \check{B}_{ik} = -\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}$.*

Proof. By the definition, it is straightforward to verify that

$$\begin{aligned}\hat{B}_{ik} - \check{B}_{ik} &= \frac{d^2\tilde{r}_{ik}}{d\tilde{x}_{ik}^2}(1-\varpi_{ik}) - \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\left\{\left(1-\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\right)+\left(\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}+\varrho_{ik}\right)\varpi_{ik}\right\} \\ &\quad - \frac{d^2\tilde{r}_{ik}}{d\tilde{x}_{ik}^2}(1-\varpi_{ik}) + \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\left\{\left(-\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\right)+\left(\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}+\varrho_{ik}\right)\varpi_{ik}\right\} \\ &= -\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}},\end{aligned}$$

as claimed. □

C.8.3 Systematic Patterns Induced by Production- and Demand-Side Assumptions

Finally, I am in a position to derive key results for Section 4.2.1. With the production- and demand-side assumptions combined, the following proposition states that the elasticity of the firm's quantity is constant for all firms in the same sector.

Proposition C.10 (Elasticity of Firm-Level Quantity)**.** *Suppose that Assumptions 2.4, 4.3, 4.4 and A.1 hold. Then, for each $i \in \mathbf{N}$, $\frac{dq_{ik}^*/d\tau_n}{q_{ik}^*} = \bar{c}_i^q$ for all $k \in \mathbf{N}_i$ with $\bar{c}_i^q := -\frac{\beta_i^\ell \frac{dW^*}{d\tau_n} + \beta_i^m \frac{dP_i^{M*}}{d\tau_n}}{\beta_i^\ell W^* + \beta_i^m P_i^{M*}}$, where β_i^ℓ and β_i^ℓ are constants appearing in Lemma C.9.*

Proof. Observe that (31) (for the realized ℓ_{ik}^* and m_{ik}^*) can equivalently be rewritten as

$$\begin{bmatrix} \frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i1}} & \frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i2}} & \dots & \frac{\partial mr_{i1}(\cdot)^*}{\partial q_{iN_i}} \\ \frac{\partial mr_{i2}(\cdot)^*}{\partial q_{i1}} & \frac{\partial mr_{i2}(\cdot)^*}{\partial q_{i2}} & \dots & \frac{\partial mr_{i2}(\cdot)^*}{\partial q_{iN_i}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial mr_{iN_i}(\cdot)^*}{\partial q_{i1}} & \frac{\partial mr_{iN_i}(\cdot)^*}{\partial q_{i2}} & \dots & \frac{\partial mr_{iN_i}(\cdot)^*}{\partial q_{iN_i}} \end{bmatrix} \begin{bmatrix} q_{i1} & 0 & \dots & 0 \\ 0 & q_{i2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & q_{iN_i} \end{bmatrix} \begin{bmatrix} \frac{1}{q_{i1}} & 0 & \dots & 0 \\ 0 & \frac{1}{q_{i2}} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \frac{1}{q_{iN_i}} \end{bmatrix} \begin{bmatrix} \frac{dq_{i1}^*}{d\tau_n} \\ \frac{dq_{i2}^*}{d\tau_n} \\ \vdots \\ \frac{dq_{iN_i}^*}{d\tau_n} \end{bmatrix}$$
$$= \begin{bmatrix} \frac{1}{q_{i1}} & 0 & \dots & 0 \\ 0 & \frac{1}{q_{i2}} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \frac{1}{q_{iN_i}} \end{bmatrix} \begin{bmatrix} \ell_{i1}^* & m_{i1}^* \\ \ell_{i2}^* & m_{i2}^* \\ \vdots & \vdots \\ \ell_{iN_i}^* & m_{iN_i}^* \end{bmatrix} \begin{bmatrix} \frac{dW^*}{d\tau_n} \\ \frac{dP_i^{M*}}{d\tau_n} \end{bmatrix},$$

which can further be rearranged as

$$\begin{bmatrix} q_{i1} & 0 & \dots & 0 \\ 0 & q_{i2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & q_{iN_i} \end{bmatrix} \begin{bmatrix} \frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i1}} & \frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i2}} & \dots & \frac{\partial mr_{i1}(\cdot)^*}{\partial q_{iN_i}} \\ \frac{\partial mr_{i2}(\cdot)^*}{\partial q_{i1}} & \frac{\partial mr_{i2}(\cdot)^*}{\partial q_{i2}} & \dots & \frac{\partial mr_{i2}(\cdot)^*}{\partial q_{iN_i}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial mr_{iN_i}(\cdot)^*}{\partial q_{i1}} & \frac{\partial mr_{iN_i}(\cdot)^*}{\partial q_{i2}} & \dots & \frac{\partial mr_{iN_i}(\cdot)^*}{\partial q_{iN_i}} \end{bmatrix} \begin{bmatrix} q_{i1} & 0 & \dots & 0 \\ 0 & q_{i2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & q_{iN_i} \end{bmatrix} \begin{bmatrix} \frac{dq_{i1}^*/d\tau_n}{q_{i1}^*} \\ \frac{dq_{i2}^*/d\tau_n}{q_{i2}^*} \\ \vdots \\ \frac{dq_{iN_i}^*/d\tau_n}{q_{iN_i}^*} \end{bmatrix}$$
$$= \begin{bmatrix} \ell_{i1}^* & m_{i1}^* \\ \ell_{i2}^* & m_{i2}^* \\ \vdots & \vdots \\ \ell_{iN_i}^* & m_{iN_i}^* \end{bmatrix} \begin{bmatrix} \frac{dW^*}{d\tau_n} \\ \frac{dP_i^{M*}}{d\tau_n} \end{bmatrix}. \tag{117}$$

Due to the invertibility (Assumption A.1), (117) can uniquely be solved for $[\frac{dq_{i1}^*/d\tau_n}{q_{i1}^*}\ \frac{dq_{i2}^*/d\tau_n}{q_{i2}^*}\ \dots\ \frac{dq_{iN_i}^*/d\tau_n}{q_{iN_i}^*}]^T$. Thus, it suffices to very that

$$\begin{bmatrix} \frac{dq_{i1}^*/d\tau_n}{q_{i1}^*} \\ \frac{dq_{i2}^*/d\tau_n}{q_{i2}^*} \\ \vdots \\ \frac{dq_{iN_i}^*/d\tau_n}{q_{iN_i}^*} \end{bmatrix} = -\frac{\beta_i^\ell \frac{dW^*}{d\tau_n} + \beta_i^m \frac{dP_i^{M*}}{d\tau_n}}{\beta_i^\ell W^* + \beta_i^m P_i^{M*}} \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} \tag{118}$$

satisfies (117).

Now, provided (118), the left hand side of (117) boils down to

$$
\begin{aligned}
&\bar{c}_i^q \begin{bmatrix} q_{i1} & 0 & \dots & 0 \\ 0 & q_{i2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & q_{iN_i} \end{bmatrix} \begin{bmatrix} \frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i1}} & \frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i2}} & \dots & \frac{\partial mr_{i1}(\cdot)^*}{\partial q_{iN_i}} \\ \frac{\partial mr_{i2}(\cdot)^*}{\partial q_{i1}} & \frac{\partial mr_{i2}(\cdot)^*}{\partial q_{i2}} & \dots & \frac{\partial mr_{i2}(\cdot)^*}{\partial q_{iN_i}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial mr_{iN_i}(\cdot)^*}{\partial q_{i1}} & \frac{\partial mr_{iN_i}(\cdot)^*}{\partial q_{i2}} & \dots & \frac{\partial mr_{iN_i}(\cdot)^*}{\partial q_{iN_i}} \end{bmatrix} \begin{bmatrix} q_{i1} & 0 & \dots & 0 \\ 0 & q_{i2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & q_{iN_i} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} \\
&= \bar{c}_i^q \begin{bmatrix} \frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i1}} q_{i1}^* q_{i1}^* & \frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i2}} q_{i1}^* q_{i2}^* & \dots & \frac{\partial mr_{i1}(\cdot)^*}{\partial q_{iN_i}} q_{i1}^* q_{iN_i}^* \\ \frac{\partial mr_{i2}(\cdot)^*}{\partial q_{i1}} q_{i2}^* q_{i1}^* & \frac{\partial mr_{i2}(\cdot)^*}{\partial q_{i2}} q_{i2}^* q_{i2}^* & \dots & \frac{\partial mr_{i2}(\cdot)^*}{\partial q_{iN_i}} q_{i2}^* q_{iN_i}^* \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial mr_{iN_i}(\cdot)^*}{\partial q_{i1}} q_{iN_i}^* q_{i1}^* & \frac{\partial mr_{iN_i}(\cdot)^*}{\partial q_{i2}} q_{iN_i}^* q_{i2}^* & \dots & \frac{\partial mr_{iN_i}(\cdot)^*}{\partial q_{iN_i}} q_{iN_i}^* q_{iN_i}^* \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}, \quad (119)
\end{aligned}
$$

where $\bar{c}_i^q := -\frac{\beta_i^\ell \frac{dW^*}{d\tau_n} + \beta_i^m \frac{dP_i^{M*}}{d\tau_n}}{\beta_i^\ell W^* + \beta_i^m P_i^{M*}}$. Notice here that

$$
\begin{aligned}
\frac{\partial mr_{ik}(\cdot)^*}{\partial q_{ik}} q_{ik}^* q_{ik}^* &= p_{ik}^* q_{ik}^* (1 - \varpi_{ik}) \hat{B}_{ik} + \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}} p_{ik}^* q_{ik}^* \varpi_{ik} q_{ik}^* t_{ik} \\
\frac{\partial mr_{ik}(\cdot)^*}{\partial q_{ik'}} q_{ik}^* q_{ik'}^* &= -p_{ik}^* q_{ik}^* \varpi_{ik'} \check{B}_{ik} + \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}} p_{ik}^* q_{ik}^* \varpi_{ik} q_{ik'}^* t_{ik'}.
\end{aligned}
$$

(i) the 1st row. The first row of (119), denoted as LHS_1, reads

$$
\begin{aligned}
LHS_1 = \bar{c}_i^q \Big\{ & p_{i1}^* q_{i1}^* (1 - \varpi_{i1}) \hat{B}_{i1} + \frac{d\tilde{r}_{i1}}{d\tilde{x}_{i1}} p_{i1}^* q_{i1}^* \varpi_{i1} q_{i1}^* t_{i1} \\
& - p_{i1}^* q_{i1}^* \varpi_{i2} \check{B}_{i1} + \frac{d\tilde{r}_{i1}}{d\tilde{x}_{i1}} p_{i1}^* q_{i1}^* \varpi_{i1} q_{i2}^* t_{i2} \\
& - \dots \\
& - p_{i1}^* q_{i1}^* \varpi_{iN_i} \check{B}_{i1} + \frac{d\tilde{r}_{i1}}{d\tilde{x}_{i1}} p_{i1}^* q_{i1}^* \varpi_{i1} q_{iN_i}^* t_{iN_i} \Big\} \\
= \bar{c}_i^q & p_{i1}^* q_{i1}^* (1 - \varpi_{i1}) (\hat{B}_{i1} - \check{B}_{i1}) \\
= -\bar{c}_i^q & mc_i z_{i1}^{-1} q_{i1}^* \\
= \Big(& \beta_i^\ell \frac{dW^*}{d\tau_n} + \beta_i^m \frac{dP_i^{M*}}{d\tau_n} \Big) z_{i1}^{-1} q_{i1}^*,
\end{aligned}
$$

where the second equality is a consequence of Fact C.9, the third equality is due to Lemma C.10 and Fact C.10, and the fourth equality follows from Corollary C.4.

The first row of the right hand side of (117), denoted as RHS_1, is

$$
RHS_1 = \ell_{i1}^* \frac{dW^*}{d\tau_n} + m_{i1}^* \frac{dP_i^{M*}}{d\tau_n} = \Big(\beta_i^\ell \frac{dW^*}{d\tau_n} + \beta_i^m \frac{dP_i^{M*}}{d\tau_n} \Big) z_{i1}^{-1} q_{i1}^*,
$$

where the second equality comes from Lemma C.9.

Clearly, $LHS_1 = RHS_1$, meaning that (118) is true for the first row of (117).

(ii) the N_ith row. The last row of (119), denoted as LHS_{N_i}, reads

$$
\begin{aligned}
LHS_{N_i} = \bar{c}_i^q \Big\{ & - p_{iN_i}^* q_{iN_i}^* \varpi_{i1} \check{B}_{iN_i} + \frac{d\tilde{r}_{iN_i}}{d\tilde{x}_{i1}} p_{iN_i}^* q_{iN_i}^* \varpi_{iN_i} q_{i1}^* t_{i1} \\
& - \dots \\
& - p_{iN_i}^* q_{iN_i}^* \varpi_{i,N_i-1} \check{B}_{iN_i} + \frac{d\tilde{r}_{iN_i}}{d\tilde{x}_{iN_i}} p_{iN_i}^* q_{iN_i}^* \varpi_{iN_i} q_{i,N_i-1}^* t_{i,N_i-1} \\
& + p_{iN_1}^* q_{iN_1}^* (1 - \varpi_{iN_1}) \hat{B}_{iN_1} + \frac{d\tilde{r}_{iN_1}}{d\tilde{x}_{iN_1}} p_{iN_1}^* q_{iN_1}^* \varpi_{iN_1} q_{iN_1}^* t_{iN_1} \Big\} \\
= {} & \bar{c}_i^q p_{iN_i}^* q_{iN_i}^* (1 - \varpi_{iN_i})(\hat{B}_{iN_i} - \check{B}_{iN_i}) \\
= {} & -\bar{c}_i^q mc_i z_{iN_i}^{-1} q_{iN_i}^* \\
= {} & \Big(\beta_i^\ell \frac{dW^*}{d\tau_n} + \beta_i^m \frac{dP_i^{M*}}{d\tau_n} \Big) z_{iN_i}^{-1} q_{iN_i}^*,
\end{aligned}
$$

while the right hand side of (117), denoted as RHS_{N_i}, is

$$
RHS_{N_i} = \ell_{iN_i}^* \frac{dW^*}{d\tau_n} + m_{iN_i}^* \frac{dP_i^{M*}}{d\tau_n} = \Big(\beta_i^\ell \frac{dW^*}{d\tau_n} + \beta_i^m \frac{dP_i^{M*}}{d\tau_n} \Big) z_{iN_i}^{-1} q_{iN_i}^*.
$$

Clearly, $LHS_{N_i} = RHS_{N_i}$, meaning that (118) is true for the last row of (117).

(iii) the kth row ($k = 2, 3, \dots, N_i - 1$). The kth row of (119), denoted as LHS_k, reads

$$
\begin{aligned}
LHS_k = \bar{c}_i^q \Big\{ & - p_{ik}^* q_{ik}^* \varpi_{i1} \check{B}_{ik} + \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}} p_{ik}^* q_{ik}^* \varpi_{ik} q_{i1}^* t_{i1} \\
& - \dots \\
& - p_{ik}^* q_{ik}^* \varpi_{i,k-1} \check{B}_{ik} + \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}} p_{ik}^* q_{ik}^* \varpi_{ik} q_{i,k-1}^* t_{i,k-1} \\
& + p_{ik}^* q_{ik}^* (1 - \varpi_{i,k}) \hat{B}_{ik} + \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}} p_{ik}^* q_{ik}^* \varpi_{ik} q_{i,k}^* t_{i,k} \\
& - p_{ik}^* q_{ik}^* \varpi_{i,k+1} \check{B}_{ik} + \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}} p_{ik}^* q_{ik}^* \varpi_{ik} q_{i,k+1}^* t_{i,k+1} \\
& - \dots \\
& - p_{ik}^* q_{ik}^* \varpi_{iN_i} \check{B}_{ik} + \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}} p_{ik}^* q_{ik}^* \varpi_{ik} q_{iN_i}^* t_{iN_i} \Big\} \\
= {} & \bar{c}_i^q p_{ik}^* q_{ik}^* (1 - \varpi_{ik})(\hat{B}_{ik} - \check{B}_{ik}) \\
= {} & -\bar{c}_i^q mc_i z_{ik}^{-1} q_{ik}^* \\
= {} & \Big(\beta_i^\ell \frac{dW^*}{d\tau_n} + \beta_i^m \frac{dP_i^{M*}}{d\tau_n} \Big) z_{ik}^{-1} q_{ik}^*,
\end{aligned}
$$

where the second equality is a consequence of Fact C.9, the third equality is due to Lemma C.10 and Fact C.10, and the fourth equality follows from Corollary C.4.

The kth row of the right hand side of (117), denoted as RHS_k, is

$$RHS_k = \ell_{ik}^* \frac{dW^*}{d\tau_n} + m_{ik}^* \frac{dP_i^{M*}}{d\tau_n} = \Big(\beta_i^\ell \frac{dW^*}{d\tau_n} + \beta_i^m \frac{dP_i^{M*}}{d\tau_n}\Big) z_{ik}^{-1} q_{ik}^*,$$

where the second equality comes from Lemma C.9.

Clearly, $LHS_k = RHS_k$, meaning that (118) is true for the kth row of (117) for $k = 2, \dots, N_i - 1$. Hence, I have shown that (118) is certainly a unique solution for (117), completing the proof. $\square$

Corollary C.5. *Suppose that assumptions of Proposition C.10 are satisfied. Then, for each* $i \in \mathbf{N}$, *(i)* $\frac{dp_{ik}^*/d\tau_n}{p_{ik}^*} = \bar{c}_i^p$, *where* $\bar{c}_i^p = -\bar{c}_i^q$; *and (ii)* $\frac{dp_{ik}^*}{d\tau_n} q_{ik}^* + p_{ik}^* \frac{dq_{ik}^*}{d\tau_n} = 0$ *for all* $k \in \mathbf{N}_i$.

Proof. (i) By construction,

$$\begin{aligned}
\frac{dp_{ik}^*}{d\tau_n} &= \sum_{k'=1}^{N_i} \frac{\partial \wp_{ik}(\cdot)}{\partial q_{ik'}} \frac{dq_{ik'}}{d\tau_n} \\
&= \frac{\partial \wp_{ik}(\cdot)}{\partial q_{ik}} \frac{dq_{ik}}{d\tau_n} + \sum_{k' \neq k} \frac{\partial \wp_{ik}(\cdot)}{\partial q_{ik'}} \frac{dq_{ik'}}{d\tau_n} \\
&= -p_{ik}^* \Big\{1 - \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}(1 - \varpi_{ik})\Big\} \frac{dq_{ik}^*/d\tau_n}{q_{ik}^*} - \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}} p_{ik}^* \sum_{k' \neq k} \varpi_{ik'} \frac{dq_{ik'}^*/d\tau_n}{q_{ik'}^*} \\
&= \bar{c}_i^q \Big[- p_{ik}^* \Big\{1 - \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}(1 - \varpi_{ik})\Big\} - \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}} p_{ik}^* (1 - \varpi_{ik}) \Big] \\
&= -\bar{c}_i^q p_{ik}^*,
\end{aligned}$$

where the third equality utilizes the analytical expressions for the price elasticities derived in Lemma C.4, and the fourth equality is a consequence of Proposition C.10. Rearranging this leads to

$$\frac{dp_{ik}^*/d\tau_n}{p_{ik}^*} = -\bar{c}_i^q.$$

By setting $\bar{c}_i^p = -\bar{c}_i^q$, the claim is proved.

(ii) It is straightforward to show that

$$\frac{dp_{ik}^*}{d\tau_n} q_{ik}^* + p_{ik}^* \frac{dq_{ik}^*}{d\tau_n} = p_{ik}^* q_{ik}^* \left(\frac{dp_{ik}^*/d\tau_n}{p_{ik}^*} + \frac{dq_{ik}^*/d\tau_n}{q_{ik}^*} \right) = p_{ik}^* q_{ik}^* (-\bar{c}_i^q + \bar{c}_i^q) = 0,$$

where the second equality is due to part (i) of this corollary and Proposition C.10. This completes the proof of this corollary. $\square$

Notice that it follows from the second part of this corollary that for each $i \in \mathbf{N}$,

$$\frac{dp_{ik}^*/p_{ik}^*}{dq_{ik}^*/q_{ik}^*} = -1, \tag{120}$$

for all $k \in \mathbf{N}_i$. That is, the price elasticity is unit elastic.

Proof of Proposition 4.1. Proposition 4.1 is an immediate consequence of Proposition C.10 and Corollary C.5. $\square$

C.8.4 Systematic Patterns Induced by Production- and Demand-Side Assumptions (Monopolistic Competition)

The following lemma is a monopolistic competition counterpart of Lemma C.10.

Lemma C.11 (Firm's Pricing Equations in Monopolistic Competition)**.** *Assume that firms in each sector is engaged in monopolistic competition in the output market. Suppose that Assumption 4.4 holds. Then, for each $i \in \mathbf{N}$, $p_{ik} = (\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}})^{-1} mc_{ik}$ for all $k \in \mathbf{N}_i$.*

Proof. Firm's profit-maximization with respect to quantity implies

$$mr_{ik} = mc_{ik} \tag{121}$$

for each firm $k \in \mathbf{N}_i$. Under Assumption 4.4, the left hand side of (121) reads (with a slight abuse of notation)

$$mr_{ik} = \frac{dr_{ik}}{dq_{ik}} = \exp\{\tilde{\varphi}_i(\tilde{x}_{ik})\} \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}} \frac{d\tilde{x}_{ik}}{dx_{ik}} \frac{\partial x_{ik}(\cdot)}{\partial q_{ik}} = \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}} p_{ik}.$$

Thus, (121) implies

$$p_{ik} \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}} = mc_{ik},$$

so that

$$p_{ik} = \left(\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}} \right)^{-1} mc_{ik},$$

proving the statement of this lemma. $\square$

Proposition C.11 (Elasticity of Firm-Level Quantity in Monopolistic Competition)**.** *Assume that firms in each sector are engaged in monopolistic competition in the output market. Suppose that Assumptions 2.4, 4.3 and 4.4 hold. Then, for each $i \in \mathbf{N}$, there exists a sector-specific constant $\bar{c}_i^q \in \mathbb{R}\backslash\{0\}$ such that $\frac{dq_{ik}^*/d\tau_n}{q_{ik}^*} = \bar{c}_i^q$ for all $k \in \mathbf{N}_i$, if and only if there exists a sector-specific constant $\bar{d}_i^q \in \mathbb{R}\backslash\{0\}$ such that $\left(\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\right)^{-1} \left\{ \frac{d^2\tilde{r}_{ik}}{d\tilde{x}_{ik}^2} - \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}} \left(1 - \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\right) \right\} = \bar{d}_i^q$ for all $k \in \mathbf{N}_i$.*

Proof. First of all, in monopolistic competition, the equation corresponding to (117) can be written as

$$\begin{bmatrix} q_{i1} & 0 & \dots & 0 \\ 0 & q_{i2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & q_{iN_i} \end{bmatrix} \begin{bmatrix} \frac{\partial mr_{i1}(\cdot)^*}{\partial q_{i1}} & 0 & \dots & 0 \\ 0 & \frac{\partial mr_{i2}(\cdot)^*}{\partial q_{i2}} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \frac{\partial mr_{iN_i}(\cdot)^*}{\partial q_{iN_i}} \end{bmatrix} \begin{bmatrix} q_{i1} & 0 & \dots & 0 \\ 0 & q_{i2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & q_{iN_i} \end{bmatrix} \begin{bmatrix} \frac{dq^*_{i1}/d\tau_n}{q^*_{i1}} \\ \frac{dq^*_{i2}/d\tau_n}{q^*_{i2}} \\ \vdots \\ \frac{dq^*_{iN_i}/d\tau_n}{q^*_{iN_i}} \end{bmatrix} = \begin{bmatrix} \ell^*_{i1} & m^*_{i1} \\ \ell^*_{i2} & m^*_{i2} \\ \vdots & \vdots \\ \ell^*_{iN_i} & m^*_{iN_i} \end{bmatrix} \begin{bmatrix} \frac{dW^*}{d\tau_n} \\ \frac{dP_i^{M*}}{d\tau_n} \end{bmatrix}, \tag{122}$$

where $\frac{\partial mr_{ik}(\cdot)^*}{\partial q_{ik}} = \frac{p^*_{ik}}{q^*_{ik}}\Big\{\frac{d^2\tilde{r}_{ik}}{d\tilde{x}^2_{ik}} - \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\big(1 - \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\big)\Big\}$.[133]

($\Longrightarrow$). Suppose that for each sector $i \in \mathbf{N}$, there exists a sector-specific constant $\bar{c}^q_i \in \mathbb{R}\backslash\{0\}$ such that $\frac{dq^*_{ik}/d\tau_n}{q^*_{ik}} = \bar{c}^q_i$ for all $k \in \mathbf{N}_i$. Then, it follows from (122) that for each $i \in \mathbf{N}$,

$$\bar{c}^q_i \frac{\partial mr_{ik}(\cdot)^*}{\partial q_{ik}} q^*_{ik} q^*_{ik} = \ell^*_{ik}\frac{dW^*}{d\tau_n} + m^*_{ik}\frac{dP_i^{M*}}{d\tau_n},$$

for all $k \in \mathbf{N}_i$. In view of Lemma C.9, Corollary C.4 and Lemma C.11, this yields

$$\Big(\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\Big)^{-1}\Big\{\frac{d^2\tilde{r}_{ik}}{d\tilde{x}^2_{ik}} - \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\big(1 - \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\big)\Big\} = (\bar{c}^q_i)^{-1}\frac{\beta^\ell_i \frac{dW^*}{d\tau_n} + \beta^m_i \frac{dP_i^{M*}}{d\tau_n}}{\beta^\ell_i W^* + \beta^m_i P_i^{M*}},$$

where β^ℓ_i and β^m_i are constants appearing in Lemma C.9. Since the right hand side of this expression is free from the firm-specific index k, the implication is true by setting $\bar{d}^q_i := (\bar{c}^q_i)^{-1}\frac{\beta^\ell_i \frac{dW^*}{d\tau_n} + \beta^m_i \frac{dP_i^{M*}}{d\tau_n}}{\beta^\ell_i W^* + \beta^m_i P_i^{M*}}$.

($\Longleftarrow$). Suppose that for each sector $i \in \mathbf{N}$, there exists a sector-specific constant $\bar{d}^q_i \in \mathbb{R}\backslash\{0\}$ such that $\Big(\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\Big)^{-1}\Big\{\frac{d^2\tilde{r}_{ik}}{d\tilde{x}^2_{ik}} - \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\big(1 - \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\big)\Big\} = \bar{d}^q_i$ for all $k \in \mathbf{N}_i$. Then, it follows from (122) that for each $i \in \mathbf{N}$,

$$\frac{\partial mr_{ik}(\cdot)^*}{\partial q_{ik}} q^*_{ik} q^*_{ik} \frac{dq^*_{ik}/d\tau_n}{q^*_{ik}} = \ell^*_{ik}\frac{dW^*}{d\tau_n} + m^*_{ik}\frac{dP_i^{M*}}{d\tau_n},$$

for all $k \in \mathbf{N}_i$. In view of Lemma C.9, Corollary C.4 and Lemma C.11, this yields

$$\frac{dq^*_{ik}/d\tau_n}{q^*_{ik}} = (\bar{d}^q_i)^{-1}\frac{\beta^\ell_i \frac{dW^*}{d\tau_n} + \beta^m_i \frac{dP_i^{M*}}{d\tau_n}}{\beta^\ell_i W^* + \beta^m_i P_i^{M*}},$$

[133] See Remark C.4.

where β_i^ℓ and β_i^m are constants appearing in Lemma C.9. Since the right hand side of this expression is free from the firm-specific index k, the implication is true by setting $\bar{c}_i^q := (\bar{d}_i^q)^{-1}\frac{\beta_i^\ell \frac{dW^*}{d\tau_n}+\beta_i^m\frac{dP_i^{M*}}{d\tau_n}}{\beta_i^\ell W^*+\beta_i^m P_i^{M*}}$.

This completes the proof of this proposition. □

The following corollary corresponds to but is not quite the same as Corollary C.5 (i).

Corollary C.6. *Assume that firms in each sector are engaged in monopolistic competition in the output market. Suppose that assumptions in Proposition C.11 are satisfied. In addition, assume that for each $i \in \mathbf{N}$, (i) there exists a sector-specific constant $\bar{d}_i^q \in \mathbb{R}\backslash\{0\}$ such that $\left(\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\right)^{-1}\left\{\frac{d^2\tilde{r}_{ik}}{d\tilde{x}_{ik}^2} - \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\left(1-\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\right)\right\} = \bar{d}_i^q$ for all $k \in \mathbf{N}_i$; and (ii) there exists a sector-specific constant $\bar{e}_i^q \in \mathbb{R}$ such that $1-\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}} = \bar{e}_i^q$ for all $k \in \mathbf{N}_i$. Then, there exists a sector-specific constant $\bar{c}_i^p \in \mathbb{R}$ such that $\frac{dp_{ik}^*/d\tau_n}{p_{ik}^*} = \bar{c}_i^p$ for all $k \in \mathbf{N}_i$.*

Proof. By construction,

$$\frac{dp_{ik}^*}{d\tau_n} = \frac{\partial\wp_{ik}(\cdot)^*}{\partial q_{ik}}\frac{dq_{ik}^*}{d\tau_n} = -\bar{e}_i^q\frac{p_{ik}^*}{q_{ik}^*}\frac{dq_{ik}^*}{d\tau_n},$$

where the second equality is the result of Remark C.2 and the hypothesis (ii) of this corollary. This can be rearranged to

$$\frac{dp_{ik}^*/d\tau_n}{p_{ik}^*} = -\bar{e}_i^q\frac{dq_{ik}^*/d\tau_n}{q_{ik}^*}.$$

In view of Proposition C.11, the hypothesis (i) of this corollary implies that there exists a sector-specific constant $\bar{c}_i^q \in \mathbb{R}\backslash\{0\}$ such that $\frac{dq_{ik}^*/d\tau_n}{q_{ik}^*} = \bar{c}_i^q$ for all $k \in \mathbf{N}_i$. Hence,

$$\frac{dp_{ik}^*/d\tau_n}{p_{ik}^*} = -\bar{e}_i^q\bar{c}_i^q.$$

Since the right hand side of this expression is free from the firm-specific index k, the claim of this corollary is true by choosing $\bar{c}_i^p := -\bar{c}_i^q\bar{e}_i^q$. □

This corollary means that the elasticity of the firm's price might be constant for all firms, but it is not the same in magnitude as the elasticity of the firm's quantity. In regard to the added term $\bar{e}_i^q$, it is worth noting that in equilibrium, $\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}$ dictates the inverse of the firm's markup (see the proof of Lemma C.2). Hence, the hypothesis (ii) of this corollary essentially requires that the equilibrium markup is the same for all firms. (This can be true in the case of a CES sectoral aggregator as shown in Example C.1.) An intuition is that each monopolist can exercise market power against the demand side.

The next corollary appears similar to Corollary C.5 (ii), but its implication is quite the opposite: it states that it is effectively impossible to have a version of it in monopolistic competition.

Corollary C.7. *Assume that firms in each sector are engaged in monopolistic competition in the output market. Suppose that assumptions in Proposition C.11 are satisfied. In addition, assume that for each $i \in \mathbf{N}$, (i) there exists a sector-specific constant $\bar{d}_i^q \in \mathbb{R}\backslash\{0\}$ such that $\left(\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\right)^{-1}\left\{\frac{d^2\tilde{r}_{ik}}{d\tilde{x}_{ik}^2} - \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\left(1 - \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}\right)\right\} = \bar{d}_i^q$ for all $k \in \mathbf{N}_i$; and (ii) there exists a sector-specific constant $\bar{e}_i^q \in \mathbb{R}$ such that $1 - \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}} = \bar{e}_i^q$ for all $k \in \mathbf{N}_i$. Then, for each sector $i \in \mathbf{N}$, $\frac{dp_{ik}^*}{d\tau_n}q_{ik}^* + p_{ik}^*\frac{dq_{ik}^*}{d\tau_n} = 0$ for all $k \in \mathbf{N}_i$ if and only if $\bar{e}_i^q = 1$.*

Proof. In view of Proposition C.11, it follows from the hypothesis (i) that for each $i \in \mathbf{N}$, there exists a sector-specific constant $\bar{c}_i^q \in \mathbb{R}\backslash\{0\}$ such that $\frac{dq_{ik}^*/d\tau_n}{q_{ik}^*} = \bar{c}_i^q$ for all $k \in \mathbf{N}_i$. Moreover, it holds by Corollary C.6 that there exists a sector-specific constant $\bar{c}_i^p \in \mathbb{R}$ such that $\frac{dp_{ik}^*}{d\tau_n}p_{ik}^* = \bar{c}_i^p$ for all $k \in \mathbf{N}_i$. In particular, $\bar{c}_i^p = -\bar{c}_i^q\bar{e}_i^q$.

Now, pick an arbitrary k. It is then straightforward to show that

$$\frac{dp_{ik}^*}{d\tau_n}q_{ik}^* + p_{ik}^*\frac{dq_{ik}^*}{d\tau_n} = 0 \Longleftrightarrow p_{ik}^*q_{ik}^*\left(\frac{dp_{ik}^*/d\tau_n}{p_{ik}^*} + \frac{dq_{ik}^*/d\tau_n}{q_{ik}^*}\right) \Longleftrightarrow p_{ik}^*q_{ik}^*\bar{c}_i^q(1 - \bar{e}_i^q) = 0 \Longleftrightarrow \bar{e}_i^q = 1.$$

The proof is completed as soon as noticing that this equivalence result does not depend on the particular choice of k. ☐

Notice that $\bar{e}_i^q = 1$ means that the firm's markup is infinity and so is the firm's output price, a case that is unlikely be interesting both on theoretical and empirical grounds. Because of this, Corollary C.7 effectively tells us that the firm-level price effect will never exactly offsets the quantity effect, leaving a non-zero revenue effect. This observation is summarized in the following corollary.

Corollary C.8. *Assume that firms in each sector are engaged in monopolistic competition in the output market. Suppose that the assumptions in Corollary C.6 are satisfied. Then, $\frac{dp_{ik}^*/p_{ik}^*}{dq_{ik}^*/q_{ik}^*} \in (-1, 0)$.*

Proof. It follows from Corollary C.6 that $\frac{dp_{ik}^*/d\tau_n}{p_{ik}^*} = -\bar{e}_i^q\bar{c}_i^q$. Since $\frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}}$ is equal to the inverse of the firm's markup, it holds that $0 < \frac{d\tilde{r}_{ik}}{d\tilde{x}_{ik}} < 1$, so that $0 < \bar{e}_i^q < 1$. Combining these leads to

$$\frac{dp_{ik}^*/d\tau_n}{p_{ik}^*}(\bar{c}_i^q)^{-1} \in (-1, 0).$$

Noticing that $\bar{c}_i^q = \frac{dq_{ik}^*/d\tau_n}{q_{ik}^*}$ completes the proof. ☐

This corollary implies that the price elasticity in monopolistic competition is inelastic due to the firm's market power, marking a sharp contrast with the unitary elasticity in the oligopolistic environment (120). Specifically, Corollary C.8 means that the monopolistic firms experience positive revenue effects.

D Extensions

D.1 Dynamic Environment

The CHIPS and Science Act consists of two parts: *i*) Investment in construction, expansion, or modernization of facilities producing semiconductors, and *ii*) tax credit for capital investments in semiconductors. In the main text, I focus on the second part only; as far as the tax credits and the static analysis are concerned, the empirical analysis of this paper is consistent with the model. The empirical analysis of this paper can be reconciled with the notion of capital by simply viewing it to be an endowment and to be incorporated into the firms' production capacities (see Appendix B.3.5). To explicitly account for the investment part, the model of this paper needs to be extended to include the firms' dynamic capital accumulation, which is left for future work.

D.2 Long-Run Perspective

This paper focuses on the short-run effects of policies, excluding the firms' endogenous entry and exit decisions in reaction to a change in policy. At first glance, this might appear to be restrictive because the present paper studies merely a "special case" of the "full-fledged model." In practice, however, the short-run analysis deserves separate attention in its own right mainly for two reasons. First, the short-run analysis *per se* is useful as a tool for "validation" of the policy under consideration.[134] In the short run, the model prediction can be compared to what has actually happened in the data. If the data turn out to be substantially different from the model prediction, the policymaker can/should revise and update the model. In contrast, when the observed outcomes are largely in line with the model prediction, it is a strong indication that the model is plausible, granting the policymaker confidence about the policy in place. Second, the short-run analysis is a necessary step to separately identify the intensive and extensive margin causal effects.[135] While the short-run analysis identifies the intensive margin causal effect as explored in the main text, the long-run analysis directly identifies the total causal effect. Thus, the extensive margin causal effect is only identified as a residual between the intensive margin and total causal effects.

To illustrate the idea, I briefly sketch the definition and identification of the extensive margin causal effects.

D.2.1 Illustrative Example

Definition. Consider the same setup as in the main text but deviate by allowing for the firm's endogenous entry and exit. Consider a policy reform from $\boldsymbol{\tau}^0$ to $\boldsymbol{\tau}^1$. Let $\mathcal{N}_i^0$ and $\mathcal{N}_i^1$ be the index sets for firms in sector i under $\boldsymbol{\tau}^0$ and $\boldsymbol{\tau}^1$, respectively. Let u signify the competitiveness of the

[134] This insight is employed in empirical microeconomic literature. See Low and Meghir (2017) and references therein.

[135] For example, the international trade literature studies the "trade elasticities" for the both intensive and extensive margins (e.g., Chaney 2008; Adão et al. 2020; Boehm et al. 2023). Other works decompose the total growth/difference in the value of trade into the intensive and extensive margins (e.g., Feenstra 1994; Hummels and Klenow 2005; Kehoe and Ruhl 2013). My framework separately defines the intensive and extensive margin causal policy effects.

market under $\mathcal{N}_i^u$, thereby $y_{ik}^u(\boldsymbol{\tau})$ representing the firm-level value-added of firm k in sector i under u and $\boldsymbol{\tau}$. The competitiveness is determined by the membership of firms in the same sector. The total causal effect of the policy reform is defined as

$$\Delta Y(\boldsymbol{\tau}^0, \boldsymbol{\tau}^1) \coloneqq \sum_{i=1}^{N} \sum_{k \in \mathcal{N}_i^1} y_{ik}^1(\boldsymbol{\tau}^1) - \sum_{i=1}^{N} \sum_{k \in \mathcal{N}_i^0} y_{ik}^0(\boldsymbol{\tau}^0).$$

By the technique of add and subtract, it can be decomposed into the intensive and extensive margin causal effects:

$$\underbrace{\Delta Y(\boldsymbol{\tau}^0, \boldsymbol{\tau}^1)}_{\text{the total causal effect}} = \underbrace{\sum_{i=1}^{N} \sum_{k \in \mathcal{N}_i^1} y_{ik}^1(\boldsymbol{\tau}^1) - \sum_{i=1}^{N} \sum_{k \in \mathcal{N}_i^0} y_{ik}^0(\boldsymbol{\tau}^1)}_{\text{the extensive margin causal effect}} + \underbrace{\sum_{i=1}^{N} \sum_{k \in \mathcal{N}_i^0} y_{ik}^0(\boldsymbol{\tau}^1) - \sum_{i=1}^{N} \sum_{k \in \mathcal{N}_i^0} y_{ik}^0(\boldsymbol{\tau}^0)}_{\text{the intensive margin causal effect}}.$$

The first term of the right-hand side of this expression is a *ceteris paribus* difference in GDP due to a change in the number of firms, thus presenting the *extensive margin causal effects.* The second term fixes the number of firms at the status quo level while only changing the level of subsidy; thus, this term is the *intensive margin causal effects*, as discussed in the main text.

Identification. Notice here that the second half (the intensive margin causal effect) is identified by the short-run analysis of this paper. As shown below, the long-run analysis directly identifies the total causal effect. Hence, the extensive margin causal effect is identified as a residual.

To simplify the exposition, suppose that the market competitiveness is summarized in a single variable: let $\mathbf{a}^u \in \mathbb{R}$ be the index of the market competitiveness corresponding to u. Under the assumption of the HSA demand system, I can write as

$$y_{ik}(\boldsymbol{\tau}, \mathbf{a}^u) = y_{ik}^u(\boldsymbol{\tau}),$$

for any $\boldsymbol{\tau} \in \{\boldsymbol{\tau}^0, \boldsymbol{\tau}^1\}$. Assume that the “within-the-support condition” (a version of Assumption 4.2) holds for $[\mathbf{a}^0, \mathbf{a}^1]$ as well. The total causal effect can be expressed as

$$\Delta Y(\boldsymbol{\tau}^0, \boldsymbol{\tau}^1) = \sum_{i=1}^{N} \sum_{k \in \mathcal{N}_i^1} y_{ik}(\boldsymbol{\tau}^1, \mathbf{a}^1) - \sum_{i=1}^{N} \sum_{k \in \mathcal{N}_i^0} y_{ik}(\boldsymbol{\tau}^0, \mathbf{a}^0).$$

From this expression, the identification analysis can further be broken down into four components as

$$\Delta Y(\boldsymbol{\tau}^0, \boldsymbol{\tau}^1) = \sum_{i=1}^{N} \Bigg\{ \underbrace{\sum_{k \in \mathcal{N}_i^0 \cap \mathcal{N}_i^1} \big(y_{ik}(\boldsymbol{\tau}^1, \mathbf{a}^1) - y_{ik}(\boldsymbol{\tau}^0, \mathbf{a}^0) \big)}_{\text{continuing firms}}$$

$$+ \underbrace{\sum_{k \in \mathcal{N}_i^1 \backslash \mathcal{N}_i^0} \big(y_{ik}(\boldsymbol{\tau}^1, \mathbf{a}^1) - y_{ik}(\boldsymbol{\tau}^0, \mathbf{a}^0) \big)}_{\text{new entrants}} + \underbrace{\sum_{k \in \mathcal{N}_i^0 \backslash \mathcal{N}_i^1} \big(y_{ik}(\boldsymbol{\tau}^1, \mathbf{a}^1) - y_{ik}(\boldsymbol{\tau}^0, \mathbf{a}^0) \big)}_{\text{exiting firms}}$$
$$+ \underbrace{\sum_{k \in \mathcal{N}_i^1 \backslash \mathcal{N}_i^0} y_{ik}(\boldsymbol{\tau}^0, \mathbf{a}^0) - \sum_{k \in \mathcal{N}_i^0 \backslash \mathcal{N}_i^1} y_{ik}(\boldsymbol{\tau}^1, \mathbf{a}^1)}_{\text{a normalization constant}} \Bigg\}$$

The first term is the causal effect that stems from the continuing firms' (i.e., firms that operate both before and after the policy reform) moving from the current state of the economy $(\boldsymbol{\tau}^0, \mathbf{a}^0)$ to an alternative state of the economy $(\boldsymbol{\tau}^1, \mathbf{a}^1)$. The second and third terms represent the causal effect arising from new entrants (i.e., firms that do not operate before the policy reform but become active after the policy reform) and from exiting firms (i.e., firms that are active before the policy reform but cease to operate after the policy reform), respectively. Note that these terms involve counterfactual outcomes because $\{y_{ik}(\boldsymbol{\tau}^0, \mathbf{a}^0) : k \in \mathcal{N}_i^1 \backslash \mathcal{N}_i^0\}$ and $\{y_{ik}(\boldsymbol{\tau}^1, \mathbf{a}^1) : k \in \mathcal{N}_i^0 \backslash \mathcal{N}_i^1\}$ are not observed in the data. This fact points to the importance of a structural model in defining and identifying the causal policy effects. The last term is the difference between the sum of firm-level value-added that would have been created by the entering firms if they were to be operative before the policy reform, and the sum of firm-level value-added that would have been yielded by the exiting firms if they were to continue to operate under the post-policy environment. This term acts as a normalization constant, reflecting the free entry condition as well as other model specifications.

For the first three terms (i.e., for continuing firms, new entrants and exiting firms), the summand can be rearranged as

$$\begin{aligned} y_{ik}(\boldsymbol{\tau}^1, \mathbf{a}^1) - y_{ik}(\boldsymbol{\tau}^0, \mathbf{a}^0) &= y_{ik}(\boldsymbol{\tau}^1, \mathbf{a}^1) - y_{ik}(\boldsymbol{\tau}^0, \mathbf{a}^1) + y_{ik}(\boldsymbol{\tau}^0, \mathbf{a}^1) - y_{ik}(\boldsymbol{\tau}^0, \mathbf{a}^0) \\ &= \int_{\boldsymbol{\tau}^0}^{\boldsymbol{\tau}^1} \frac{\partial y_{ik}(s, \mathbf{a}^1)}{\partial s} ds + \int_{\mathbf{a}^0}^{\mathbf{a}^1} \frac{\partial y_{ik}(\boldsymbol{\tau}^0, s)}{\partial s} ds. \end{aligned}$$

The left hand side of this equation is identified as soon as both $\frac{\partial y_{ik}(s,\mathbf{a}^1)}{\partial s}$ and $\frac{\partial y_{ik}(\boldsymbol{\tau}^0,s)}{\partial s}$ are identified. It depends on the specification of the market competitiveness $\mathbf{a}$ and is beyond the scope of this paper. The identification of the fourth term (i.e., the normalization constant) hinges on the formulation of the free entry condition, which determines the number of firms $\mathcal{N}_i^1$. Further investigation is left for future work.

D.3 Other Causal Parameters of Interest

The discussion of the main text of this paper concentrated around the policy parameter (14) (i.e., the *ceteris paribus* difference in GDP as a result of a policy reform) for the sake of exposition. However, the approach of this paper apply more broadly. In this subsection, I thus explore the versatility of my framework by showing how it can be used to define other economically interesting causal policy parameters studied in the literature. All the parameters in this subsection are identified under the same set of assumptions as in Theorem 4.1.

D.3.1 Various Formulations

First, the researcher may want to restrict attention to a subset $\mathbf{N}^{sub} \subset \mathbf{N}$ of sectors (e.g., broadly defined sectors). In such a case, the object of interest takes the form of

$$\sum_{i \in \mathbf{N}^{sub}} Y_i(\boldsymbol{\tau}^1) - \sum_{i \in \mathbf{N}^{sub}} Y_i(\boldsymbol{\tau}^0).$$

Second, under Assumption 2.1, the policy parameter (14) is essentially equivalent to writing as

$$\frac{1}{N}\sum_{i=1}^{N} Y_i(\boldsymbol{\tau}^1) - \frac{1}{N}\sum_{i=1}^{N} Y_i(\boldsymbol{\tau}^0).$$

This expression allows for the interpretation as the average treatment effect (ATE) of the policy change on sectoral GDP.

Another economically interesting policy parameter would be the growth rate $\%\Delta Y(\tau_n^0, \tau_n^1)$ of the kind studied in Arkolakis et al. (2012) and Adão et al. (2017). This is just a version of (14) and can be defined as

$$\%\Delta Y(\tau_n^0, \tau_n^1) \coloneqq \frac{1}{Y^{\boldsymbol{\tau}^0}} \Delta Y(\tau_n^0, \tau_n^1).$$

Furthermore, the elasticity-type policy parameter $\frac{d \ln Y}{d\tau_n}$ around $\boldsymbol{\tau}^0$ (e.g., Caliendo and Parro (2015), Liu (2019), Baqaee and Farhi (2022)) can also be viewed as a version of (14) at the limit of $\boldsymbol{\tau}^1 \to \boldsymbol{\tau}^0$, i.e.,

$$\left.\frac{d \ln Y^{\tau}}{d\tau_n}\right|_{\boldsymbol{\tau}=\boldsymbol{\tau}^0} = \lim_{\boldsymbol{\tau}^1 \to \boldsymbol{\tau}^0} \%\Delta Y(\tau_n^0, \tau_n^1).$$

D.3.2 Aggregate Variables

Consumption. The causal policy effect on final consumption is given by

$$\Delta C(\tau_n^0, \tau_n^1) \coloneqq C(\boldsymbol{\tau}^1) - C(\boldsymbol{\tau}^0) = \int_{\tau_n^0}^{\tau_n^1} \frac{dC}{d\tau_n} d\tau_n,$$

where $C(\boldsymbol{\tau})$ represents the equilibrium consumption under policy regime $\boldsymbol{\tau}$. Assuming that government spending G is fixed, it can be rewritten as

$$\frac{dC}{d\tau_n} = \frac{dY}{d\tau_n} = \sum_{i=1}^{N} \frac{dY_i}{d\tau_n},$$

where the identification of $\frac{dY_i}{d\tau_n}$ is studied in the main text.

Labor, material and output quantity. In equilibrium, labor employed in sector i is defined as

$$L_i^* \coloneqq \sum_{k=1}^{N_i} \ell_{ik}^*.$$

The policy effect on labor employed in sector i, $\Delta L_i(\tau_n^0, \tau_n^1)$, is given by

$$\Delta L_i(\tau_n^0, \tau_n^1) \coloneqq L_i(\boldsymbol{\tau}^1) - L_i(\boldsymbol{\tau}^0) = \sum_{k=1}^{N_i} \int_{\tau_n^0}^{\tau_n^1} \frac{d\ell_{ik}^*}{d\tau_n} d\tau_n,$$

where $L(\boldsymbol{\tau})$ denotes the total labor employed in sector i under policy $\boldsymbol{\tau}$. From this equality, $\Delta L_i(\tau_n^0, \tau_n^1)$ is identified as soon as $\frac{d\ell_{ik}^*}{d\tau_n}$ is identified for all $k \in \mathbf{N}_i$ and $\tau_n \in [\tau_n^0, \tau_n^1]$.[136]

Analogous arguments hold for quantities of material input and output.

Unilateral and bilateral trade flows. The equilibrium volume of unilateral trade flow from sector j to i is defined as

$$U_{i,j}^* \coloneqq \sum_{k=1}^{N_i} m_{ik,j}^*.$$

The policy effect on the unilateral trade flow is given by

$$\Delta U_{i,j}(\tau_n^0, \tau_n^1) \coloneqq U_{i,j}(\boldsymbol{\tau}^0) - U_{i,j}(\boldsymbol{\tau}^1) = \sum_{k=1}^{N_i} \int_{\tau_n^0}^{\tau_n^1} \frac{dm_{ik,j}^*}{d\tau_n} d\tau_n,$$

where $U_{i,j}(\boldsymbol{\tau})$ represents the unilateral trade flow from sector j to i under policy $\boldsymbol{\tau}$. It follows from this expression that the causal effect $\Delta U_{i,j}(\tau_n^0, \tau_n^1)$ is recovered through the identification of $\frac{dm_{ik,j}^*}{d\tau_n}$.[137]

The policy effect on the bilateral trade flow between sector i and j, denoted by $B_{i,j}$, can be similarly analyzed by noticing $B_{i,j} = U_{i,j} + U_{j,i}$.

D.3.3 Various Treatment Effects

As stated in the main text, the construction of the policy parameter (14) shares the common vein with the treatment effects. In fact, multitudes of "treatment effects" can be analyzed within my framework. As an example, consider the net profit of individual firm k, defined by

$$\pi_{ik}^* \coloneqq p_{ik}^* q_{ik}^* - (W^* \ell_{ik}^* + P_i^{M^*} m_{ik}^*).$$

This represents the firm's profit after all taxes and subsidies are applied.

[136] This is established in Proposition C.8.
[137] This is established in Proposition C.9.

Individual-level treatment effects. Individual-level treatment effect is given by

$$\Delta\pi_{ik}(\tau_n^0, \tau_n^1) := \pi_{ik}(\boldsymbol{\tau}^1) - \pi_{ik}(\boldsymbol{\tau}^0) = \int_{\tau_n^0}^{\tau_n^1} \frac{d\pi_{ik}^*}{d\tau_n} d\tau_n,$$

where $\pi_{ik}(\boldsymbol{\tau})$ denotes the firm k's equilibrium profit π_{ik}^* under policy regime $\boldsymbol{\tau}$. Here, it is straightforward to verify that $\frac{d\pi_{ik}^*}{d\tau_n}$ is identified under the same set of assumptions as Theorem 4.1, and thus so is the individual treatment effect $\Delta\pi_{ik}(\tau_n^0, \tau_n^1)$.

Average treatment effects. For each sector $i \in \mathbf{N}$, the sector-level average treatment effect is given by

$$\Delta\Pi_i(\tau_n^0, \tau_n^1) := \frac{1}{N_i}\sum_{k=1}^{N_i} \pi_{ik}(\boldsymbol{\tau}^1) - \frac{1}{N_i}\sum_{k=1}^{N_i} \pi_{ik}(\boldsymbol{\tau}^0) = \frac{1}{N_i}\sum_{k=1}^{N_i} \Delta\pi_{ik}(\tau_n^0, \tau_n^1).$$

Moreover, the economy-wide average treatment effect (i.e., producer surplus) is given by

$$\Delta\Pi(\tau_n^0, \tau_n^1) := \frac{1}{N}\sum_{i=1}^{N} \frac{1}{N_i}\sum_{k=1}^{N_i} \pi_{ik}(\boldsymbol{\tau}^1) - \frac{1}{N}\sum_{i=1}^{N} \frac{1}{N_i}\sum_{k=1}^{N_i} \pi_{ik}(\boldsymbol{\tau}^0) = \frac{1}{N}\sum_{i=1}^{N} \Delta\Pi_i(\tau_n^0, \tau_n^1).$$

Given the identification of the individual-level treatment effect $\Delta\pi_{ik}(\tau_n^0, \tau_n^1)$, the sector-level average treatment effect $\Delta\Pi_i(\tau_n^0, \tau_n^1)$ is also identified, which in turn recovers the economy-wide average treatment effect $\Delta\Pi(\tau_n^0, \tau_n^1)$.

Remark D.1. *The recent international trade literature has applied the statistical treatment effect approach to study the average treatment effects of a trade policy change on the bilateral international trade flows (e.g., Baier and Bergstrand 2007, 2009; Egger et al. 2008, 2011). Such an estimand can be mirrored in my framework by incorporating the observations in Appendices D.3.1 and D.3.2.*

Distributional treatment effects. Given that individual-level treatment effects $\Delta\pi_{ik}(\tau_n^0, \tau_n^1)$ are identified and the firm-level profits under the current policy regime $\pi_{ik}(\boldsymbol{\tau}^0)$ are directly observed in the data, it is possible to recover the firms' profits under an alternative policy $\boldsymbol{\tau}^1$:

$$\pi_{ik}(\boldsymbol{\tau}^1) = \pi_{ik}(\boldsymbol{\tau}^0) + \Delta\pi_{ik}(\tau_n^0, \tau_n^1).$$

This means that one can recover the joint distribution of $\pi_{ik}(\boldsymbol{\tau}^0)$ and $\pi_{ik}(\boldsymbol{\tau}^1)$, a basis on which a variety of distributional criteria for policy evaluation are defined and identified. For example, the policymaker may be interested in the proportion of firms that benefit from policy $\boldsymbol{\tau}^1$ compared to $\boldsymbol{\tau}^0$.[138] In such a case, the object of interest is given by

$$Prop_i(\boldsymbol{\tau}^0, \boldsymbol{\tau}^1) := Pr(\pi_{ik}(\boldsymbol{\tau}^1) \geq \pi_{ik}(\boldsymbol{\tau}^0)).$$

[138] This is called the voting criteria (Heckman et al. 1999; Heckman and Vytlacil 2007).

Another distributional policy parameter that is often of practical interest is the (unconditional) quantile treatment effect for quantile $u \in (0,1)$, which is defined as

$$QTW_i^u(\boldsymbol{\tau}^0, \boldsymbol{\tau}^1) \coloneqq F^{-1}_{\Pi(\boldsymbol{\tau}^1)}(u) - F^{-1}_{\Pi(\boldsymbol{\tau}^0)}(u),$$

where $F^{-1}_{\Pi(\boldsymbol{\tau})}(\cdot)$ stands for the inverse of the probability distribution of π^*_{ik} under policy regime $\boldsymbol{\tau}$.

See Heckman et al. (1999) for an extensive catalog of distributional treatment effects. It is immediate to show that these distributional criteria are identified when Theorem 4.1 holds.

D.4 Changing Subsidies to Multiple Sectors

In the main text, I restrict attention to the case where only subsidy to a single sector is manipulated. In practice, however, subsidies to other sectors are also more or less subject to changes, regardless whether they are purposefully targeted. Thus, it is practically very important to accommodate changes in multiple subsidies at once. For ease of exposition, suppose that there are only two sectors. Consider a policy reform from $\boldsymbol{\tau}^0 \coloneqq (\tau_1^0, \tau_2^0)$ to $\boldsymbol{\tau}^1 \coloneqq (\tau_1^1, \tau_2^1)$, where $\boldsymbol{\tau}^0, \boldsymbol{\tau}^1 \in \mathscr{T}$ with $\mathscr{T}$ representing the observed support (i.e., both τ_1 and τ_2 satisfy the "within-support condition" of the form of Assumption 4.2).

The object of interest can be written as

$$\begin{aligned}
\Delta Y(\boldsymbol{\tau}^0, \boldsymbol{\tau}^1) &\coloneqq \sum_{i=1}^N Y_i((\tau_1^1, \tau_2^1)) - \sum_{i=1}^N Y_i((\tau_1^0, \tau_2^0)) \\
&= \underbrace{\sum_{i=1}^N Y_i((\tau_1^1, \tau_2^1)) - \sum_{i=1}^N Y_i((\tau_1^1, \tau_2^0))}_{\text{one-sector problem (the effect of } \tau_2^0 \to \tau_2^1)} + \underbrace{\sum_{i=1}^N Y_i((\tau_1^1, \tau_2^0)) - \sum_{i=1}^N Y_i((\tau_1^0, \tau_2^0))}_{\text{one-sector problem (the effect of } \tau_1^0 \to \tau_1^1)} .
\end{aligned}$$

The first term indicates the causal effect of moving from a counterfactual policy regime (τ_1^1, τ_2^0) to another counterfactual policy regime (τ_1^1, τ_2^1). This is nothing but the causal effect of changing only τ_2 from τ_2^0 to τ_2^1 while keeping τ_1 fixed at τ_1^1, which is identified by the analysis of this paper. The second term represents the causal effect of moving from the current policy regime (τ_1^0, τ_2^0) to a counterfactual policy regime (τ_1^1, τ_2^0), which is also identified by the analysis of this paper. Again, this is the causal effect of changing only τ_1 from τ_1^0 to τ_1^1 with τ_2 fixed at τ_2^0. That is, a multiple-subsidy problem can be broken down to multiple one-subsidy problems, each of which is independently identified by the method of this paper.

This observation marks a remarkable distinction between the empirical treatment effects literature and my framework. In my framework, policy interventions that affect all units (i.e., universal treatments) can be well defined and identified, while the effects of such treatments are not generally identifiable in the treatment effect paradigm.

D.5 Optimal Policy Design

Definition. My model can be used to formulate an optimal policy design problem:

$$\tau_n^{1*} \in \arg\max_{\tau_n^1} \quad \Delta Y(\tau_n^0, \tau_n^1) \qquad s.t. \qquad \mathcal{C}(\boldsymbol{\tau}^0, \boldsymbol{\tau}^1) \geq \mathbf{0}, \tag{123}$$

where $\mathcal{C}(\boldsymbol{\tau}^0, \boldsymbol{\tau}^1) \geq \mathbf{0}$ represents a set (vector) of constraints faced by the policymaker. This embodies, for example, political economy considerations about equality and fairness among sectors and/or firms.

It should be noted that (123) is distinct from the canonical formulation of optimal-policy problems or normative analysis (e.g., Liu 2019; Gaubert et al. 2021; Lashkaripour and Lugovskyy 2023). The canonical formulation only gives the values of the policy variables that maximize outcome variables of interest; it does not necessarily yield the policy values that lead to maximum causal impacts on outcome variables. By contrast, τ_n^{1*} in (123) maximizes the causal policy effect $\Delta Y(\tau_n^0, \tau_n^1)$.

E Estimation Strategies

Given that the firm-level revenue functions and share regressions are nonparametrically identified (Appendix C), I employ polynomial regressions to nonparametrically estimate these functions. The degrees of polynomials are chosen adaptively on the basis of the root-mean squared errors (RMSE).

E.1 Firm-Level Quantities & Prices

To estimate $\tilde{\phi}_i(\cdot)$ in Step 1 of Lemma C.2, I consider a polynomial regression specification. For instance, the approximation by a second-order polynomial takes the form of

$$\tilde{r}_{ik} = b_{i,0} + b_{i,1}\tilde{\ell}_{ik} + b_{i,2}\tilde{m}_{ik} + b_{i,3}\tilde{\ell}_{ik}^2 + b_{i,4}\tilde{m}_{ik}^2 + b_{i,5}\tilde{\ell}_{ik}\tilde{m}_{ik} + \tilde{\eta}_{ik} = \tilde{x}_{ik}\boldsymbol{b}_i + \tilde{\eta}_{ik}, \tag{124}$$

where $\tilde{x}_{ik} \coloneqq [\tilde{\ell}_{ik}, \tilde{m}_{ik}, \tilde{\ell}_{ik}^2, \tilde{m}_{ik}^2, \tilde{\ell}_{ik}\tilde{m}_{ik}]^T$ and $\mathbf{b}_i \coloneqq [b_{i,0}, b_{i,1}, b_{i,2}, b_{i,3}, b_{i,4}, b_{i,5}]^T$, where T denotes the transpose of a vector. Stacking in matrix form, I obtain $\tilde{\mathbf{r}}_i = \tilde{\mathbf{x}}_i\mathbf{b}_i + \tilde{\boldsymbol{\eta}}_i$, where $\tilde{\mathbf{r}}_i \coloneqq [\tilde{r}_{i1}, \dots, \tilde{r}_{iN_i}]^T$. The estimator is defined as the minimizer of the mean squared error of (124) under the monotonicity of $\phi(\cdot)$ with respect to both labor and material inputs. Letting $\hat{\mathbf{b}}_i$ be the estimator, the fitted value of the log-revenue $\tilde{r}_{ik}$ is $\hat{\tilde{\phi}}_i(\tilde{x}_{ik}) \coloneqq \tilde{x}_{ik}\hat{\mathbf{b}}_i$. Moreover, given the estimator $\hat{\mathbf{b}}_i$, the specification (124) naturally gives rise to the estimator for the first-order partial derivatives of $\tilde{\phi}_i(\cdot)$ with respect to $\tilde{\ell}_{ik}$ and $\tilde{m}_{ik}$:

$$\widehat{\frac{\partial\tilde{\phi}_i}{\partial\tilde{\ell}_{ik}}}(\tilde{\ell}_{ik}, \tilde{m}_{ik}) \coloneqq \hat{b}_{i,1} + 2\hat{b}_{i,3}\tilde{\ell}_{ik} + \hat{b}_{i,5}\tilde{m}_{ik}$$
$$\widehat{\frac{\partial\tilde{\phi}_i}{\partial\tilde{m}_{ik}}}(\tilde{\ell}_{ik}, \tilde{m}_{ik}) \coloneqq \hat{b}_{i,2} + 2\hat{b}_{i,4}\tilde{m}_{ik} + \hat{b}_{i,5}\tilde{\ell}_{ik}.$$

E.2 Second-Order Derivatives of the Firm-Level Production Function

To construct a nonparametric estimator for the derivatives of firm-level production functions, I consider approximating (110) by polynomials and solve the following minimization problem as proposed in Gandhi et al. (2019): for instance, the case of second order polynomial approximation solves

$$\hat{\boldsymbol{\zeta}} \in \arg\min_{\boldsymbol{\zeta}^\circ} \sum_{k=1}^{N_i} \left\{ \tilde{s}_{ik}^{\ell,\tilde{\mu}} - \ln\left\{ \zeta_{i,0}^\circ + \zeta_{i,1}^\circ\tilde{\ell}_{ik} + \zeta_{i,2}^\circ\tilde{m}_{ik} + \zeta_{i,3}^\circ\tilde{\ell}_{ik}^2 + \zeta_{i,4}^\circ\tilde{m}_{ik}^2 + \zeta_{i,5}^\circ\tilde{\ell}_{ik}\tilde{m}_{ik} \right\} \right\}^2.$$

Note that this optimization subject to the implications by Euler's theorem for homogeneous functions. Specifically, I impose equality constraints for the first- and second-order partial derivatives.

E.3 Adaptive Choice of Degrees of Polynomials

In estimating these functions, I fit polynomial regressions of degree one and two.[139] For each of these, the root mean squared error (RMSE) is calculated. I then choose the one with the lowest MSE as the optimal polynomial degree. Throughout this adaptive choice, the sample size stays well above the number of the parameters of the polynomials (see Table 4).

[139] My analysis only needs the first-order derivative of the revenue function, and the first- and second-order derivatives of the production function, this setup is sufficient. Note that the function that is recovered by the share regression is already a derivative of the production function. Allowing for potentially higher degree of polynomials requires considerable computational cost, and may even deteriorate the prediction accuracy (even for interpolation).

F Monte Carlo Simulations

In this section, I examine the finite-sample properties of my nonparametric estimation approach described in Section 4 through Monte Carlo simulations. For the ease of exposition, I focus on estimating $\frac{dY_i(s)}{ds}\Big|_{s=\boldsymbol{\tau}}$ given in (16).

F.1 Simulation Design

I assume that there are only two sectors in the economy (i.e., $\mathbf{N} = \{1, 2\}$), each of which is populated by an identical set of firms with the number of firms being N_i for all $i \in \mathbf{N}$. I consider two scenarios for the current policy regimes (Scenarios A and B). In Scenario A, the values for the policies in place are all set equal to zero; that is, $\tau_i = 0$ for all $i \in \mathbf{N}$. Scenario B assumes that there are nonzero pre-existing policies. I set $\tau_i = 0.1$ for all $i \in \mathbf{N}$.

For each scenario, I consider four specifications, referred to as Specifications I, II, III and IV. In Specifications I and II firms are monopolistically competitive in the output market in each sector. By contrast, firms in Specifications III and IV are oligopolistic and engaged in a Cournot competition. While Specification I and III assume away from production networks, Specification II and IV admit a production network across sectors. For Specification I and III, the adjacency matrix is equivalent to an identity matrix; that is, $\Omega = I$. In Specification II and IV, I assume that sectors 1 and 2 are symmetric in terms of the input-output linkages with the adjacency matrix:

$$\Omega = \begin{bmatrix} 0.8 & 0.2 \\ 0.2 & 0.8 \end{bmatrix}.$$

Using a parametric model described below, I first generate simulation data for firm-level revenues, labor and material inputs, productivity, prices, quantity, and other aggregate variables (these are used as a *status quo* environment). Next, to obtain outcomes under an alternative policy regime, I repeat the same simulation with an increased value of the policy variable, and then calculate the change in GDP to measure the policy effects with respect to the policy change (the estimates based on this method are referred to as simulated policy effects). Then, I also compute the policy effects based on my estimation method (the estimates obtained by this approach are called estimated policy effects). To make the estimation problem as close to reality as possible, the estimated policy effects are calculated without directly using the realization of productivity, prices and quantity, as these are not observed in the real data either (see Section 3). In this experiment, I focus on the impacts of increasing only the subsidy to sector 1 (i.e., $n = 1$). For example, the simulated policy effects for Specification I are calculated by first generating outcome variables under $\boldsymbol{\tau}^0 = 0$, followed by the same simulation with the subsidy level changed to $\tau_1^1 = \tau_1^0 + d\tau_1$,[140] where I set $d\tau_1 = 0.001$. These results can be used to compute the total derivatives of the endogenous variables.[141]

[140] The subsidy to sector 2 is fixed constant, i.e., $\tau_2^1 = \tau_2^0$.

[141] Let x^0 and x^1 be endogenous variables obtained in the first and second simulations, respectively. Then, the total derivative of x is approximated as $\frac{dx}{d\tau_1} = \frac{x^1 - x^0}{d\tau_1}$.

The number of Monte Carlo simulations is set to $R = 500$. For each Monte Carlo sample, I generate $S = 99$ bootstrap samples. The performance of the proposed estimator is evaluated in terms of mean, bias, root mean square errors and empirical coverage probability.

F.1.1 Model

Following Grassi (2017), I posit that the sectoral aggregator takes the form of The parametric functional-form assumptions used in this section is akin to . This setup is also an extension of The sectoral aggregator is assumed to be a constant elasticity of substitution (CES) production function:

$$Q_i = \left(\sum_{k=1}^{N_i} \delta q_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}},$$

where σ is elasticity of substitution and δ_i stands for a demand shifter. The corresponding price index is given by $P_i = \left(\sum_{k=1}^{N_i} \delta^{\sigma} {p_{ik}}^{1-\sigma} \right)^{\frac{\sigma}{1-\sigma}}$.

In each sector i, individual firm k transforms labor ℓ_{ik} and material m_{ik} into output q_{ik} using a Cobb-Douglas production function:

$$q_{ik} = z_{ik} \ell_{ik}^{\alpha} m_{ik}^{1-\alpha},$$

where the output elasticity represents α and z_{ik} is productivity. Material input is composed of sectoral intermediate goods $\{m_{ik,j}\}_{j \in \mathbf{N}}$ according to the Cobb-Douglas production:

$$m_{ik} = \prod_{j=1}^{N} m_{ik,j}^{\gamma_{i,j}},$$

where $\gamma_{i,j}$ corresponds to the input share of sector j's intermediate good, reflecting the production network $\boldsymbol{\Omega}$.

To put the insight of Corollary C.3 into perspective, I consider monopolistic competition for a benchmark case along with oligopolistic competition.

Monopolistic competition. For each sector $i \in \mathbf{N}$, the optimal pricing for a monopolistic firm k is given by

$$p_{ik}^{*} = \frac{\sigma}{\sigma - 1} mc_{ik}^{*},$$

where $mc_{ik}^* = z_{ik}^{-1}\alpha^{-\alpha}(1-\alpha)^{1-\alpha}W^{*\alpha}P_i^{M*1-\alpha}$. The associated optimal input choices are

$$\ell_{ik}^* = z_{ik}^{-1}\left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha}\left(\frac{P_i^{M*}}{W^*}\right)^{1-\alpha}q_{ik}^*$$
$$m_{ik}^* = z_{ik}^{-1}\left(\frac{\alpha}{1-\alpha}\right)^{-\alpha}\left(\frac{P_i^{M*}}{W^*}\right)^{-\alpha}q_{ik}^*,$$

with the optimal quantity $q_{ik}^* = \left(\frac{p_{ik}^*}{P_i^*}\right)Q_i^*$. See Grassi (2017) for the detail.

Oligopolistic competition. When firms engage in Cournot competition in the output market, the Cournot-Nash equilibrium prices satisfy the following system of equations: for each sector $i \in \mathbf{N}$,

$$p_{ik}^* = \frac{\sigma}{(1-\sigma)(1-s_{ik}^*)}mc_{ik}^*$$
$$s_{ik}^* = \delta^\sigma\left(\frac{p_{ik}^*}{P_i^*}\right),$$

where s_{ik}^* is a firm's equilibrium market share. See Atkeson and Burstein (2008), Grassi (2017), Gaubert and Itskhoki (2020) for the detail. The input problem is identical to the monopolistic case.

F.1.2 Parameter Values

Parameter values are chosen in such a way that a Cournot-Nash equilibrium is well-defined. First, firms' heterogeneous productivities are drawn from a log normal distribution: $z_{ik} \sim log(\mathcal{N}(0, 0.1))$. I set $\alpha = 0.6$, $\sigma = 1.1$ (i.e., firms' products are substitutes), and $\delta_i = (1/N_i)^{1/\sigma_i} = 0.0285$ for all $i \in \{1, 2\}$.

The researcher has access to firm-level revenue, labor and material inputs, as well as aggregate variables; no access to firm-level productivities, prices and quantities. Consistent with my framework, the observed revenue is contaminated with a measurement error $\eta_{ik} \sim log(\mathcal{N}(0, 0.001))$.[142] Lastly, I fix the wage rage at $W^* = 1$ throughout the simulation study, meaning that I focus on a partial equilibrium exercise.[143]

To facilitate comparison, truncations of the polynomials are fixed throughout the simulations; I use degree the two polynomial specifications for both estimating the revenue functions and share regressions — as described in Appendices E.1 and E.2, respectively.

[142] The measurement error is assumed to enter in a linear, additive fashion in logs, i.e., $\log r_{ik} = \log \bar{r}_{ik} + \log \eta_{ik}$, where r_{ik} and $\bar{r}_{ik}$ are the observed and true (simulated) revenue, respectively. It is also assumed that $E[\log \eta_{ik} \mid \tilde{\ell}_{ik}, \tilde{m}_{ik}] = 0$. See Section C.2.2.

[143] For the first simulation that generates the status quo outcomes, I solve the aggregate equilibrium problem (with W exogenous fixed). Taking the aggregate variables and marginal costs as given, the second simulations, which computes the outcome under a counterfactual policy environment, only solves the sectoral equilibrium problem.

F.2 Asymptotic Theory

The goal of this subsection is to derive asymptotic theories relating to my nonparametric estimator. The theories in this subsection are mostly focused on sector-level outcomes accounting for dependence between random variables arising from firms' strategic interactions.[144]

Let $y_{N_i,k} := \frac{dy^{\circ}_{ik}}{d\tau_n}$, where $y^{\circ}_{ik} := p^*_{ik}q^*_{ik} - \sum_{j=1}^{N} P^*_j m^*_{ik,j}$. Notice that the $y_{N_i,k}$'s form a triangular array of dependent, identically distributed random variables, as emphasized in their double indices.[145] Here, $\frac{dY_i(s)}{ds}$ in (15) can be written as a sum of $\{y_{N_i,k}\}_{k=1}^{N_i}$:

$$\frac{dY_i}{d\tau_n} = \sum_{k=1}^{N_i} y_{N_i,k}.$$

Observe that $y_{N_i,k}$ can be viewed as a responsiveness of firm-level value-added by definition; hence $\sum_{k=1}^{N_i} y_{ik}$ can be thought of as the responsiveness of sector-level value-added. To study asymptotic properties, I also consider the average of firm-level value-added, i.e., $\frac{1}{N_i}\sum_{k=1}^{N_i} y_{ik}$.

The following assumption requires the finite existence of the second moments.

Assumption F.1. *For every $N_i > 0$ and every $k \in \mathbf{N}_i$, (i) $E[y_{N_i,k}]$ exists and is finite; and (ii) $Var(y_{N_i,k})$ and $Cov(y_{N_i,k'}, y_{N_i,k''})$ exist and are finite.*

Remark F.1. *Assumption F.1 (ii) implies the finite existence of $Var(\sum_{k=1}^{N_i} y_{N_i,k})$.*

F.2.1 Consistency

To obtain a consistency result, I impose the following assumption.

Assumption F.2. $\max_{\{k',k''\}\in\mathbf{N}_i} |Cov(y_{N_i,k'}, y_{N_i,k''})| \to 0$ *as $N_i \to \infty$.*

This assumption, in the context of this paper, states that as the number of firms increases, correlations between firms' responsiveness stemming from firms' strategic interactions vanish. This means that strategic forces become less relevant as there are more firms. In other words, this assumption excludes the presence of "superstar" firms that remain dominant for good.

The following theorem shows a law of large number for the sectoral average of firm-level responsivenesses of value-added.

Theorem F.1 (Consistency)**.** *Suppose that Assumption F.2 holds. Then,*

$$\frac{1}{N_i}\sum_{k=1}^{N_i} y_{N_i,k} \xrightarrow{p} \frac{1}{N_i}\sum_{k=1}^{N_i} E[y_{N_i,k}]$$

as $N_i \to \infty$.

[144] Investigating asymptotic properties that accommodate the other dependence — network spillovers between sectors — is at the frontier of recent econometrics and statistics literature, and thus goes well beyond the scope of this paper.

[145] The ultimate source of randomness of the $x_{N_i,k}$ is the random realization of firms' productivity, which follows an identical distribution. The dependence arises due to the firms' strategic interactions in each sector.

Proof. Denote

$$\bar{V}_{N_i} := \max_k Var(y_{N_i,k})$$
$$\bar{C}_{N_i} := \max_{\{k',k''\}\in \mathbf{N}_i} |Cov(y_{N_i,k'}, y_{N_i,k''})|.$$

By the Chebyshev's inequality, it holds that for every $\epsilon > 0$,

$$\begin{aligned} Pr\left(\left|\frac{1}{N_i}\sum_{k=1}^{N_i} y_{N_i,k} - \frac{1}{N_i}\sum_{k=1}^{N_i} E[y_{N_i,k}]\right| > \varepsilon\right) &\leq \frac{1}{\varepsilon^2} Var\left(\frac{1}{N_i}\sum_{k=1}^{N_i} y_{N_i,k}\right) \\ &= \frac{1}{\varepsilon^2}\frac{1}{N_i^2}\left(\sum_{k=1}^{N_i} Var(y_{N_i,k}) + 2\sum_{k'<k''} Cov(y_{N_i,k'}, y_{N_i,k''})\right) \\ &\leq \frac{1}{\varepsilon^2}\frac{1}{N_i^2}\left(\sum_{k=1}^{N_i} \bar{V}_{N_i} + 2\sum_{k'<k''} \bar{C}_{N_i}\right) \\ &= \frac{1}{\varepsilon^2}\left(\frac{1}{N_i}\bar{V}_{N_i} + \frac{1}{2}(1 - \frac{1}{N_i})\bar{C}_{N_i}\right) \\ &\to 0 \end{aligned}$$

as $N_i \to \infty$. This proves the statement. $\square$

F.2.2 Asymptotic Normality

Next, I explore the asymptotic normality of $\frac{1}{N_i}\sum_{k=1}^{N_i} y_{ik}$. To do so, I leverage the results developed by Dvoretzky (1970, 1972). This requires some notational overhead. To begin with, define

$$x_{N_i,k} := \frac{y_{N_i,k} - E[y_{N_i,k}]}{Var(\sum_{k=1}^{N_i} y_{N_i,k})^{\frac{1}{2}}}$$
$$S_{N_i} := \sum_{k=1}^{N_i} x_{N_i,k}.$$

I assume that the conditional mean and variance of $x_{N_i,k}$ are well-defined.

Assumption F.3. *For each $N_i > 0$ and each $k \in \mathbf{N}_i$, the conditional means $\mu_{N_i,k} := E[x_{N_i,k} \mid \mathcal{D}_{N_i,k-1}]$ and the conditional variances, $\sigma^2_{N_i,k} := Var(x_{N_i,k} \mid \mathcal{D}_{N_i,k-1})$, exist and are finite almost surely.*

Assumption F.3 means that the triangular array has finite conditional second moments. In my context, this means that responses of firm-level value added are "not too large" both in mean and variance, conditional on changes of the competitors' value added.

Remark F.2. *It is immediate to establish $\sigma^2_{N_i,k} = E[x^2_{N_i,k} \mid \mathcal{D}_{N_i,k-1}] - \mu^2_{N_i,k}$.*

To derive a central limit theorem, I follow Dvoretzky (1972) in further imposing the following conditions, each of which can be rationalized in the present context.

Assumption F.4. *As $N_i \to \infty$, (i) $\sum_{k=1}^{N_i} \mu_{N_i,k} \xrightarrow{p} 0$; (ii) $\sum_{k=1}^{N_i} \sigma^2_{N_i,k} \xrightarrow{p} 1$; and (iii) $\sum_{k=1}^{N_i} E\big[x^2_{N_i,k} \mathbb{1}_{\{|x_{N_i,k}|>\epsilon\}} \mid \mathcal{D}_{N_i,k-1}\big] \xrightarrow{p} 0$ for every $\epsilon > 0$.*

To assess the economic content of these restrictions, it is helpful to consider them in terms of the responsiveness of firm-level value added $y_{N_i,k}$. Assumption F.4 (i) is equivalent to

$$\sum_{k=1}^{N_i} \big(E[y_{N_i,k} \mid \mathcal{D}_{N_i,k-1}] - E[y_{N_i,k}]\big) \xrightarrow{p} 0 \qquad as \qquad N_i \to \infty.$$

Analogously, Assumption F.4 (ii) can be written as

$$\frac{\sum_{k=1}^{N_i} Var(y_{N_i,k} \mid \mathcal{D}_{N_i,k-1})}{Var(\sum_{k=1}^{N_i} y_{N_i,k})} \xrightarrow{p} 1 \qquad as \qquad N_i \to \infty.$$

To grasp an intuition behind this expression, it proves useful to consider a sufficient condition: it is satisfied, for example, when $(ii\text{-}a)$ $\max_{\{k',k''\}\in \mathbf{N}_i} |Cov(y_{N_i,k'}, y_{N_i,k''})| \xrightarrow{p} 0$ and $(ii\text{-}b)$ $\sup_{k\in\mathbf{N}_i} |Var(y_{N_i,k}) - Var(y_{N_i,k} \mid \mathcal{D}_{N_i,k-1})| \xrightarrow{p} 0$ as $N_i \to \infty$.[146] Condition $(ii\text{-}a)$ is maintained in Assumption F.2, while part $(ii\text{-}b)$ means that the competitors' actions become unrelated to the variability of $y_{N_i,k}$. Loosely speaking, these conditions jointly require that the market competition, which is supposed to be strategic, eventually turns to monopolistic. Assumption (iii) is a generalization of the canonical Lindberg's condition (see Dvoretzky (1972)). In the context of strategic competition, it requires that the number of firms whose $y_{N_i,k}$ deviates, conditional on the competitors actions, from its expectation by a certain amount ε eventually goes to zero, whatever the value of ε is.

Under these conditions, Dvoretzky (1972) shows a central limit theorem for a sum of dependent random variables.

Theorem F.2 (Theorem 2.2 of Dvoretzky (1972))**.** *Suppose that Assumptions F.3 and F.4 are satisfied. Then,*

$$S_{N_i} \xrightarrow{d} \mathcal{N}(0,1) \qquad as \qquad N_i \to \infty.$$

This theorem gives a CLT result for sector-level value-added. In fact, it can be read as

$$\frac{\sum_{k=1}^{N_i} y_{N_i,k} - \sum_{k=1}^{N_i} E[y_{N_i,k}]}{Var(\sum_{k=1}^{N_i} y_{N_i,k})^{\frac{1}{2}}} \xrightarrow{d} \mathcal{N}(0,1) \qquad as \qquad N_i \to \infty.$$

Moreover, this result can also be interpreted as stating a CLT for the sectoral average of firm-level value-added, i.e.,

$$\frac{\frac{1}{N_i}\sum_{k=1}^{N_i} y_{N_i,k} - \frac{1}{N_i}\sum_{k=1}^{N_i} E[y_{N_i,k}]}{Var(\frac{1}{N_i}\sum_{k=1}^{N_i} y_{N_i,k})^{\frac{1}{2}}} \xrightarrow{d} \mathcal{N}(0,1) \qquad as \qquad N_i \to \infty.$$

[146] These conditions could be relaxed, respectively, to $(ii\text{-}a)$' $\sum_{k'<k''} Cov(y_{N_i,k'}, y_{N_i,k''}) \xrightarrow{p} 0$ and $(ii\text{-}b)$' $\sum_{k\in\mathbf{N}_i} Var(y_{N_i,k}) - \sum_{k\in\mathbf{N}_i} Var(y_{N_i,k} \mid \mathcal{D}_{N_i,k-1}) \xrightarrow{p} 0$ as $N_i \to \infty$.

These results allow the researcher to calculate the standard errors of the estimates and confidence intervals for the policy parameters, preparing a ground for statistical hypothesis testing.[147]

F.3 Results

F.3.1 Scenario A

Table 5 compares the simulation results for sectoral average of firm-level value added for different sample sizes, i.e., $N_i = 50, 100, 150$.

[147] Consistently estimating the standard errors accounting for both strategic interactions and network dependence is of great interest in its own right, and goes beyond the scope of this paper.

Table 5: Results: Simulated and Estimated Policy Effects

N_i	Specifications	Sectors	True	Estimates			95% coverage
				Mean	Bias	RMSE	
50	Specification I	Sector 1	5.3709	5.3202	-0.0506	0.1363	0.9680
		Sector 2	—	—	—	—	—
	Specification II	Sector 1	5.5960	5.5271	-0.0689	0.1464	0.9740
		Sector 2	1.9016	1.8940	-0.0076	0.0446	0.9940
	Specification III	Sector 1	-6.1124	-6.2453	-0.1329	0.2015	0.5480
		Sector 2	—	—	—	—	—
	Specification IV	Sector 1	-8.5308	-8.7088	-0.1780	0.2692	0.5680
		Sector 2	-0.0006	-0.0230	-0.0224	0.0225	0.0000
100	Specification I	Sector 1	5.3682	5.3302	-0.0380	0.1379	0.9760
		Sector 2	—	—	—	—	—
	Specification II	Sector 1	5.5932	5.5164	-0.0768	0.1231	0.9400
		Sector 2	1.9006	1.8907	-0.0100	0.0344	0.9920
	Specification III	Sector 1	-6.0681	-6.1501	-0.0819	0.1348	0.5720
		Sector 2	—	—	—	—	—
	Specification IV	Sector 1	-8.4689	-8.5921	-0.1231	0.1934	0.5840
		Sector 2	-0.0006	-0.0161	-0.0155	0.0155	0.0000
150	Specification I	Sector 1	5.3655	5.3204	-0.0451	0.0888	0.9680
		Sector 2	—	—	—	—	—
	Specification II	Sector 1	5.5904	5.5134	-0.0770	0.1104	0.9240
		Sector 2	1.8997	1.8897	-0.0100	0.0289	0.9900
	Specification III	Sector 1	-6.0515	-6.1065	-0.0550	0.3030	0.5500
		Sector 2	—	—	—	—	—
	Specification IV	Sector 1	-8.4458	-8.5511	-0.1054	0.1608	0.5500
		Sector 2	-0.0006	-0.0139	-0.0133	0.0134	0.0000

Note: This table evaluates the performance of the proposed estimator in terms of the mean, bias, root mean square error and empirical coverage probability for 95% nominal level. The true value is computed as the average of the simulated policy effects over Monte Carlo simulations. For each sample size (N_i), the table compares the results across different specifications.

G Empirical Illustration

G.1 Details of CHIPS and Science Act of 2022

CHIPS stands for Creating Helpful Incentives to Produce Semiconductors (White House 2022). This act was passed into law in 2022 and aims to invest nearly \$53 billion in the U.S. semiconductor manufacturing, research and development, and workforce (White House 2023). This policy also includes a 25% tax credit for manufacturing investment, which is projected to provide up to \$24.25 billion for the next 10 years (Congressional Budget Office 2022).

G.2 Main Results

G.2.1 Robustness

To explore robustness of my estimation procedure, I run the same algorithm for different choices of the number of bins ($\bar{v}$ in (24a)). Given that results in the main text are based on the choice $\bar{v} = 20$, this subsection examines the variability of the estimates with respect to increasing and decreasing the number of bins. Specifically, I consider $\bar{v} = 10$ for the former and $\bar{v} = 30$ for the latter. Table 6 shows the estimates of the policy effect $\widehat{\Delta Y}(\tau_n^0, \tau_n^1)$ for both situations. Clearly, the estimates do not vary significantly relative to my main result (Table 1). The robustness is further illuminated by comparing Figures 2 and 4, which depicts the trajectories of the responsiveness of GDP. Overall, the estimates remain stable both qualitatively and quantitatively across the different choices of the number of bins.

Table 6: The estimates of the object of interest

(i) $\bar{v} = 10$

(billion U.S. dollars)	Monopolistic competition	Oligopolistic competition
Estimates based on (24a)	14.51	-2.05
Estimates based on (24b)	38.39	-1.98

(ii) $\bar{v} = 30$

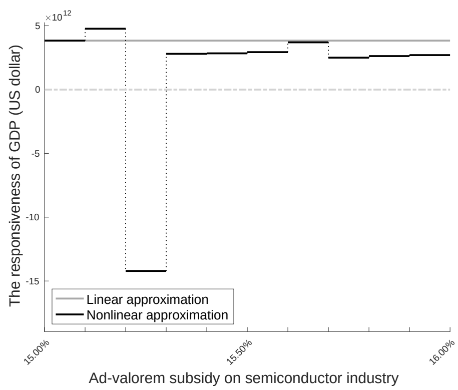
(billion U.S. dollars)	Monopolistic competition	Oligopolistic competition
Estimates based on (24a)	23.51	-1.80
Estimates based on (24b)	38.39	-1.98

Note: This table compares the estimates for the object of interest (14) based on the benchmark and my method. The estimates are measured in billions of U.S. dollars.

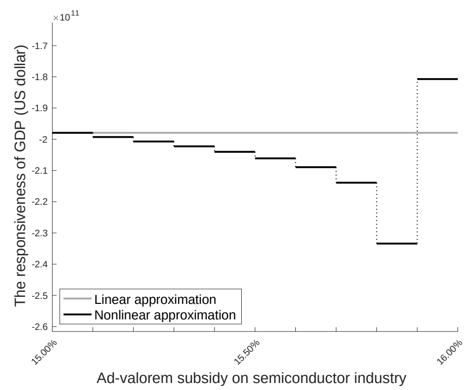
G.3 Mechanism

Figure 4: The total derivative of Y with respect to τ_n

(i) $\bar{v} = 10$

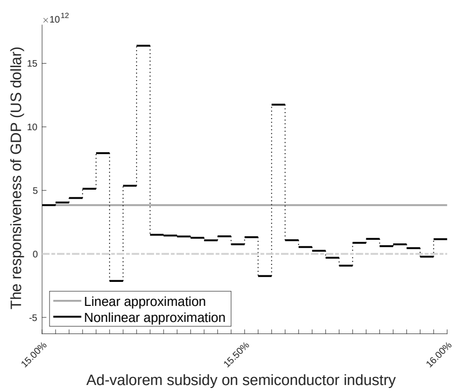


(a) Monopolistic Competition

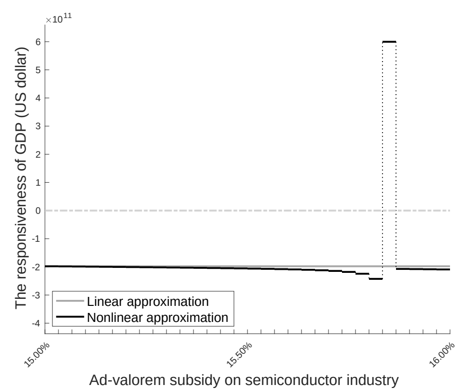


(b) Oligopolistic Competition

(ii) $\bar{v} = 30$



(a) Monopolistic Competition



(b) Oligopolistic Competition

Note: This figure illustrates the estimates of the total derivative of (economy-wide) GDP with respect to the semiconductor subsidy between $\tau_n = 15.03\%$ and 16.03%. Panel (a) shows the result for the case of monopolistic competition and panel (b) for the case of oligopolistic competition. The solid black line represents the estimates based on the nonlinear approximation (24a). The solid medium grey line indicates the estimates based on the linear approximation (24b). The dash-dotted light grey line stands for zero. Hence, the part surrounded by the light grey line and back line above it measures the total increment of GDP over the course of the policy reform, while the other part gives the total decrement of GDP. The difference between these two areas delivers the estimated value of the policy effect according to (24a). Similarly, the area surrounded by the light grey line and medium grey line gives the estimated value of the policy effect according to (24b).

G.3.1 Responsiveness of Sectoral GDP

Tables 7 and 8 report the detailed results of the empirical illustration for monopolistic and oligopolistic competition, respectively. These tables break down the responsiveness of sectoral GDP into four components, as explained in Section 5.2, and display the estimates in descending order of the total effects.

Table 7: Responsiveness of Sectoral GDP: Monopolistic Competition (in Billions of U.S. Dollars)

Industry	Total Effect	Effects on Revenue		Effects on Material Cost	
		p.effect	q.effect	w.effect	s.effect
Wholesale trade	960.32	-265.02	1557.61	147.31	-479.57
Retail trade	755.47	-200.98	1492.76	203.19	-739.50
Construction	483.18	-83.89	970.66	76.00	-479.60
Motor vehicles and other transportation equipment	440.74	-98.58	653.08	81.01	-194.78
Fabricated metal products	204.44	-78.01	333.48	19.82	-70.85
Computer and electronic products	171.43	-147.55	553.50	46.54	-281.05
Air and ground transportation	161.18	-60.98	365.95	55.21	-199.00
Machinery	109.60	-41.77	180.49	28.39	-57.50
Electrical equipment, appliances, and components	91.45	-29.03	171.10	22.64	-73.25
Paper products and printing-related services	82.42	-18.37	116.77	13.72	-29.70
Miscellaneous manufacturing	67.86	-13.61	94.28	7.36	-20.17
Administrative and waste services	57.10	-40.46	129.50	25.95	-57.89
Wood and nonmetallic mineral products	41.76	-16.73	66.10	11.67	-19.28
Plastics and rubber products	39.79	-12.96	63.48	9.59	-20.31
Publishing industries and information services	33.30	-29.47	76.17	14.88	-28.28
Educational services	31.68	-21.31	83.77	14.36	-45.14
Chemical products	28.95	-22.56	65.89	11.01	-25.39
Mass media and telecommunications	25.93	-53.32	121.72	27.82	-70.29
Arts, Entertainment, and Recreation	24.97	-20.03	59.32	13.61	-27.93
Support activities for mining	22.20	-4.60	32.04	3.47	-8.72
Furniture and related products	21.25	-4.51	32.83	3.98	-11.06
Accommodation	17.75	-28.35	161.85	32.64	-148.39
Food and beverage and tobacco products	14.49	-40.17	135.89	28.96	-110.19
Primary metals	10.75	-4.44	18.81	2.90	-6.53
Petroleum and coal products	10.00	-6.78	26.95	1.98	-12.15
Other transportation and support activities	3.69	-43.13	106.17	28.82	-88.16
Textile-related mills and apparel products	2.22	-1.61	5.41	1.15	-2.73
Legal, scientific, and technical services	-8.00	-259.48	-95.74	209.49	137.73
Water transportation	-16.07	-0.96	5.70	0.30	-21.12
Oil and gas extraction, and mining	-51.18	-13.40	28.83	7.08	-73.70
Total	3838.66				

Note: This table reports the estimates of the total effects (i.e., the marginal change in sectoral GDP in the order of a billion dollars) for the case of monopolistic competition. The industries are arranged in descending order in terms of the total effects, which are in turn broken down into the effects on revenue and material input costs. They are further decomposed into four effects according to (25), namely, *p.effect* stands for the price effects, *q.effect* the quantity effects, *w.effect* the wealth effects, and *s.effect* the switching effects. Note that the first column in each panel indicates names of industries based on the segmentation given in Table B.2.

Table 8: Responsiveness of Sectoral GDP: Oligopolistic Competition (in Billions of U.S. Dollars)

Industry	Total Effect	Effects on Revenue		Effects on Material Cost	
		p.effect	q.effect	w.effect	s.effect
Petroleum and coal products	0.10	1.32	-1.32	-0.02	0.11
Support activities for mining	0.00	0.03	-0.03	0.04	-0.04
Paper products and printing-related services	-0.01	-0.12	0.12	0.29	-0.31
Wood and nonmetallic mineral products	-0.02	0.05	-0.05	0.25	-0.27
Plastics and rubber products	-0.02	-0.08	0.08	0.22	-0.24
Primary metals	-0.03	0.22	-0.22	0.05	-0.09
Textile-related mills and apparel products	-0.04	-0.02	0.02	0.04	-0.07
Fabricated metal products	-0.04	-0.27	0.27	0.42	-0.46
Furniture and related products	-0.04	-0.06	0.06	0.10	-0.14
Water transportation	-0.06	0.02	-0.02	0.00	-0.06
Oil and gas extraction, and mining	-0.09	0.50	-0.50	0.01	-0.10
Arts, Entertainment, and Recreation	-0.17	-0.03	0.03	0.17	-0.34
Other transportation and support activities	-0.19	0.90	-0.90	0.17	-0.36
Miscellaneous manufacturing	-0.22	-0.31	0.31	0.26	-0.48
Machinery	-0.25	-0.66	0.66	0.78	-1.02
Chemical products	-0.29	0.41	-0.41	0.30	-0.59
Air and ground transportation	-0.38	2.43	-2.43	0.06	-0.44
Food and beverage and tobacco products	-0.46	0.55	-0.55	0.21	-0.67
Educational services	-0.50	-0.33	0.33	0.39	-0.89
Administrative and waste services	-0.61	-0.22	0.22	0.72	-1.33
Electrical equipment, appliances, and components	-0.63	-0.92	0.92	0.82	-1.45
Publishing industries and information services	-0.68	-0.75	0.75	1.02	-1.70
Accommodation	-0.75	-0.22	0.22	0.47	-1.22
Motor vehicles and other transportation equipment	-1.13	-1.74	1.74	2.36	-3.49
Construction	-2.53	2.49	-2.49	0.70	-3.23
Mass media and telecommunications	-2.84	-2.50	2.50	2.14	-4.98
Retail trade	-6.60	0.21	-0.21	5.27	-11.87
Wholesale trade	-12.63	7.81	-7.81	7.26	-19.89
Legal, scientific, and technical services	-25.82	-8.43	8.43	12.24	-38.06
Computer and electronic products	-141.01	-97.61	97.61	6.61	-147.61
Total	-197.93				

Note: This table reports the estimates of the total effects (i.e., the marginal change in sectoral GDP in the order of a billion dollars) for the case of oligopolistic competition. The industries are arranged in descending order in terms of the total effects, which are in turn broken down into the effects on revenue and material input costs. They are further decomposed into four effects according to (25), namely, *p.effect* stands for the price effects, *q.effect* the quantity effects, *w.effect* the wealth effects, and *s.effect* the switching effects. Note that the first column in each panel indicates names of industries based on the segmentation given in Table B.2.

Table 9: Comovement of Sectoral Price Indices

Industry	Monopolistic	Oligopolistic
Oil and gas extraction, and mining	0.14	0.00
Support activities for mining	0.35	0.01
Construction	0.34	0.00
Food and beverage and tobacco products	0.14	0.00
Textile-related mills and apparel products	0.16	0.01
Wood and nonmetallic mineral products	0.16	0.01
Paper products and printing-related services	0.35	0.01
Petroleum and coal products	0.02	-0.00
Chemical products	0.07	0.00
Plastics and rubber products	0.17	0.01
Primary metals	0.06	0.00
Fabricated metal products	0.20	0.01
Machinery	0.44	0.02
Computer and electronic products	1.00	1.00
Electrical equipment, appliances, and components	0.38	0.02
Motor vehicles and other transportation equipment	0.32	0.01
Furniture and related products	0.31	0.01
Miscellaneous manufacturing	0.54	0.03
Wholesale trade	0.13	0.01
Retail trade	0.13	0.01
Air and ground transportation	0.10	0.00
Water transportation	0.11	0.00
Other transportation and support activities	0.16	0.00
Publishing industries and information services	0.15	0.02
Mass media and telecommunications	0.14	0.02
Legal, scientific, and technical services	0.17	0.02
Administrative and waste services	0.17	0.01
Educational services	0.24	0.01
Arts, Entertainment, and Recreation	0.12	0.00
Accommodation	0.16	0.00

Note: .

G.3.2 Macro and Micro Complementarities

Tables 10 and 11 exhibit the full results for the changes in sectoral price indices and material cost indices, accompanied by the estimates for macro and micro complementarities. In these tables, the industries are arranged in the order consistent with Tables 7 and 8. Table 10 summarizes the results for monopolistic competition, while Table 11 shows those for oligopolistic competition.

Table 10: Changes in Sectoral Output Price and Material Cost Indices: Monopolistic Competition

Industry	h_i^L	$h_{i,n}^M$	$\frac{dP_i^{M*}}{d\tau_n}$	$\bar{\lambda}_{i\cdot}^L$	$\bar{\lambda}_{i\cdot}^M$	$\frac{dP_i{}^*}{d\tau_n}$
Wholesale trade	437.68	0.18	-236.48	20.94	0.05	-16.87
Retail trade	501.99	0.17	-241.91	29.29	0.19	-53.20
Construction	1437.26	0.46	-675.59	22.26	0.08	-58.50
Motor vehicles and other transportation equipment	956.02	0.40	-518.27	21.93	0.15	-81.26
Fabricated metal products	751.02	0.25	-357.32	37.61	0.39	-148.23
Computer and electronic products	475.83	1.60	-1371.65	17.45	0.10	-142.47
Air and ground transportation	386.85	0.10	-164.40	10.23	0.13	-23.62
Machinery	1468.05	0.57	-768.94	15.65	0.09	-70.59
Electrical equipment, appliances, and components	1166.78	0.49	-638.72	15.77	0.17	-110.63
Paper products and printing-related services	1205.45	0.42	-591.28	21.72	0.11	-70.51
Miscellaneous manufacturing	1711.50	0.65	-884.51	57.31	0.10	-98.11
Administrative and waste services	615.22	0.22	-302.67	10.18	0.07	-22.48
Wood and nonmetallic mineral products	704.80	0.24	-343.69	14.73	0.09	-32.51
Plastics and rubber products	602.00	0.20	-289.37	14.97	0.15	-45.40
Publishing industries and information services	360.37	0.17	-214.59	9.18	0.04	-11.27
Educational services	869.22	0.31	-434.76	12.69	0.11	-48.41
Chemical products	237.64	0.08	-113.00	12.08	0.09	-12.62
Mass media and telecommunications	296.31	0.17	-198.63	7.77	0.06	-12.74
Arts, Entertainment, and Recreation	495.22	0.13	-212.83	10.61	0.09	-20.95
Support activities for mining	1112.14	0.36	-526.51	22.57	0.11	-62.18
Furniture and related products	1135.45	0.40	-557.30	20.08	0.12	-73.12
Accommodation	600.89	0.18	-274.23	8.80	0.25	-70.26
Food and beverage and tobacco products	529.84	0.15	-230.44	15.27	0.09	-24.94
Primary metals	230.26	0.06	-100.02	18.46	0.09	-12.63
Petroleum and coal products	54.51	0.01	-20.91	9.24	0.04	-2.84
Other transportation and support activities	657.19	0.19	-294.85	6.32	0.07	-22.10
Textile-related mills and apparel products	524.64	0.19	-265.10	10.67	0.08	-22.45
Legal, scientific, and technical services	506.39	0.22	-285.36	6.45	-0.02	4.31
Water transportation	441.43	0.13	-196.29	32.99	0.23	-51.99
Oil and gas extraction, and mining	464.44	0.13	-203.02	6.87	0.03	-8.13

Note: This table displays the estimates for the macro and micro complementarities for those industries listed in Table 7. The subscript n on the variables denotes the targeted industry, i.e., the computer and electronic product industry.

Table 11: The Changes in Sectoral Price Indices and Material Cost Indices: Oligopolistic Competition

Industry	h_i^L	$h_{i,n}^M$	$\frac{dP_i^{M*}}{d\tau_n}$	$\bar{\lambda}_{i\cdot}^L$	$\bar{\lambda}_{i\cdot}^M$	$\frac{dP_i^*}{d\tau_n}$
Petroleum and coal products	7.72	0.00	0.18	2.81	0.02	0.14
Support activities for mining	56.94	0.01	-5.93	3.08	0.02	0.06
Paper products and printing-related services	50.70	0.02	-12.64	3.31	0.02	-0.07
Wood and nonmetallic mineral products	43.56	0.01	-7.47	4.44	0.03	0.02
Plastics and rubber products	37.06	0.01	-6.53	3.69	0.04	-0.06
Primary metals	18.30	0.00	-1.84	4.01	0.03	0.15
Textile-related mills and apparel products	39.10	0.01	-9.12	3.26	0.03	-0.09
Fabricated metal products	29.92	0.01	-7.65	3.31	0.04	-0.12
Furniture and related products	54.00	0.02	-13.71	3.50	0.02	-0.13
Water transportation	35.47	0.00	-0.54	3.84	0.02	0.18
Oil and gas extraction, and mining	28.66	0.00	-0.30	2.96	0.01	0.14
Arts, Entertainment, and Recreation	46.47	0.01	-2.65	3.36	0.07	-0.01
Other transportation and support activities	63.83	0.01	-1.77	6.31	0.07	0.19
Miscellaneous manufacturing	68.82	0.04	-31.15	2.98	0.01	-0.32
Machinery	44.48	0.03	-21.00	3.25	0.02	-0.26
Chemical products	18.70	0.01	-3.06	3.42	0.03	0.08
Air and ground transportation	49.17	0.00	-0.17	3.30	0.02	0.16
Food and beverage and tobacco products	25.25	0.00	-1.66	3.15	0.03	0.10
Educational services	58.28	0.02	-11.85	3.30	0.03	-0.19
Administrative and waste services	48.57	0.01	-8.42	3.38	0.02	-0.04
Electrical equipment, appliances, and components	38.47	0.03	-23.26	3.09	0.03	-0.59
Publishing industries and information services	36.87	0.02	-14.75	3.00	0.02	-0.11
Accommodation	38.18	0.01	-3.94	2.49	0.05	-0.10
Motor vehicles and other transportation equipment	27.33	0.02	-15.12	3.24	0.02	-0.22
Construction	52.92	0.01	-6.18	4.27	0.01	0.15
Mass media and telecommunications	26.80	0.02	-15.27	2.90	0.03	-0.26
Retail trade	47.66	0.01	-6.28	3.37	0.02	0.01
Wholesale trade	42.39	0.02	-11.65	3.74	0.01	0.08
Legal, scientific, and technical services	47.41	0.02	-16.67	2.60	0.03	-0.38
Computer and electronic products	24.94	1.11	-874.19	2.54	0.03	-25.12

Note: This table displays the estimates for the macro and micro complementarities for those industries listed in Table 8. The subscript n on the variables denotes the targeted industry, i.e., the computer and electronic product industry.

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